

Contents

Preface	ix
1 Complex and Real Algebras	1
1.1 Complex and Real C^* -Algebras	1
1.2 Real Extensions of Complex $*$ -Algebras	4
1.3 Uniqueness of Involution in Real Extensions of Complex $*$ -Algebras	7
1.4 Real and Complex Spectrum. Inverse Closedness	15
1.5 Moore-Penrose Invertibility in Algebra $\tilde{\mathcal{A}}$	20
1.6 Operator Sequences: Stability	24
1.7 Asymptotic Moore-Penrose Invertibility	31
1.8 Approximation Methods in Para-Algebras	38
1.9 Local Principles	41
1.9.1 Gohberg-Krupnik Local Principle	41
1.9.2 Allan's Local Principle	44
1.9.3 Local Principle for Para-algebras	45
1.10 Singular Integral and Mellin Operators	47
1.10.1 Singular Integrals	47
1.10.2 The Algebra $\sum^{\mathcal{P}}(\alpha)$	49
1.10.3 Integral Representations	51
1.10.4 The Algebra $\sum^{\mathcal{P}}(\alpha, \beta, \nu)$	52
1.10.5 Singular Integral Operators with Piecewise Continuous Coefficients	53
1.10.6 Singular Integral Operators with Conjugation	56
1.11 Comments and References	57
2 Integral Operators. Smooth Curves	59
2.1 Polynomial Galerkin Method	60
2.2 Polynomial Collocation Method	65
2.3 Equations with Differential Operators	69
2.3.1 Singular Integro-Differential Equations	69
2.3.2 Approximate Solution of Singular Integro-Differential Equations with Conjugation	72

2.4	A C^* -Algebra of Operator Sequences	73
2.4.1	Moore-Penrose Invertibility	86
2.5	The ε -Collocation Method	86
2.6	Spline Galerkin Method	91
2.7	Quadrature Methods on the Unit Circle	93
2.8	Polynomial Qualocation Method	100
2.9	Spline Qualocation Method	104
2.10	Quadrature Methods on Closed Smooth Curves	113
2.11	A Remark on Tensor Product Techniques	117
2.12	Comments and References	119
3	Riemann-Hilbert Problem	123
3.1	Galerkin Method	123
3.2	Interpolation Method. Continuous Coefficients	131
3.3	Local Principle for the Para-Algebra \mathcal{A}/\mathcal{J}	136
3.4	Discontinuous Coefficients	147
3.5	Generalized Riemann-Hilbert-Poincaré Problem	152
3.6	Comments and References	154
4	Piecewise Smooth and Open Contours	157
4.1	Stability of Approximation Methods	157
4.1.1	Quadrature Methods for Singular Integral Equations with Conjugation	158
4.1.2	The ε -Collocation Method	168
4.1.3	Qualocation Method	170
4.2	Fredholm Properties of Local Operators	171
4.2.1	Quadrature Methods	172
4.2.2	The ε -Collocation Method	176
4.2.3	Qualocation Method	179
4.3	L_2 -Spaces with Weight	180
4.3.1	A Quadrature Method and Its Stability	180
4.3.2	Local Operators and Their Indices	184
4.3.3	The Vanishing of the Index of Local Operators	196
4.3.4	Modification of the Quadrature Method by Cutting Off Corner Singularities	199
4.4	Double Layer Potential Equation	202
4.4.1	Quadrature Methods	203
4.4.2	The ε -Collocation Method	206
4.4.3	Qualocation Method	207
4.5	Mellin Operators with Conjugation	208
4.5.1	A Quadrature Method	210
4.5.2	Fredholm Properties of Local Operators	213
4.5.3	Index of the Local Operators	218
4.5.4	Examples of the Essential Spectrum of Local Operators	226

4.6	Comments and References	230
5	Muskhelishvili Equation	233
5.1	Boundary Problems for the Biharmonic Equation	233
5.1.1	Reduction of Biharmonic Problems to a Boundary Problem for Two Analytic Functions	234
5.1.2	Reduction of Boundary Problems for Two Analytic Functions to Integral Equations	241
5.2	Fredholm Properties, Invertibility	244
5.3	Approximations on Smooth Contours	256
5.3.1	Galerkin Method	257
5.3.2	The ϵ -Collocation Method	259
5.3.3	Qualocation Method	261
5.3.4	Biharmonic Problem	262
5.4	Approximation Methods on Special Contours	265
5.5	Galerkin Method, Piecewise Smooth Contour	270
5.6	Comments and References	272
6	Numerical Examples	275
6.1	Muskhelishvili Equation	275
6.2	Biharmonic Problem	281
	Bibliography	285
	Index	303



<http://www.springer.com/978-3-7643-8750-1>

Approximation of Additive Convolution-Like Operators

Real C*-Algebra Approach

Didenko, V.; Silbermann, B.

2008, XII, 306 p., Softcover

ISBN: 978-3-7643-8750-1

A product of Birkhäuser Basel