Preface

Applying Exponential Dichotomy

Exponentially dichotomous operators are the natural evolution operators of first-order linear homogeneous differential equations in an arbitrary Banach space in which causal effects can have impact on both future and past events. When incorporated as the differential equation describing the state of a linear system, these systems are called noncausal or forward-backward or of mixed type. Exponentially dichotomous operators can be viewed as direct sums

$$S = S_+ \dot{+} S_-,$$

where S_+ and S_- are the infinitesimal generators of exponentially decaying strongly continuous semigroups on a Banach space, one forward in time and the other backward in time. This means that its resolvent $(\lambda - S)^{-1}$ exists on a vertical strip

$$C_\varepsilon = \{\lambda \in \mathbb{C} : |\mathrm{Re}\, \lambda| \leq \varepsilon\}$$

for some $\varepsilon > 0$, is bounded on C_ε, and is the Fourier transform of a so-called bisemigroup $E(t)$, composed of a semigroup forward in time on the first component space and minus a semigroup backward in time on the second component space. Both of these semigroups are exponentially decaying. The Cauchy problem governed by S now has the form

$$\begin{cases} u'(t) = Su(t) + f(t), & t \in \mathbb{R}, \\ u(0^+) - u(0^-) = x_0, \end{cases}$$

where both the inhomogeneous term f and the solution u are assumed Bochner integrable to guarantee the existence of a unique solution.

The author's interest in exponentially dichotomous operators has been sparked by his past involvement in four research areas, where it has been deemed convenient to employ exponentially dichotomous operators. These four areas are linear kinetic equations (including those governed by a Sturm-Liouville differential operator with indefinite weight function), forward-backward systems of Pritchard-Salamon type, inverse scattering on the line, and algebraic Riccati equations related to so-called block operators of Hamiltonian type. Exponentially dichotomous operators have also arisen in the study of linear integral equations with semi-separable kernels and, more recently, in the study of functional differential equations of mixed type. We discuss each of these research areas briefly.

The research area most familiar to the author has been the mathematical modeling of stationary particle transport or radiative transfer in a spatially homogeneous plane parallel domain. Typically the boundary conditions describe the incoming particle density or incident radiative flux, which naturally requires distinguishing between the contributions of a forward and a backward direction. Here distance from the boundary takes the place of forward and backward time. Further, repeated single scattering events lead to a coupling between the contributions in the forward and backward directions. This has culminated in an extensive theory of abstract kinetic equations. A closely related application has been the use of exponentially dichotomous differential operators in the study of Sturm-Liouville equations with an indefinite weight function. We deal with kinetic equations in Chapter 5 and indefinite Sturm-Liouville problems in Chapter 6.

Linear integral equations of the second kind on intervals of the real line often have a so-called semi-separable integral kernel. This means that the kernel is separable, but the separation of variables depends on the sign of the difference between the independent variables. When the integral equation is of convolution or Hankel type in that it depends on either the difference or the sum of its arguments, its solutions can be obtained using a linear system of forward-backward type. Here the role of time is played by the independent variable. The basic results, where the linear noncausal system is finite-dimensional, were developed in the mid-1980s (cf. [17]). The theory has been refined to deal with a more extensive class of integral kernels, where the principal objective has been the investigation of a class of forward-backward systems with minimal emphasis on integral equations. We mention in particular forward-backward Pritchard-Salamon systems, but in principle even more general systems (such as natural generalizations of the well-posed linear systems studied in [148]) could be studied. We discuss two basic types of forward-backward systems in Chapter 7.

Block operators, i.e., 2×2 matrices whose entries are linear operators, constitute another area where exponentially dichotomous operators play an important role. Viewing such operators as additive perturbations of block diagonal operators, where the decomposition underlying the block structure renders the latter exponentially dichotomous, we are naturally led to additive (bounded) perturbation theory of exponentially dichotomous operators. Viewing the perturbed block operator as a Hamiltonian operator, its invariant subspace requirements naturally lead to algebraic Riccati equations. We thus have in hand a powerful tool for studying existence of its solutions and even approximation properties. We treat block operators and algebraic Riccati equations in detail in Chapter 4.

Delay equations have traditionally been a major source of exponentially dichotomous operators. The situation is rather special, because the component semigroup exponentially decaying backward in time is in fact a strongly continuous group. In other words, the exponentially dichotomous operators involved in treating delay equations are generators of hyperbolic semigroups. It has only been in recent years that there have been serious attempts to extend the theory of delay equations to equations with both positive and negative delays, the so-called functional differential equations of mixed type. In this case the exponentially dichotomous operators are no longer generators of hyperbolic semigroups. Another complicating factor is the apparent impossibility to apply perturbation theory for exponentially dichotomous operators. We have therefore decided to discuss functional differential equations of mixed type only in the final Chapter 8.

Exponentially dichotomous operators and the bisemigroups they generate have been introduced in the study of linear transport equations in L^p-spaces by the author [154], but the treatment fell far short of a formal definition of exponentially dichotomous operators and bisemigroups. Bart, Gohberg, and Kaashoek [16] have pioneered abstract exponential dichotomy by giving such a formal definition, by deriving some basic properties, and applying them to the realization problem for certain infinite-dimensional systems, subsequently called BGK realizations. Applications to linear integral equations with semi-separable kernels soon followed [17]. Ever since, bisemigroups have been applied in various contexts: linear transport theory, diffusion equations of indefinite Sturm-Liouville type, extended Pritchard-Salamon realizations, block operators and their various applications, and functional differential equations of mixed type. Bisemigroups appeared in the explicit expressions for the solutions of the inverse scattering problem for the matrix Zakharov-Shabat system on the line [5, 156], but it turned out later that bisemigroups could have been avoided and increased transparency been reached. Although some characterizations of exponential dichotomy were derived right from the dawn of its theory [16], it has been quite recent that more implementable characterizations have been derived [134, 38, 157].

The theory of exponential dichotomy as presented in this monograph and in its major input publications has been developed with almost total disregard of the theory of exponential dichotomy prevailing in the study of ordinary differential equations and functional differential equations. The latter theory consists of a plethora of applications to nonautonomous ordinary differential equations (see the bibliography of [142]) and functional differential equations [119, 84, 120] with various degrees of generality. Using the language of dynamical systems, Sacker and Sell have developed an umbrella theory of exponential dichotomy of linear evolution families, first in the finite-dimensional case [139, 140, 141] and more recently in infinite-dimensional Banach spaces [142]. At present, a theory of exponential dichotomy of linear evolution families within the tradition of the monograph by Chicone and Latushkin [44] on linear evolution families in complex Banach spaces awaits development.

Purpose, Limitations, and Readership

The purpose of this monograph is to provide a unified treatment of exponentially dichotomous operators and to discuss its major applications in detail. In Chapter 1 we introduce exponentially dichotomous operators, discuss their spectral properties, and outline the special cases pertaining to specific types of constituent semigroups. We also characterize (special kinds of) exponentially dichotomous operators in terms of the operator-valued function having its resolvent as a Fourier transform. In Chapter 2 we address the problem of proving that, under reasonable assumptions, a bounded additive perturbation of an exponentially dichotomous operator is exponentially dichotomous itself. This requires discussing Fourier transforms of Bochner and Pettis integrable functions with values in general Banach algebras. The most general perturbation results will be obtained in a Hilbert space setting, but still elude us in general Banach spaces (unless the perturbation is small enough in the operator norm). In Chapter 3 we generalize the theory of Cauchy problems governed by the infinitesimal generator of a strongly continuous semigroup to the bisemigroup setting. Chapters 4–8 are devoted to applications of exponentially dichotomous operators to algebraic Riccati equations, transport theory, indefinite Sturm-Liouville diffusion equations, noncausal infinite-dimensional systems, and functional differential equations of mixed type.

In this monograph we limit ourselves to linear autonomous equations with exponential dichotomy. Strongly continuous semigroups will be discussed only as a portal to their bisemigroup counterpart. Thus we reduce to the bare minimum the discussion of results on bisemigroups which can be transcribed directly from semigroup theory by passing through the constituent semigroups. We refrain from discussing discrete-time counterparts of bisemigroups (such as those introduced in [12]), linear evolution families, their exponentially dichotomous generalizations, and any applications to nonautonomous differential and functional differential equations altogether. We have selected applications, where (i) bisemigroups are really the way to go (thus excluding a discussion of inverse scattering on the line and linear integral equations with semi-separable kernels), and (ii) there exists enough established knowledge to formulate an umbrella theory of an extensive family of applications.

Though we have made a strenuous effort to make the book self-contained, it still requires a nonnegligible basic knowledge of functional analysis. Some of the necessary material on closed linear operators, strongly continuous semigroups, Banach algebras, selfadjoint operators, integration of vector-valued functions, and compactness in spaces of bounded continuous functions is outlined in the first chapter of this monograph. In Subsection 2.3.2 we outline Bochner and Pettis integration, although by necessity vector-valued integrals will already appear in Chapter 1 in a rather intuitive way. We refer to various textbooks for details.

The audience we have in mind consists of researchers and graduate students interested in acquiring basic knowledge on exponentially dichotomous operators and their major applications. Providing the material for a graduate course has not been our primary objective.

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