

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Categories</b>	<b>7</b>
2.1	Basic Concepts . . . . .	7
2.2	Limits and Co-limits . . . . .	11
2.3	Adjunctions . . . . .	16
2.4	2-categories . . . . .	18
2.5	Indexed Categories and Fibrations . . . . .	20
<b>3</b>	<b>Institutions</b>	<b>23</b>
3.1	From concrete logic to Institutions . . . . .	23
3.2	Examples of institutions . . . . .	28
3.3	Morphisms and Comorphisms . . . . .	38
3.4	Institutions as Functors . . . . .	45
<b>4</b>	<b>Theories and Models</b>	<b>49</b>
4.1	Theories and Presentations . . . . .	50
4.2	Theory (co-)limits . . . . .	57
4.3	Model Amalgamation . . . . .	60
4.4	The method of Diagrams . . . . .	65
4.5	Inclusion Systems . . . . .	74
4.6	Free Models . . . . .	82
<b>5</b>	<b>Internal Logic</b>	<b>91</b>
5.1	Logical Connectives . . . . .	92
5.2	Quantifiers . . . . .	94
5.3	Substitutions . . . . .	97
5.4	Representable Signature Morphisms . . . . .	102
5.5	Satisfaction by Injectivity . . . . .	107
5.6	Elementary Homomorphisms . . . . .	114

<b>6</b>	<b>Model Ultraproducts</b>	<b>121</b>
6.1	Filtered Products . . . . .	121
6.2	Fundamental Theorem . . . . .	124
6.3	Łoś Institutions . . . . .	132
6.4	Compactness . . . . .	134
6.5	Finitely Sized Models . . . . .	137
<b>7</b>	<b>Saturated Models</b>	<b>141</b>
7.1	Elementary Co-limits . . . . .	141
7.2	Existence of Saturated Models . . . . .	144
7.3	Uniqueness of Saturated Models . . . . .	152
7.4	Saturated Ultraproducts . . . . .	157
<b>8</b>	<b>Preservation and Axiomatizability</b>	<b>163</b>
8.1	Preservation by Saturation . . . . .	163
8.2	Axiomatizability by Ultraproducts . . . . .	168
8.3	Quasi-varieties and Initial Models . . . . .	170
8.4	Quasi-Variety Theorem . . . . .	174
8.5	Birkhoff Variety Theorem . . . . .	178
8.6	General Birkhoff Axiomatizability . . . . .	181
<b>9</b>	<b>Interpolation</b>	<b>189</b>
9.1	Semantic interpolation . . . . .	192
9.2	Interpolation by Axiomatizability . . . . .	197
9.3	Interpolation by Consistency . . . . .	204
9.4	Craig-Robinson Interpolation . . . . .	211
9.5	Borrowing Interpolation . . . . .	215
<b>10</b>	<b>Definability</b>	<b>223</b>
10.1	Explicit implies implicit definability . . . . .	226
10.2	Definability by Interpolation . . . . .	228
10.3	Definability by Axiomatizability . . . . .	230
<b>11</b>	<b>Possible Worlds</b>	<b>235</b>
11.1	Internal Modal Logic . . . . .	236
11.2	Ultraproducts of Kripke models . . . . .	242
<b>12</b>	<b>Grothendieck Institutions</b>	<b>253</b>
12.1	Fibred and Grothendieck Institutions . . . . .	254
12.2	Theory Co-limits and Model Amalgamation . . . . .	260
12.3	Interpolation . . . . .	267

<b>13 Institutions with Proofs</b>	<b>275</b>
13.1 Free Proof Systems . . . . .	278
13.2 Compactness . . . . .	284
13.3 Proof-theoretic Internal Logic . . . . .	288
13.4 The Entailment Institution . . . . .	297
13.5 Birkhoff Completeness . . . . .	302
<b>14 Specification</b>	<b>317</b>
14.1 Structured Specifications . . . . .	318
14.2 Specifications with Proofs . . . . .	327
14.3 Predefined Types . . . . .	331
<b>15 Logic Programming</b>	<b>337</b>
15.1 Herbrand Theorems . . . . .	338
15.2 Unification . . . . .	340
15.3 Modularization . . . . .	344
15.4 Constraints . . . . .	346
<b>A Table of Notation</b>	<b>351</b>
<b>Bibliography</b>	<b>355</b>
<b>Index</b>	<b>368</b>



<http://www.springer.com/978-3-7643-8707-5>

Institution-independent Model Theory

Diaconescu, R.

2008, XI, 376 p., Softcover

ISBN: 978-3-7643-8707-5

A product of Birkhäuser Basel