

Contents

Preface	ix
Introduction	xi
Highlights of the chapters	xv
1 Preliminaries	1
1.1 General local principle for C^* -algebras	1
1.2 C^* -Algebras generated by orthogonal projections	14
2 Prologue	33
2.1 On the term “symbol”	33
2.2 Bergman space and Bergman projection	34
2.3 Representation of the Bergman kernel function	38
2.4 Some integral operators and representation of the Bergman projection	42
2.5 “Continuous” theory and local properties of the Bergman projection	45
2.6 Model discontinuous case	50
2.7 Symbol algebra	53
2.8 Toeplitz operators	57
2.9 Some further results on compactness	61
3 Bergman and Poly-Bergman Spaces	65
3.1 Bergman space and Bergman projection	66
3.2 Connections between Bergman and Hardy spaces	71
3.3 Poly-Bergman spaces, decomposition of $L_2(\Pi)$	73
3.4 Projections onto the poly-Bergman spaces	76
3.5 Poly-Bergman spaces and two-dimensional singular integral operators	82
4 Bergman Type Spaces on the Unit Disk	89
4.1 Bergman space and Bergman projection	89
4.2 Poly-Bergman type spaces, decomposition of $L_2(\mathbb{D})$	96

5	Toeplitz Operators with Commutative Symbol Algebras	101
5.1	Semi-commutator versus commutator	102
5.2	Infinite dimensional representations	105
5.3	Spectra and compactness	110
5.4	Finite dimensional representations	114
5.5	General case	116
6	Toeplitz Operators on the Unit Disk with Radial Symbols	121
6.1	Toeplitz operators with radial symbols	122
6.2	Algebras of Toeplitz operators	132
7	Toeplitz Operators on the Upper Half Plane with Homogeneous Symbols	135
7.1	Representation of the Bergman space	135
7.2	Toeplitz operators with homogeneous symbols	138
7.3	Bergman projection and homogeneous functions	146
7.4	Algebra generated by the Bergman projection and discontinuous coefficients	151
7.5	Some particular cases	158
7.6	Toeplitz operator algebra. A first look	162
7.7	Toeplitz operator algebra. Some more analysis	165
8	Anatomy of the Algebra Generated by Toeplitz Operators with Piece-wise Continuous Symbols	175
8.1	Symbol class and operators	177
8.2	Algebra $\mathcal{T}(PC(\overline{\mathbb{D}}, T))$	178
8.3	Operators of the algebra $\mathcal{T}(PC(\overline{\mathbb{D}}, T))$	180
8.4	Toeplitz operators of the algebra $\mathcal{T}(PC(\overline{\mathbb{D}}, T))$	183
8.5	More Toeplitz operators	187
8.6	Semi-commutators involving unbounded symbols	198
8.7	Toeplitz or not Toeplitz	206
8.8	Technical statements	209
9	Commuting Toeplitz Operators and Hyperbolic Geometry	215
9.1	Bergman metric	216
9.2	Basic properties of Möbius transformations	217
9.3	Fixed points and commuting Möbius transformations	220
9.4	Elements of hyperbolic geometry	221
9.5	Action of Möbius transformations	224
9.6	Classification theorem	226
9.7	Proof of the classification theorem	228

10 Weighted Bergman Spaces	233
10.1 Unit disk	233
10.2 Upper half-plane	237
10.3 Representations of the weighted Bergman space	240
10.4 Model classes of Toeplitz operators	250
10.5 Boundedness, spectra, and invariant subspaces	260
11 Commutative Algebras of Toeplitz Operators	263
11.1 On symbol classes	264
11.2 Commutativity on a single Bergman space	267
11.3 Commutativity on each weighted Bergman space	270
11.4 First term: common gradient and level lines	272
11.5 Second term: gradient lines are geodesics	275
11.6 Curves with constant geodesic curvature	278
11.7 Third term: level lines are cycles	285
11.8 Commutative Toeplitz operator algebras and pencils of geodesics	290
12 Dynamics of Properties of Toeplitz Operators with Radial Symbols	293
12.1 Boundedness and compactness properties	294
12.2 Schatten classes	305
12.3 Spectra of Toeplitz operators, continuous symbols	314
12.4 Spectra of Toeplitz operators, piece-wise continuous symbols	318
12.5 Spectra of Toeplitz operators, unbounded symbols	324
13 Dynamics of Properties of Toeplitz operators on the Upper Half Plane: Parabolic case	329
13.1 Boundedness of Toeplitz operators with symbols depending on $y = \text{Im } z$	329
13.2 Continuous symbols	339
13.3 Piece-wise continuous symbols	341
13.4 Oscillating symbols	343
13.5 Unbounded symbols	345
14 Dynamics of Properties of Toeplitz operators on the Upper Half Plane: Hyperbolic case	349
14.1 Boundedness of Toeplitz operators with symbols depending on $\theta = \arg z$	349
14.2 Continuous symbols	353
14.3 Piece-wise continuous symbols	355
14.4 Unbounded symbols	358

Appendices

A Coherent states and Berezin transform	361
A.1 General approach to coherent states	361
A.2 Numerical range and spectra	365
A.3 Coherent states in the Bergman space	367
A.4 Berezin transform	368
B Berezin Quantization on the Unit Disk	373
B.1 Definition of the quantization	373
B.2 Quantization on the unit disk	375
B.3 Two first terms of asymptotic of the Wick symbol	376
B.4 Three first terms of asymptotic in a commutator	380
Bibliographical Remarks	391
Bibliography	397
List of Figures	413
Index	415

Commutative Algebras of Toeplitz Operators on the
Bergman Space

Vasilevski, N.

2008, XXIX, 418 p., Hardcover

ISBN: 978-3-7643-8725-9

A product of Birkhäuser Basel