Preface

In 1953, Grothendieck [G] characterized locally convex Hausdorff spaces which have the Dunford-Pettis property and used this property to characterize weakly compact operators $u : C(K) \to F$, where K is a compact Hausdorff space and F is a locally convex Hausdorff space (briefly, lcHs) which is complete. Among other results, he also showed that there is a bijective correspondence between the family of all F-valued weakly compact operators u on $C(K)$ and that of all F-valued σ-additive Baire measures on K. But he did not develop any theory of integration to represent these operators.

Later, in 1955, Bartle, Dunford, and Schwartz [BDS] developed a theory of integration for scalar functions with respect to a σ-additive Banach-space-valued vector measure $\mathbf{m}$ defined on a σ-algebra of sets and used it to give an integral representation for weakly compact operators $u : C(S) \to X$, where S is a compact Hausdorff space and X is a Banach space. A modified form of this theory is given in Section 10 of Chapter IV of [DS1]. In honor of these authors, we call the integral introduced by them as well as its variants given in Section 2.2 of Chapter 2 and in Section 4.2 of Chapter 4, the Bartle-Dunford-Schwartz integral or briefly, the BDS-integral.

About fifteen years later, in [L1,L2] Lewis studied a Pettis type weak integral of scalar functions with respect to a σ-additive vector measure $\mathbf{m}$ having range in an lcHs X. This type of definition has also been considered by Kluvánek in [K2]. In honor of these mathematicians we call the integral introduced in [L1,L2] as well as its variants given in Section 2.1 of Chapter 2 and in Section 4.1 of Chapter 4, the Kluvánek-Lewis integral or briefly, the (KL)-integral. When the domain of the σ-additive vector measure $\mathbf{m}$ is a σ-algebra Σ and X is a Banach space, Theorem 2.4 of [L1] asserts that the (KL)-integral is the same as the (BDS)-integral. Though this result is true, its proof in [L1] lacks essential details. See Remark 2.2.6 of Chapter 2.

Let T be a locally compact Hausdorff space and $\mathcal{K}(T)$ (resp. $\mathcal{K}(T, \mathbb{R})$) be the vector space of all complex- (resp. real-) valued continuous functions on T with compact support, endowed with the inductive limit locally convex topology as in §1, Chapter III of [B]. In 1970, using the results of [G], Thomas [T] developed a theory of vectorial Radon integration with respect to a weakly compact bounded (resp. a prolongable) Radon operator u on $\mathcal{K}(T, \mathbb{R})$ with values in a real Banach space and more generally, with values in a real quasicomplete lcHs. (Thomas [T] calls them bounded weakly compact (resp. prolongable) Radon vector measures.) However, by making some modifications, it can be shown that his theory is equally valid for such operators u defined on $\mathcal{K}(T)$ with range contained in a complex Banach space or in a quasicomplete complex lcHs. See Section 7.1 of Chapter 7. Functions f integrable with respect to u are called u-integrable and the integral of f with respect to u is denoted by $\int_T f du$ in [T].

If S is a compact Hausdorff space, X a Banach space and $u : C(S) \to X$ is a weakly compact operator, then by the Bartle-Dunford-Schwartz representation theorem in [BDS] and in [DU] there exists a unique X-valued Borel regular σ-additive vector measure $\mathbf{m}_u$ on the σ-algebra $\mathcal{B}(S)$ of the Borel sets in S such that $uf = \int_S f d\mathbf{m}_u$ for $f \in C(S)$. If X is a real Banach space and if $u : C^r(S) \to X$ (where $C^r(S) = \{f \in C(S) : f \text{ real-valued}\}$) is a weakly compact operator, Thomas showed in [T] that a real function f on S is u-integrable if and only if it is $\mathbf{m}_u$-integrable in the sense of Section 10 of Chapter IV of [DS1] and in that case, $\int_S f du =$ (BDS) $\int_S f d\mathbf{m}_u$. Moreover, such f is $\mathbf{m}_u$-measurable in the sense of Section 10 of Chapter IV of [DS1] though it is not necessarily $\mathcal{B}(S)$-measurable.

On the other hand, in [P3, P4] we have shown that there is a bijective correspondence Γ between the dual space $\mathcal{K}(T)^*$ and the family of all $\delta(\mathcal{C})$-regular complex measures on $\delta(\mathcal{C})$ such that $\Gamma\theta = \mu_\theta|_{\delta(\mathcal{C})}$ for $\theta \in \mathcal{K}(T)^*$, where μ_θ is the complex Radon measure determined by θ (in the sense of [P4]) and $\delta(\mathcal{C})$ is the δ-ring generated by the family $\mathcal{C}$ of all compact sets in T. As observed in [P13], the scalar-valued prolongable Radon operators on $\mathcal{K}(T)$ are precisely the continuous linear functionals on $\mathcal{K}(T)$ (i.e., the elements of $\mathcal{K}(T)^*$).

From the above results of Thomas [T] and of Panchapagesan [P3, P4, and P13] the following questions arise:

(Q1) Similar to Section 10 of Chapter IV of [DS1], can a theory of integration of scalar functions be developed with respect to a σ-additive quasicomplete lcHs-valued vector measure $\mathbf{m}$ defined on a σ-algebra or on a σ-ring $\mathcal{S}$ of sets, permitting the integration of scalar functions (with respect to $\mathbf{m}$) which are not necessarily $\mathcal{S}$-measurable?

(Q2) The same as in (Q1), excepting that the domain of $\mathbf{m}$ is a δ-ring $\mathcal{P}$ of sets and the $\mathcal{S}$-measurability of functions is replaced by $\sigma(\mathcal{P})$-measurability (where $\sigma(\mathcal{P})$ denotes the σ-ring generated by $\mathcal{P}$).

(Q3) The Bartle-Dunford-Schwartz representation theorem has been generalized in [P9] for weakly compact operators on $C_0(T)$ and hence for weakly compact bounded Radon operators u on $\mathcal{K}(T)$ with values in a quasicomplete lcHs X asserting that u determines a unique $\mathcal{B}(T)$-regular X-valued σ-additive vector measure $\mathbf{m}_u$ on $\mathcal{B}(T)$, the σ-algebra of Borel sets in T. The question is: Is it possible to give a similar representation theorem for X-valued prolongable Radon operators u on $\mathcal{K}(T)$?

(Q4) If (Q3) has an affirmative answer, does the prolongable operator u determine a σ-additive $\delta(\mathcal{C})$-regular (suitably defined) vector measure $\mathbf{m}_u$ on $\delta(\mathcal{C})$? (This question is suggested by the scalar analogue in [P13].)

(Q5) Suppose (Q1) has an affirmative answer and suppose $u : \mathcal{K}(T) \to X$, X a quasicomplete lcHs, is a weakly compact bounded Radon operator determining the σ-additive vector measure $\mathbf{m}_u$ on $\mathcal{B}(T)$. Can it be shown that a scalar function f is u-integrable in T if and only if it is $\mathbf{m}_u$-integrable in T (in the sense of integration given for (Q1))? If so, is $\int_T f du = \int_T f d\mathbf{m}_u$?

(Q6) If (Q2), (Q3) and (Q4) are answered in the affirmative and if $u : \mathcal{K}(T) \to X$, X a quasicomplete lcHs, is a prolongable Radon operator determining the σ-additive vector measure $\mathbf{m}_u$ on $\delta(\mathcal{C})$, then can it be shown that a scalar function f with compact support is u-integrable in T if and only if it is $\mathbf{m}_u$-integrable in T? If so, what is the relation between $\int_T f du$ and $\int_T f d\mathbf{m}_u$?

In the literature, integration of scalar functions with respect to a Banach-space-valued or a sequentially complete lcHs-valued σ-additive vector measure defined on a σ-algebra Σ of sets has been studied for Σ-measurable scalar functions in several papers such as [C4], [Del1, Del2, Del4], [FNR], [FMNP], [FN1,FN2], [JO], [K1, K2, K4], [KK], [L1], [N], [O], [OR1, OR2, OR3, OR4, OR5, OR6], [Ri1, Ri2, Ri3, Ri4, Ri5, Ri6] and [Shu1, Shu2]. A similar study has been done in [BD1], [L2], [Del3], and [MN2] for a σ-additive vector measure defined on a δ-ring $\mathcal{P}$. Recently, the vector measure integration on σ-algebras has been used to study the representation of real Banach lattices in [C1,C2, C3, C4]. Also see [CR1, CR2, CR3], [FMNSS1,FMNSS2], [MP], [OSV], [SP1,SP2,SP3,SP4] and [St]. Some other papers which can also be referred for various aspects of vector measures defined on rings, δ-rings and σ-rings are [Br], [BD1], [BD2] and [Del4]. None of the above papers considers the possibility of integrating non-Σ-measurable or non-$\sigma(\mathcal{P})$-measurable functions and hence the integration theory developed in the literature is not suitable for answering the above questions. Though the paper [BD2] treats the integration of non-$\sigma(\mathcal{P})$-measurable functions, its results do not answer the above questions.

However, adapting some of the concepts and techniques used by Dobrakov in [Do1, Do2] and by Dobrakov and Panchapagesan in [DP2] (in the study of integration of vector functions with respect to an operator-valued measure), the present **monograph answers all the questions raised above in the affirmative**. Moreover, a nice theory of L_p-spaces, $1 \leq p < \infty$, is also developed for a σ-additive Banach space-valued (resp. quasicomplete or sequentially complete lcHs-valued) vector measure $\mathbf{m}$ defined on a δ-ring $\mathcal{P}$ of sets and results similar to those known for the abstract Lebesgue and Bochner L_p-spaces are obtained. Compare with [FMNSS2] and [SP1].

The monograph consists of seven chapters. Chapter 1 is on Preliminaries and has two sections. Section 1.1 is devoted to fixing the notation and terminology and to give some definitions and results from the literature on Banach space-valued measures defined on a δ-ring $\mathcal{P}$ of subsets of a non-void set T and includes the theorem on interchange of limit and integral with proof. (See Proposition 1.1.21.) Moreover, motivated by [DP2], we introduce the concept of $\mathbf{m}$-measurability for functions $f : T \to \mathbb{K}$ or $[-\infty, \infty]$ where $\mathbb{K} = \mathbb{R}$ *or* $\mathbb{C}$ and this concept of $\mathbf{m}$-measurability is suitably generalized in Section 4.1 of Chapter 4 when $\mathbf{m}$ assumes values in an lcHs. This concept plays a crucial role in developing the theory of integration which permits integration of certain non-$\sigma(\mathcal{P})$-measurable functions too. Moreover, when T is a locally compact Hausdorff space and the vector measure $\mathbf{m}$ satisfies certain regularity conditions, it turns out that a scalar function f is $\mathbf{m}$-measurable if and only if f is Lusin $\mathbf{m}$-measurable with $N(f) = \{t \in T : f(t) \neq 0\}$ suitably restricted. (See Theorems 6.2.5 and 6.2.6 of Chapter 6.) Section 1.2 is devoted to giving some definitions and results on lcHs which are needed in the sequel.

In Section 2.1 of Chapter 2 we introduce the concept of (KL) $\mathbf{m}$-integrability for $\mathbf{m}$-measurable functions and study the properties of the integral. We give an $\mathbf{m}$-a.e. convergence version of the Lebesgue dominated (resp. bounded) convergence theorem for $\mathbf{m}$ and we briefly refer to it as LDCT (resp. LBCT). This theorem has been given in [L2] with an incorrect proof (see Remark 2.1.12 below). In Section 2.2 we define the (BDS) $\mathbf{m}$-integral and show that an $\mathbf{m}$-measurable function is (BDS) $\mathbf{m}$-integrable in T if and only if it is (KL) $\mathbf{m}$-integrable in T and in that case, both the integrals coincide. Hence we use the terminology of $\mathbf{m}$-integrability (resp. the symbol $\int_T f d\mathbf{m}$) to denote either integrability (resp. either of the integrals). When $\mathcal{P}$ is a σ-algebra, this result is given in Theorem 2.4 of [L1], but, as mentioned above, its proof lacks essential details (see Remark 2.2.6 below). Proposition 1.1.21 is generalized to the (BDS) $\mathbf{m}$-integral in Theorem 2.2.8.

Chapter 3 consists of Sections 3.1–3.5 and is devoted to the study of the spaces $\mathcal{L}_p(\mathbf{m})$, $1 \leq p \leq \infty$, for a Banach space-valued σ-additive measure $\mathbf{m}$ defined on a δ-ring of sets. Similar to [Do2], in Section 2.1 we introduce a seminormed space $\mathcal{L}_p\mathcal{M}(\mathbf{m})$ of $\mathbf{m}$-measurable scalar functions with its seminorm being denoted by $\mathbf{m}_p^\bullet(\cdot, T)$ for $1 \leq p < \infty$ and define the subspaces $\mathcal{L}_p\mathcal{I}(\mathbf{m})$, $\mathcal{L}_p\mathcal{I}_s(\mathbf{m})$ and $\mathcal{L}_p(\mathbf{m})$ of $\mathcal{L}_p\mathcal{M}(\mathbf{m})$ and show that all these subspaces are linear and coincide with

$$\mathcal{I}_p(\mathbf{m}) = \{f : T \to \mathbb{K}, f \,\mathbf{m}\text{-measurable and } |f|^p \text{ is (KL)}\mathbf{m}\text{-integrable in } T\}$$

for $1 \leq p < \infty$. If $\mathcal{L}_p(\sigma(\mathcal{P}), \mathbf{m}) = \{f \in \mathcal{L}_p(\mathbf{m}) : f \,\sigma(\mathcal{P})\text{-measurable}\}$, then Section 3.2 deals with the completeness of the spaces $\mathcal{L}_p(\mathbf{m})$ and $\mathcal{L}_p(\sigma(\mathcal{P}), \mathbf{m})$ for $1 \leq p \leq \infty$ where $\mathcal{L}_\infty(\mathbf{m})$ and $\mathcal{L}_\infty(\sigma(\mathcal{P}), \mathbf{m})$ are suitably defined (see Definition 3.2.10). Sections 3.3 and 3.4 study various versions of LDCT, LBCT and the Vitali convergence theorem for $\mathcal{L}_p(\mathbf{m})$, $1 \leq p < \infty$, and Section 3.5 obtains relations between the spaces $\mathcal{L}_p(\mathbf{m})$, $1 \leq p \leq \infty$, similar to those in the classical case. The study of $\mathcal{L}_p(\sigma(\mathcal{P}), \mathbf{m})$ for $1 \leq p < \infty$ for a σ-additive vector measure $\mathbf{m}$ defined on a σ-algebra with values in a real Banach space has recently been done in [SP1, SP2, SP4] and in [FMNSS2], for their study of the representation theory of real Banach lattices. Our results can also be compared with those in the literature for

$p = 1$ as noted in Remarks 2.1.6, 2.1.11, 2.1.12, 2.2.6, 3.1.5, 3.2.9, 3.3.5, 3.3.9 and 3.3.15. We would also like to emphasize the fact that the results of this chapter are much influenced by the techniques adopted by Dobrakov in [Do1, Do2].

Chapter 4 consists of Sections 4.1–4.6. Let X be an lcHs and let $\mathbf{m} : \mathcal{P} \to X$ be σ-additive, $\mathcal{P}$ being a δ-ring of subsets of a set T. Sections 4.1–4.6 generalize the results of Chapter 3 to such $\mathbf{m}$ when X is quasicomplete (resp. sequentially complete). For such $\mathbf{m}$, the concepts of (KL) $\mathbf{m}$-integrability and (KL) $\mathbf{m}$-integral are generalized in Section 4.1; Theorem 4.1.8 generalizes (i)–(iv) and (viii) of Theorem 2.1.5 while Theorems 4.1.9 and 4.1.11 (resp. Theorems 4.1.9′ and 4.1.11′ in Remark 4.1.15) generalize (v)–(vii) of Theorem 2.1.5 and Theorem 2.1.7 (LDCT) when X is quasicomplete (resp. sequentially complete with the functions considered being $\sigma(\mathcal{P})$-measurable). In Section 4.2 we generalize (BDS) $\mathbf{m}$-integrability and (BDS) $\mathbf{m}$-integral given in Section 2.2 to such X-valued $\mathbf{m}$ and by Theorems 4.2.2 and 4.2.3 (resp. by Theorem 4.2.2′ in Remark 4.2.12) an $\mathbf{m}$-measurable (resp. $\sigma(\mathcal{P})$-measurable) function f is (KL) $\mathbf{m}$-integrable in T if and only if it is (BDS) $\mathbf{m}$-integrable in T (with values in X) when X is quasicomplete (resp. sequentially complete) and in that case, both the integrals coincide. In the light of this result, we use the terminology of $\mathbf{m}$-integrability (resp. the symbol $\int_T f d\mathbf{m}$) to denote either integrability (resp. either of the integrals). For $1 \le p < \infty$, we introduce in Section 4.3 a locally convex space $\mathcal{L}_p\mathcal{M}(\mathbf{m})$ (resp. $\mathcal{L}_p\mathcal{M}(\sigma(\mathcal{P}), \mathbf{m})$) of $\mathbf{m}$-measurable (resp. $\sigma(\mathcal{P})$-measurable) functions and introduce the subspaces $\mathcal{L}_p\mathcal{I}(\mathbf{m})$ and $\mathcal{L}_p(\mathbf{m})$ of $\mathcal{L}_p\mathcal{M}(\mathbf{m})$ (resp. $\mathcal{L}_p\mathcal{I}(\sigma(\mathcal{P}), \mathbf{m})$ and $\mathcal{L}_p(\sigma(\mathcal{P}), \mathbf{m})$ of $\mathcal{L}_p\mathcal{M}(\sigma(\mathcal{P}), \mathbf{m})$) and show that they are linear and coincide. When X is a Fréchet space, in Section 4.4 we show that these subspaces are pseudo-metrizable and complete. Section 4.5 is devoted to generalize the results in Sections 3.3, 3.4 and 3.5 when X is quasicomplete (resp. sequentially complete). We introduce a subspace $\mathcal{L}_p\mathcal{I}_s(\mathbf{m})$ (resp. $\mathcal{L}_p\mathcal{I}_s(\sigma(\mathcal{P}), \mathbf{m})$) of $\mathcal{L}_p\mathcal{M}(\mathbf{m})$ (resp. of $\mathcal{L}_p\mathcal{M}(\sigma\mathcal{P}), \mathbf{m})$ and show that it coincides with $\mathcal{L}_p\mathcal{I}(\mathbf{m})$ (resp. $\mathcal{L}_p\mathcal{I}(\sigma(\mathcal{P}), \mathbf{m})$). Section 4.6 gives some sufficient conditions for the separability of $\mathcal{L}_p(\mathbf{m})$ and $\mathcal{L}_p(\sigma(\mathcal{P}), \mathbf{m})$ for $1 \le p < \infty$. To compare with some of the results in the literature for $p = 1$, see Remarks 4.1.16, 4.4.10, and 4.6.15. **The theory of the m-integral developed in Sections 4.1 and 4.2 for quasicomplete lcHs-valued σ-additive measures defined on $\mathcal{P}$ answers (Q1) and (Q2) in the affirmative**.

Chapters 5 and 6 give a vector measure treatment of the results of [T]. Chapter 5 consists of Sections 5.1, 5.2 and 5.3. Let T be a locally compact Hausdorff space. Section 5.1 gives some generalizations of the Vitali-Carathéodory integrability criterion for $\mathbf{m}$-measurable (resp. $\sigma(\mathcal{R})$-measurable) real functions where $\mathcal{R} = \mathcal{B}(T)$ or $\mathcal{R} = \delta(\mathcal{C})$, $\mathbf{m} : \mathcal{R} \to X$ is σ-additive and $\mathcal{R}$-regular and X is a quasicomplete (resp. sequentially complete) lcHs. These results play a key role in the study of the duals of $\mathcal{L}_1(\mathbf{m})$ and $\mathcal{L}_1(\mathbf{n})$ in Section 6.5 of Chapter 6, where $\mathbf{m} : \mathcal{B}(T) \to X$ is σ-additive and $\mathcal{B}(T)$-regular (resp. $\mathbf{n} : \delta(\mathcal{C}) \to X$ is σ-additive and $\delta(\mathcal{C})$-regular) and X is a Banach space. Let $\{\mu_n\}_1^\infty$ be a sequence of Borel-regular complex measures on T. By proving that, for each open set U in T, there exists an open Baire set $V \subset U$ such that $\mu_n(V) = \mu_n(U)$ for all n, it is shown in Section 5.2 that the boundedness hypothesis in Corollary 1 of [P8] is redundant. Using this result and adapting the proofs of [T] in the set-up of vector measures, the improved version of Corollary 1 of [P8] is generalized in Theorem 5.2.21 (resp. Theorem 5.2.23) to Banach space-valued (resp. sequentially complete lcHs-valued) σ-additive regular Borel measures. Section 5.3 deals with the weakly compact bounded and prolongable Radon operators on $\mathcal{K}(T)$ with values in a quasicomplete lcHs X. In Theorem 5.3.9 of Chapter 5, the Bartle-Dunford-Schwartz representation theorem is generalized to such an X-valued prolongable Radon operator u on $\mathcal{K}(T)$ and it is shown that u determines a unique X-valued $\delta(\mathcal{C})$-regular σ-additive measure $\mathbf{m}_u$ on $\delta(\mathcal{C})$. **Thus (Q3) and (Q4) are answered in the affirmative.** Using the results of [P9], 22 characterizations are given for an X-valued continuous linear map u on $\mathcal{K}(T)$ to be a prolongable Radon operator.

Chapter 6 consists of Sections 6.1–6.5. Let T be a locally compact Hausdorff space. Let $\mathbf{m} : \mathcal{B}(T) \to X$ (resp. $\mathbf{n} : \delta(\mathcal{C}) \to X$) be σ-additive and Borel regular (resp. and $\delta(\mathcal{C})$-regular). Section 6.1 deals with the generalized Lusin's theorem and its variants for $\mathbf{m}$ and for $\mathbf{n}$ with some applications. In Section 6.2 several characterizations of the $\mathbf{m}$-measurability (resp. $\mathbf{n}$-measurability) of a set A in T are given. The concepts of Lusin $\mathbf{m}$-measurability and Lusin $\mathbf{n}$-measurability for scalar functions are introduced and they are characterized in terms of $\mathbf{m}$-measurability and $\mathbf{n}$-measurability, respectively. The proofs of Lemmas 3.10 and 3.14, Propositions 2.17, 2.20 and 3.7 and Theorems 3.5, 3.13 and 3.20 of [T] are adapted here in the set-up of vector measures to improve Theorem 2.2.2 (resp. Theorem 4.2.2) when X is a Banach space (resp. a quasicomplete or complete lcHs) and when $\mathbf{m} : \delta(\mathcal{C}) \to X$ is σ-additive and $\delta(\mathcal{C})$-regular. See Theorems 6.3.4, 6.3.5 and 6.3.8. Section 6.4 deals with some additional convergence theorems. Section 6.5 is devoted to the study of the duals of $L_1(\mathbf{m})$ and $L_1(\mathbf{n})$ and it is shown that $L_1(\mathbf{m})$ and $L_1(\mathbf{n})$ are weakly sequentially complete Banach spaces when X is a Banach space with $c_0 \not\subset X$.

Chapter 7 consists of Sections 7.1–7.6. In Section 7.1 we briefly indicate how the results in Section 1 of [T] can be extended to complex functions in $\mathcal{K}(T)$. Section 7.2 is devoted to integration with respect to a weakly compact bounded Radon operator, improving the complex versions of Theorems 2.2, 2.7 and 2.7 bis of [T]. In Section 7.3 we improve most of the results such as the complex versions of Theorems 3.3, 3.4, 3.11, 3.13 and 3.20 of [T]. Section 7.4 studies the complex Baire versions of Proposition 4.8 and Theorem 4.9 of [T]. In Section 7.5 we introduce the concepts of weakly compact and prolongable Radon vector measures and generalize the results of [P3,P4] to such Radon vector measures. If u is a bounded Radon operator with values in a quasicomplete lcHs, we define $M_u = \{A \subset T : \chi_A \in \mathcal{L}_1(u)\}$ and $\boldsymbol{\mu}_u(A) = \int_A du$ for $A \in M_u$. The Radon vector measure induced by u is denoted by $\boldsymbol{\mu}_u$ and M_u is called the domain of $\boldsymbol{\mu}_u$. When u is a weakly compact bounded Radon operator, we show that M_u is a σ-algebra containing $\mathcal{B}(T)$ and $\boldsymbol{\mu}_u$ is the generalized Lebesgue completion of the representing measure $\mathbf{m}_u|_{\mathcal{B}(T)}$ of u (in the sense of 5.2.10). We give several characterizations of a weakly compact bounded (resp. a prolongable) Radon operator u and study the regularity properties of $\boldsymbol{\mu}_u$ in both the cases. Following [Si] we define the outer measure $\boldsymbol{\mu}_u^*$ induced by $\boldsymbol{\mu}_u$ and study its relation with $\boldsymbol{\mu}_u$ when u is a weakly compact bounded Radon operator. Introducing the concepts of Lebesgue-Radon completion and localized Lebesgue-Radon completion, we generalize Theorems 4.4 and 4.6 of [P4]. (See Theorems 7.5.24 and 7.5.27.) Thus Theorems 9.13, 9.14 and 9.17 of [P13] are proved here. In Section 7.6, we show that when u is a weakly compact bounded Radon operator on $\mathcal{K}(T)$ with values in a quasicomplete lcHs, $\mathcal{L}_p(u)$ is the same as $\mathcal{L}_p(\mathbf{m}_u)$ for $1 \leq p < \infty$; f is u-integrable if and only if f is $\mathbf{m}_u$-integrable in T and when f is u-integrable, $\int f du = \int_T f d\mathbf{m}_u$. (See Theorem 7.6.13.) When u is a prolongable Radon operator on $\mathcal{K}(T)$ with values in a quasicomplete lcHs and ω is a relatively compact open set in T, we show that $\mathcal{L}_p(u|_{\mathcal{K}(\omega)}) = \mathcal{L}_p(\mathbf{m}_u|_{\mathcal{B}(\omega)})$ for $1 \leq p < \infty$ and for $f \in \mathcal{L}_1(\mathbf{m}_u)$ with support $K \in \mathcal{C}$ and with $K \subset \omega$, $\int_T f d\mathbf{m}_u = \int f\chi_\omega du$. (See Theorem 7.6.19.). Thus **questions (Q5) and (Q6) are also answered in the affirmative.**

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