

Preface

The articles in this volume are an outgrowth of seminars and schools of Impanga in the period 2005–2007. Impanga is an algebraic geometry group operating since 2000 at the Institute of Mathematics of Polish Academy of Sciences in Warsaw. The present volume covers, besides seminars, the following schools organized by Impanga at the Banach Center in Warsaw:

- *Moduli spaces*, April 2005,
- *Algebraic cycles and motives*, October 2005,
- *A tribute to Hoene-Wroński*, January 2007.

More information about Impanga, including complete lists of seminars, schools, and sessions, can be found at the web-page:

<http://www.impan.gov.pl/~pragacz/impanga.htm> .

Let us describe briefly the contents of the lecture notes in this volume.¹

Jean-Marc Drézet, in his first article, discusses fine moduli spaces of coherent sheaves, i.e., those endowed, at least locally, with universal sheaves. Whereas the most known fine moduli spaces appear in the theory of (semi)stable sheaves, the author constructs other, the so called “exotic” fine moduli spaces; the corresponding sheaves are sometimes not simple.

The subject of the second article of Jean-Marc Drézet is the study of moduli spaces of coherent sheaves on multiple curves embedded in a smooth projective surface. The author introduces new invariants for such curves: canonical filtrations, generalized rank and degree, and proves a Riemann-Roch theorem. A more detailed study of coherent sheaves on double curves is presented.

Tomas L. Gomez gives an outline of constructions of different moduli spaces. His starting point is the Jacobian of a smooth projective curve, and the final aims are moduli spaces of principal sheaves. A pretty complete account of the theory of principal bundles and sheaves is presented; a special emphasis is put on their stability properties. Orthogonal and symplectic sheaves serve as instructing examples.

¹The lecture notes by J.-M. Drézet, T.L. Gomez, A.H.W. Schmitt, and Ngo Dac Tuan stem from the first school, the article by V. Srinivas from the second school, the opening article of P. Pragacz from the third school, and finally the articles by A. Langer, P. Pragacz, and that by P. Pragacz-A. Weber from the seminars of Impanga.

Adrian Langer gives a comprehensive introduction to torsion free sheaves and the moduli spaces of (semi)stable sheaves in any dimension and arbitrary characteristic. The author discusses carefully the (semi)stability conditions and restriction theorems. One of the main goals is to give the boundedness results, which are crucial to construct moduli spaces using the techniques of the Quot-schemes. Line bundles on the moduli spaces are also described, and generic smoothness of the moduli spaces of sheaves on surfaces is showed.

Piotr Pragacz discusses some topological, algebraic, and geometric properties of the zero schemes of sections of vector bundles, namely the connectedness and the “point” and “diagonal” properties. An overview of recent results by Vasudevan Srinivas, Vishwambhar Pati, and the author on these properties is presented.

Piotr Pragacz and Andrzej Weber generalize Thom polynomials from singularities of maps to invariant cones in representations of products of linear groups. With the help of the Fulton-Lazarsfeld theory of positivity of ample vector bundles, they show that the coefficients of Thom polynomials expanded in the basis of the products of the Schur functions, are nonnegative.

Alexander H.W. Schmitt gives an account of classical and new results in Geometric Invariant Theory (especially the theory relative to a base curve), and present a recent progress in the construction of moduli spaces of vector bundles and principal bundles with extra structure (called augmented or “decorated” vector or principal bundles). The problems of taking various quotients and stability conditions are widely discussed and illustrated by numerous examples.

Vasudevan Srinivas shows some applications of the intersection theory of algebraic cycles to commutative algebra. A special emphasis is put on the study of the groups of zero-dimensional cycles, modulo rational equivalence, on smooth projective or affine varieties (in particular, surfaces). Their applications to embedding and immersion of affine varieties, indecomposable projective modules, and the complete intersection property are given.

Ngo Dac Tuan presents a “friendly” introduction to shtukas, the stacks of shtukas, and their compactifications. The notion of a “shtuka” was first introduced by Drinfeld and used in his proof of the Langlands correspondence for GL_2 over function fields. It recently has been used by Lafforgue in his proof of the Langlands correspondence for higher groups GL_r over function fields.

We dedicate the whole volume to the memory of **Józef Maria Hoene-Wroński** – one of the most original figures in the history of science. The opening article by Piotr Pragacz discusses some aspects of his life and work.

Acknowledgments. The Editor thanks the authors for their scientific contributions, to Adrian Langer and Halszka Gasińska-Tutaj for their help with the school on moduli spaces, and finally to Dr. Thomas Hempfling from Birkhäuser-Verlag for a pleasant editorial cooperation.

Warszawa, July 2007

The Editor



<http://www.springer.com/978-3-7643-8536-1>

Algebraic Cycles, Sheaves, Shtukas, and Moduli

Impanga Lecture Notes

Pragacz, P. (Ed.)

2008, VIII, 236 p., Hardcover

ISBN: 978-3-7643-8536-1

A product of Birkhäuser Basel