Preface

This book is designed to present some recent results on some nonlinear parabolic-hyperbolic coupled systems arising from physics, mechanics and material science such as the compressible Navier-Stokes equations, thermo(visco)elastic systems and elastic systems. Some of the content of this book is based on research carried out by the author and his collaborators in recent years. Most of it has been previously published only in original papers, and some of the material has never been published until now. Therefore, the author hopes that the book will benefit both the interested beginner in the field and the expert.

All the models under consideration in Chapters 2–10 are built on nonlinear evolution equations that are parabolic-hyperbolic coupled systems of partial differential equations with time t as one of the independent variables. This type of partial differential equations arises not only in many fields of mathematics, but also in other branches of science such as physics, mechanics and materials science, etc. For example, some models studied in this book, such as the compressible Navier-Stokes equations (a $1D$ heat conductive viscous real gas and a polytropic ideal gas) from fluid mechanics, and thermo(visco)elastic systems from materials science, are typical examples of nonlinear evolutionary equations.

It is well known that the properties of solutions to nonlinear parabolic-hyperbolic coupled systems are very different from those of parabolic or hyperbolic equations. Since the 1970s, more and more mathematicians have begun to focus their interests on the study of local well-posedness, global well-posedness and blow-up of solutions in a finite time. Local well-posedness means that, for any given initial datum, a solution exists locally in time, and if it exists locally in time, it is unique and stable in some sense in the considered class. Generally speaking, we have two powerful tools to derive the local existence of solutions to a wide of class of nonlinear evolutionary equations, i.e., the contraction mapping theorem and the Leray-Schauder fixed point theorem. Once a local solution in some sense has been established, we may talk about the global well-posedness of solutions, i.e., the global-in-time existence, uniqueness and stability of global solutions. Since the 1960s, many methods of studying global well-posedness have been developed, among which are two powerful tools to derive the global existence of solutions; one is continuation of local solutions, the other is the global iteration method.

In the 1980s, more interest was focused on the global existence of “small solutions”. However, knowledge about the global existence of a “small solution” is usually far from being enough for physical and mechanical problems. Thus we have to look for global solutions with arbitrary (not necessarily small) initial data. It turns out that an important step is to derive uniform a priori estimates on the solutions by using the special constitutive relations of the equations under consideration. Once global existence and uniqueness have been established, then the main interest should be focused on topics related to the asymptotic behavior of solutions, multiplicity of equilibria, convergence to an equilibrium, dynamical systems such as absorbing sets, the maximal compact attractor, etc. The study of asymptotic behavior of solutions can be divided into two categories. The first category comprises investigations of asymptotic behavior of the global solution for any *given* initial datum. The second category comprises investigations of asymptotic behavior of all solutions when the initial data vary in any bounded set. There are essential differences between these two categories. The first category deals with only one orbit starting from the datum in the phase space, while the second category deals with a family of orbits starting from any bounded set in the phase space.

For the basic theories of infinite-dimensional dynamical systems, we refer readers to the works by Babin [16], Babin and Vishik [17, 18], Ball [22, 23], Bernard and Wang [38], Chepyzhov, Gatti, Grasselli, Miranville and Pata [56], Chepyzhov and Vishik [57], Constantin and Foias [63], Constantin, Foias and Temam [64], Dlotko [84], Eden and Kalantarov [90], Edfendiev, Zelik and Miranville [92], Feireisl [97, 98, 100], Feireisl and Petzeltova [101, 102], Ghidaglia [117, 118], Ghidaglia and Temam [119], Goubet [125], Goubet and Moise [126], Hale [135], Hale and Perissinotto [136], Haraux [138], Hoff and Ziane [150, 151], Ladyzhenskaya [207], Liu and Zheng [240], Lu, Wu and Zhong [242], Ma, Wang and Zhong [246], Miranville [265, 266], Miranville and Wang [267], Moise and Rosa [269], Moise, Rosa and Wang [270], Pata and Zelik [307], Robinson [362], Rosa [363], Sell [369], Sell and You [370, 371], Temam [407], Vishik and Chepyzhov [413, 414], Wang [421], Wang, Zhong and Zhou [422], Wu and Zhong [429], Zhao and Zhou [445], Zheng [450], Zheng and Qin [451, 452], Zhong, Yang and Sun [457], and references therein.

There are 10 chapters in this book. Chapter 1 is a preliminary chapter in which we collect some basic results from nonlinear functional analysis, basic properties of Sobolev spaces, some differential and integral inequalities in analysis, the basic theory of semigroups of linear operators and the basic theory for global attractors. Some results in this chapter will be used in the subsequent chapters, other results, though not used in the subsequent chapters, will be very beneficial to the readers for further study.

The first topic studied in this book is compressible Navier-Stokes equations which describe the fluid motion of conservation of mass, momentum and energy. Chapters 2–5 are devoted to the study of this challenging topic. Chapter 2 will concern the global existence, asymptotic behavior of solutions and the existence of universal attractors for the compressible Navier-Stokes equations of a nonlinear $1D$ viscous and heat-conductive real gas. In Chapter 3, we shall establish the global existence, asymptotic behavior of solutions to initial boundary value problems and the Cauchy problem of the compressible Navier-Stokes equations of a $1D$ polytropic viscous and heat-conductive gas. In Chapter 4, we shall investigate the global existence, asymptotic behavior of solutions and the existence of maximal attractors for the compressible Navier-Stokes equations of a polytropic viscous and heat-conductive gas in bounded annular domains in $\mathbb{R}^n$ $(n = 2, 3)$. Chapter 5 will be concerned with the global existence and asymptotic behavior of solutions to a polytropic viscous and heat-conductive gas with cylinder symmetry in $\mathbb{R}^3$.

For the compressible Navier-Stokes equations, we consult the works by Duan, Yang and Zhu [87], Ducomet and Zlotnik [88], Feireisl and Petzeltova [103], Feireisl, Novotny and Petzeltova [104], Frid and Shelukhin [106], Fujita-Yashima and Benabidallah [110, 111], Fujita-Yashima, Padula and Novotny [112], Galdi [115], Hoff [142–146], Hoff and Serre [147], Hoff and Smoller [148], Hoff and Zarnowski [149], Hsiao and Luo [158], Huang, Matsumura and Xin [160], Itaya [161], Jiang [164–167, 169–171], Jiang and Zhang [174–177], Jiang and Zlotnik [178], Kanel [182], Kawashima [188, 189], Kawashima, Nishibata and Zhu [190], Kawashima and Nishida [191], Kawohl [192], Kazhikhov [193–195], LeFloch and Shelukhin [219], Lions [235], Matsumura [252], Matsumura and Nishida [253–257], Nagasawa [283–287], Novotny and Straškraba [301, 302], Okada and Kawashima [303], Padula [305], Qin [323, 325, 326], Qin and Hu [329], Qin, Huang and Ma [330], Qin and Jiang [331], Qin and Kong [332], Qin, Ma, Cavalcanti and Andrade [335], Qin, Ma and Huang [336], Qin, Muñoz Rivera [337, 339], Qin and Song [343], Qin and Wen [344], Qin, Wu and Liu [345], Qin and Zhao [346], Valli and Zajaczkowski [412], and the references therein.

The second topic studied in this book is a $1D$ thermoviscoelastic system which describes the motion of conservation of mass, momentum and energy in the thermoviscoelastic media. Chapter 6 will be devoted to the study of global existence, asymptotic behavior and the existence of universal attractors for a $1D$ thermoviscoelastic model in materials science.

The third topic considered in this book is that of some viscoelastic models. In Chapter 10, we shall obtain the large-time behavior of energy of multi-dimensional nonhomogeneous anisotropic elastic system.

For the related (thermo)(visco)elastic models, we refer to Andrews [12], Andrews and Ball [13], Chen and Hoffmann [54], Coleman and Gurtin [62], Dafermos [69, 75, 76], Dafermos and Nohel [79, 80], Fabrizio and Lazzari [95], Giorgi and Naso [121], Greenberg and MacCamy [129], Guo and Zhu [132], Kim [197], Lagnese [209], Liu and Zheng [239, 240], Niezgódka and Sprekels [293], Niezgódka, Zheng and Sprekels [294], Qin, Ma and Huang [336], Racke and Zheng [355], Renardy, Hrusa and Nohel [361], Shen and Zheng [373], Shen, Zheng and Zhu [376], Shibata [377], Sprekels and Zheng [390, 391], Sprekels, Zheng and Zhu [392], Watson [424], Zheng [447, 448, 450], Zheng and Shen [453, 454], Zhu [460], and the references therein.

The fourth topic under consideration is an investigation of a classical $1D$ thermoelastic model. Such a model describes the elastic and the thermal behavior of elastic, heat conductive media, in particular the reciprocal actions between elastic stresses and temperature differences. The classical thermoelastic system is such a thermoelastic model that the elastic part is the usual second-order one in the space variable and the heat flux obeys Fourier's law, which means that the heat flux is proportional to the temperature gradient. In Chapter 7, we shall establish the global existence and exponential stability of solutions to a $1D$ classical thermoelastic system of equations with a thermal memory. In

Chapter 9, we shall study the blowup phenomena of solutions to the Cauchy problem of a $1D$ non-autonomous classical thermoelastic system.

There is much literature on classical thermoelastic model; we refer the readers to Burns, Liu and Zheng [46], Dafermos [67], Dafermos and Hsiao [78], Hale and Perissinotto [136], Hansen [137], Hoffmann and Zochowski [153], Hrusa and Messaoudi [155], Hrusa and Tarabek [156], Jiang, Muñoz Rivera and Racke [172], Jiang and Racke [173], Kim [198], Kirane and Kouachi and Tatar [199], Kirane and Tatar [200], Lebeau and Zuazua [216], Liu and Zheng [238, 240], Messaoudi [260], Muñoz Rivera [274, 275], Muñoz Rivera and Barreto [277], Muñoz Rivera and Oliveira [278], Muñoz Rivera and Qin [279], Qin [315], Qin and Muñoz Rivera [341], Racke [348], Racke and Zheng [355], Slemord [378], Zheng [450], and the references therein.

Recently, Green and Naghdi [127, 128] re-examined the classical thermoelastic models and introduced the so-called models of thermoelasticity of types II and III for which the heat fluxes are different from Fourier's law. Chapter 8 will concern the global existence and exponential stability of solutions to the $1D$ thermoelastic equations of hyperbolic type, which is in fact a $1D$ thermoelastic system of type II with a thermal memory.

We consult the works by Messaoudi [261], Racke [350, 351], Racke and Wang [354] for thermoelastic models with second sound, which means that the heat flux is given by Cattaneo's law (i.e., the heat flux q satisfies $\tau q_t + q + \kappa\nabla\theta = 0$ with $\tau > 0, \kappa > 0$ constants), instead of Fourier's law of the classical thermoelastic models in which $\tau = 0$. For the thermoelastic models of type II, we refer to the works by Green and Naghdi [127, 128], Gurtin and Pipkin [133], and Qin and Muñoz Rivera [340], and the references therein. For the thermoelastic models of type III, we refer to the works by Green and Naghdi [127, 128], Quintanilla and Racke [347], Reissig and Wang [360], and Zhang and Zuazua [444], and the references therein.

I sincerely hope that readers will learn the main ideas and essence of the basic theories and methods in deriving global well-posedness, asymptotic behavior and existence of global (universal) attractors for the models under consideration in this book. Also I hope that readers will be stimulated by some ideas from this book and undertake further study and research after having read the related references.

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