Preface to the First Edition

According to Leo Breiman (1968), probability theory has a right and a left hand. The right hand refers to rigorous mathematics, and the left hand refers to 'probabilistic thinking'. The combination of these two aspects makes probability theory one of the most exciting fields in mathematics. One can study probability as a purely mathematical enterprise, but even when you do that, all the concepts that arise do have a meaning on the intuitive level. For instance, we have to define what we mean exactly by independent events as a mathematical concept, but clearly, we all know that when we flip a coin twice, the event that the first gives heads is independent of the event that the second gives tails.

Why have I written this book? I have been teaching probability for more than fifteen years now, and decided to do something with this experience. There are already many introductory texts about probability, and there had better be a good reason to write a new one. I will try to explain my reasons now.

The typical target that I have in mind is a first year student who wants or needs to learn about probability at a stage where he or she has not seen measure theory as yet. The usual and classical way to set things up in this first year, is to introduce probability as a measure on a sigma-algebra, thereby referring to measure theory for the necessary details. This means that the first year student is confronted with a theory of probability that he or she cannot possibly understand at that point. I am convinced that this is not necessary.

I do not (of course) claim that one should not base probability theory on measure theory later in the study, but I do not really see a reason to do this in the first year. One can – as I will show in this book – study discrete and continuous random variables perfectly well, and with mathematical precision, within the realm of Riemann integration.

It is not particularly difficult to write rigorously about discrete probability, but it is harder to do this more generally. There are several texts available which do promise this (no measure theory and rigorous) but I don't think that any of these texts can really say to have kept its promise. I have achieved precision without measure theory, by deviating from the classical route of defining probability measures on sigma-algebras. In this book, probabilities of events are defined as soon as a certain (Riemann) integral exists. This is, as I will show, a very natural thing to do. As a result, everything in this book can be followed with no more background than ordinary calculus.

As a result of my approach, it will become clear where the *limits* of this approach are, and this in fact forms the perfect *motivation* to study measure theory and probability theory based on measure theory later in the study. Indeed, by the end of the book, the student should be dying to learn more about measure theory.

Hence this approach to probability is fully consistent with the way mathematics works: first there is a theory and you try to see how far this gets you, and when you see that certain (advanced) things cannot be treated in this theory, you have to extend the theory. As such, by reading this book, one not only learns a lot about probability, but also about the way mathematics is discovered and developed.

Another reason to write this book is that I think that it is very important that when students start to learn about probability, they come to interesting results as soon as possible, without being disturbed by unnecessary technical complications. Probability theory is one of those fields where you can derive very interesting, important and surprising results with a minimum of technical apparatus. Doing this first is not only interesting in itself. It also makes clear what the limits are of this elementary approach, thereby motivating further developments. For instance, the first four chapters will be concerned with discrete probability, which is rich enough as to say everything about a finite number of coin flips. The very surprising arc-sine law for random walks can already be treated at this point. But it will become clear that in order to describe an *infinite* sequence of such flips, one needs to introduce more complicated models. The so called weak law of large numbers can be perfectly well stated and proved within the realm of discrete probability, but the strong law of large numbers cannot.

Finally, probability theory is one of the most useful fields in mathematics. As such, it is extremely important to point out what exactly we do when we model a particular phenomenon with a mathematical model involving uncertainty. When can we safely do this, and how do we know that the outcomes of this model do actually say something useful? These questions are addressed in the appendix. I think that such a chapter is a necessary part of any text on probability, and it provides a link between the left and right hand of Leo Breiman.

A few words about the contents of this book. The first four chapters deal with *discrete probability*, where the possible outcomes of an experiment form a finite or countable set. The treatment includes an elementary account on random walks, the weak law of large numbers and a primitive version of the central limit theorem. We have also included a number of confusing examples which make clear that it is sometimes dangerous to trust your probabilistic intuition.

After that, in the Intermezzo, we explain why discrete probability is insufficient to deal with a number of basic probabilistic concepts. Discrete probability is unable to deal with infinitely *fine* operations, such as choosing a point on a line segment, and infinitely many *repetitions* of an operation, like infinitely many coin flips.

After the Intermezzo, Chapter 5 deals with these infinitely fine operations, a subject which goes under the name *continuous probability.* In Chapter 6 we continue with infinitely many repetitions, with applications to branching processes, random walk and strong laws of large numbers.

Chapter 7 is devoted to one of the most important stochastic processes, the Poisson process, where we shall investigate a very subtle and beautiful interply between discrete and continuous random variables.

In Chapter 8, we discuss a number of limit theorems based on characteristic functions. A full proof of the central limit theorem is available at that point. Finally, in Chapter 9, we explore the limitations of the current approach. We will outline how we can extend the current theory using measure theory. This final chapter provides the link between this book and probability based on measure theory.

Previous versions of this book were read by a number of people, whose comments were extremely important. I would very much like to thank Hanneke de Boer, Lorna Booth, Karma Dajani, Odo Diekmann, Massimo Franceschetti, Richard Gill, Klaas van Harn, Rob Kooijman, Rein Nobel, Corrie Quant, Rahul Roy, Jeffrey Steif, Freek Suyver, Aad van der Vaart and Dmitri Znamenski for their valuable comments. In particular, I would also like to thank Shimrit Abraham for doing a large number of exercises.

Ronald Meester, Amsterdam, Summer 2003

Preface to the Second Edition

Teaching from a book leads to many remarks and complaints. Despite the care given to the first edition, it turned out that it still contained a fair number of small mistakes. I thank Corrie Quant and Karma Dajani (again), Rob van den Berg, Misja Nuyens and Jan Hoogendijk for their corrections and other remarks that helped me very much in preparing this second edition. Apart from correcting mistakes and typos, I also added a significant number of exercises and some examples. Finally, here and there I slightly rearranged the material. There are, however, no essential differences between the first and second edition.

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