Preface

This book is a collection of selected papers on recent trends in the study of various branches of mathematics. Most of the authors who contributed to this volume were invited speakers at the International Conference "*Geometry and Dynamics of Groups and Spaces. In Memory of Alexander Reznikov*" which was held at the Max Planck Institute for Mathematics (Bonn), Germany, in September 22–29, 2006.

Alexander (Sasha) Reznikov (1960–2003) was a brilliant mathematician who died unfortunately very early. This conference in his remembrance focused on topics Sasha made a contribution to. In particular: hyperbolic, differential and complex geometry; geometric group theory; three dimensional topology; dynamical systems.

The list of participants: Belolipetsky Mikhail; Bismut Jean-Michel; Boileau Michel; Breuillard Emmanuel; Danilenko Alexandre; Delzant Thomas; Deninger Christopher; Esnault Hélène; Franks John; Gal S.R.; Gunesch Roland; Hai Phung Ho; Kaimanovich Vadim; Kapovich Michael; Klingenberg Wilhelm; Makar-Limanov Leonid; Milman Vitali; Morishita M.; Moree Pieter; Navas Andres; Neretin Yuri; Papazoglu Panagiotis; Parker John R.; Porti Joan; Rosellen Markus; Simpson Carlos; Swenson Eric L.; Szczepanski A.; Tomanov Georges; Tsygan Boris; Verjovsky Sola Alberto; Wang Shicheng; Zakrzweski Wojtek.

Talks took place in an informal and constructive atmosphere, and it was a pleasure to see discussions taking place between groups of participants all over the Institute and at all times of day. The exceptional quality of the lectures and the great interest they generated among the conference participants gave us, the organizers of the conference, the idea of this book. The editors wish to record their thanks to the staff at the Max-Planck Institute and to the many researchers who participated for all their efforts in making this such a stimulating experience.

This volume is a collection of papers which aims at reflecting the present state of the art in a most active area of research at the intersection of several branches of mathematics. In the volume we include an unpublished manuscript "Analytic Topology of Groups, Actions, Strings and Varietes" by Sasha (this article is posted, essentially, in the form it appeared as the ArXive preprint math.DG/0001135 in January of 2000; the referee corrected obvious typos, updated and added few references, and made several comments in the form of footnotes and italicized remarks) and some short speeches/recollections about Sasha.

Topics discussed in the book include analytic topology of groups, actions, strings and varieties, category theory, homological algebra, the quantum dilogarithm, Chow groups, parabolic bundles, quantum spaces over non-archimedean fields, Nori's fundamental group scheme, Baumslag-Solitar groups, finitely generated branch groups, Serre's property (FA), hypoelliptic equations, Hodge theory, index theory and related fixed point theorems, determinants and determinant bundles, analytic torsion, Kleinian groups, hyperbolic manifolds, geometric functional analysis, Chern character and cyclic homology, Milnor invariants and l-class groups, geodesic flow, operads, algebras, inner cohomomorphisms, symmetry and deformations in noncommutative geometry, Chebyshev-Dickson polynomials, convolution operators, lattices, discrete harmonic functions, ergodic transformations and rank-one actions, hyperbolic surfaces, eigenfunctions, non-Archimedean metric spaces.

The articles collected in this volume should be of interest to specialists in such areas of mathematics as algebra, dynamical systems, geometry, group theory, functional analysis, number theory, probability theory and topology. The broad spectrum of topics covered should also present an exciting opportunity for graduate students and young researchers working in any of these areas who are willing to put their research in a wider mathematical perspective.

Alexander (Sasha) Reznikov (1960–2003)

Alexander (Sasha) Reznikov was born in Kiev (Ukraine, former USSR) on January 14, 1960. From a very early age he was fascinated with mathematics. It took him only 8 years (instead of the usual 10 years) to finish the primary and secondary schools. In 1975 (the last year of the secondary school), Sasha won the second prize at the International Mathematical Olympiad. Because of this, according to the former Soviet rules he got the right to be admitted as a student to any University of the Soviet Union without the entrance exams. For this reason, he, luckily, was able to avoid the enormous obstacles faced by other Jewish students who tried to enter prestigious Soviet universities during the 1970s and 1980s.[1] Sasha was admitted to Kiev State University at the age of 15 and was a brilliant student during his years at the University. He successfully participated in the student mathematical life and started to do research early on. Despite his success, the fact that he was a Jew effectively barred him from graduate programs and jobs at research institutes. Sasha got a job at a state planning institution which had nothing to do with mathematics. Outside of his working hours there, he visited the Kiev Institute of Mathematics and worked on his PhD thesis under the supervision of Myroslav Gorbachuk. Around that time he became interested in Jewish history and joined a small group studying the history of the state of Israel. Very soon, the Soviet secret service discovered this activity and reported Sasha to his employer. He was forced to quit his job and to leave Kiev. Sasha started to travel the country. He worked in Lithuania, Tajikistan and other remote regions, doing mostly manual work, unable to continue his research in mathematics. Fortunately, the times were changing and starting from 1988 Soviet Jews were again allowed to emigrate. Sasha emigrated to Israel in 1989 and already a year later completed his PhD thesis at the Tel Aviv University under the supervision of Vitali Milman. After spending a year as a postdoc at ICTP in Trieste, he became a lecturer at the Hebrew University in Jerusalem. He remained there until he took a chair in Durham in 1997. Sasha Reznikov, Professor of Pure Mathematics at the University of Durham, died on 5 September 2003, at the age of 43.

We will now describe briefly the most important mathematical contributions of Sasha Reznikov. He started as a classical Riemannian geometer and one of his most impressive results is a proof of the so-called weak Blaschke conjecture [Re19]. A compact Riemannian manifold is called a *Blaschke manifold* if the length of the maximal geodesic segment α starting at any point p, is independent of p and α. The problem is whether the only possible Blaschke manifolds are spheres and projective spaces over the reals, complex numbers, quaternions and Cayley numbers, equipped with their canonical metrics. Sasha proved that all Blaschke manifolds have the same volume as the spheres or projective spaces on which they are modelled.

Sasha Reznikov's most influential work is his proof of Spencer Bloch's conjecture on representations of the fundamental group of an algebraic variety. The proof

[1] This situation is described in the book by M. Shifman "You Failed Your Math Test, Comrade Einstein: Adventures and Misadventures of Young Mathematicians".

is a remarkable combination of arithmetic and analytic methods. Sasha proved that for any smooth complex projective variety, and any representation of its fundamental group into $\mathrm{SL}(2, \mathbf{C})$, the second Chern class in the Deligne cohomology of the associated holomorphic bundle is torsion [Re14]. More generally, he later showed that for all flat bundles on a smooth projective variety, all Chern classes in the Deligne cohomology, except the first one, are torsion [Re17]. These are outstanding results which opened new directions of the research in this field. C. Soulé gave a talk at the Bourbaki Séminaire devoted to this theorem of Reznikov, [So].

For these results Sasha Reznikov was awarded a sectional talk at the European Congress of Mathematics in Barcelona in 2000 [Re2].

In the middle of 1990s he became interested in the geometry and topology of 3-manifolds and geometric group theory. He attempted to prove the famous Haken–Waldhausen–Thurston conjecture that any irreducible 3-manifold with infinite fundamental group has a finite covering with positive first Betti number (such 3-manifolds are called *virtually Haken*)[2]. During these years, he wrote a series of important papers [Re4], [Re5], [Re6], [Re7] and [Re9]. In [Re9] he proved several restrictions on manifolds which are not virtually Haken. In particular, it follows from Reznikov's theorem that if the manifold M is hyperbolic and is not virtually Haken, then for every prime number p there exists a finite covering $N \to M$ such that rank $H_1(N, \mathbb{F}_p) \geq 4$. In the paper [Re4] he discussed similarities between the 3-manifold topology and the theory of number fields. These are illustrated by several interesting examples concerning Heegaard splittings of 3-manifolds. In [Re8] Sasha developed an analogy between the symplectomorphism groups and linear groups; he proved in particular that the inclusion of the compact Lie group $PSU(n+1)$ into the symplectomorphism group of the complex projective n-space is injective on the rational homology.

The final work of Sasha Reznikov "Analytic topology of groups, actions, strings and varieties" was written in 2000 and remained unpublished since then. It contains many important ideas and results, in particular it was instrumental in study of the property T (Kazhdan property) of groups of diffeomorphisms of the circle $\mathbb{S}^1$. We are very glad that this paper is included in our Proceedings and we thank the referee for his careful reading of the paper and many useful remarks and corrections. We keep the referee's comments in the form of footnotes and italicized remarks.

Sasha Reznikov wrote 34 mathematical papers. Most of them are written in a very short period during the 1990s. Their mathematical scope is enormous; the results of Sasha Reznikov belong to several different areas of mathematics: Riemannian and symplectic geometry, 3-dimensional topology, geometric group theory, algebraic geometry and dynamical systems. This diversity of Sasha's research interests is reflected by the variety of topics covered by the articles which appear in these Proceedings of the Conference dedicated to the memory of Sasha Reznikov. The Conference was held at the Max-Planck Institute of Mathematics

[2]This problem remains open.

in Bonn in 2006 from September 22 until September 29 as a satellite conference to the annual meeting of the German Mathematical Society which in 2006 took place in the period September 18–September 22 at the University of Bonn.

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