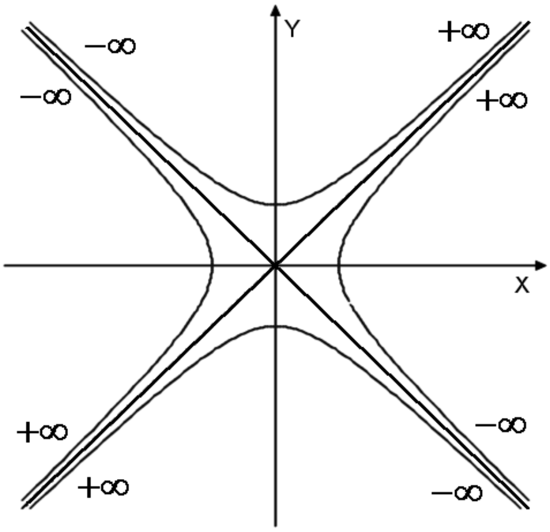

Figure 4.1: For $\rho = 1$ the x, y in Tab. 4.1 represent the four arms of equilateral hyperbolas $|x^2 - y^2| = 1$. Here we indicate how each arm is traversed as the parameter θ goes from $-\infty$ to $+\infty$. In particular there is a symmetry with respect to axis bisectors.

4.1.2 Hyperbolic Rotations as Lorentz Transformations of Special Relativity

Let us write a space-time vector as a hyperbolic variable[1], $w = t + h\,x$ and a hyperbolic constant $a = a_r + h\,a_h$ with $a_r > a_h$ in the exponential form

$$\begin{aligned} &a_r + h\,a_h \equiv \rho_a \exp[\,h\theta_a\,] \equiv \rho_a\,(\cosh\theta_a + h\sinh\theta_a) \\ \text{where} \qquad &\rho_a = \sqrt{(a_r^2 - a_h^2)}; \quad \theta_a = \tanh^{-1}(a_h/a_r). \end{aligned}$$

Then the multiplicative group, $w' \equiv\ t' + h\,x' = a\,w$ becomes

$$t' + h\,x' = \sqrt{(a_r^2 - a_h^2)}\,[t\,\cosh\theta_a + x\,\sinh\theta_a + h(t\,\sinh\theta_a + x\,\cosh\theta_a)]\,. \quad (4.1.7)$$

In this equation, by letting $(a_r^2 - a_h^2) = 1$ and considering as equal the coefficients of versors "1" and "h", as we do in complex analysis, we get the Lorentz transformation of two-dimensional special relativity [55] and [62]. It is interesting to

[1] In all the problems which refer to Special Relativity we change the symbols by indicating the variables with letters reflecting their physical meaning $x,\ y\ \Rightarrow\ t,\ x$, i.e., t is a normalized time variable (light velocity $c = 1$) and x a space variable.

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