

4.8 Lösungen zu den Aufgaben

L 4.1

$$\begin{aligned} \text{a)} \quad G(t_1, t_2) &= \langle z(t_1) z^*(t_2) \rangle \\ &= \langle \exp[-j\Phi(t_1)] \exp[j\Phi(t_2)] \rangle \\ &= \langle \exp[-j\Delta\Phi] \rangle \\ &= \int_{-\infty}^{\infty} \exp[-j\Delta\Phi] f(\Delta\Phi) d(\Delta\Phi) \\ &= \frac{1}{\sqrt{2\pi\Delta\omega|\tau|}} \int_{-\infty}^{\infty} \exp[-j\Delta\Phi] \exp\left[\frac{-\Delta\Phi^2}{2\Delta\omega|\tau|}\right] d(\Delta\Phi) \\ G(t_1, t_2) &= \exp\left[-\frac{\Delta\omega|\tau|}{2}\right] = R(\tau) \end{aligned} \quad (\text{L 4.1})$$

b)

$$\begin{aligned}
 S(\omega) &= \int_{-\infty}^{\infty} R(\tau) \exp(-j\omega\tau) d\tau \\
 &= \int_{-\infty}^{\infty} \exp\left[-\frac{\Delta\omega|\tau|}{2}\right] \exp(-j\omega\tau) d\tau \\
 &= \int_{-\infty}^0 \exp\left[\left(\frac{\Delta\omega}{2} - j\omega\right)\tau\right] d\tau \\
 &\quad + \int_0^{\infty} \exp\left[-\left(\frac{\Delta\omega}{2} + j\omega\right)\tau\right] d\tau
 \end{aligned}$$

$$\underline{\underline{S(\omega) = \frac{4}{\Delta\omega} \cdot \frac{1}{1 + \left(\frac{\omega}{\Delta\omega/2}\right)^2}}}$$

(L 4.2)

L 4.2

a)

$$\underline{G}_{\text{in}}(t_1, t_2) = \langle \vec{D}_{\text{in}}(t_1) \vec{D}_{\text{in}}^*(t_2) \rangle = \hat{D}_0^2 \exp(j\omega_0 \tau) \langle \exp[-j\Delta\Phi] \rangle \cdot$$
$$\begin{pmatrix} |e_x|^2 & |e_x||e_y| \exp[-j(\psi_x - \psi_y)] \\ |e_x||e_y| \exp[j(\psi_x - \psi_y)] & |e_y|^2 \end{pmatrix}, \quad (\text{L 4.3})$$

$$\langle \exp[-j\Delta\Phi] \rangle = \exp\left[-\frac{\Delta\omega|\tau|}{2}\right] \quad (\text{L 4.4})$$

$$\underline{R}_{in}(\tau) = \underline{G}_{in}(t_1, t_2) = \underline{G}_{in}(t_1 - t_2) = \hat{D}_0^2 \exp(j\omega_0 \tau) \exp\left[-\frac{\Delta\omega |\tau|}{2}\right].$$

$$\underbrace{\begin{pmatrix} |e_x|^2 & |e_x||e_y|\exp[-j(\psi_x - \psi_y)] \\ |e_x||e_y|\exp[j(\psi_x - \psi_y)] & |e_y|^2 \end{pmatrix}}_{=\underline{R}_{ino}}. \quad (L 4.5)$$

$$\underline{R}_{in}(\omega) = \int_{-\infty}^{\infty} \underline{R}_{in}(\tau) \exp(-j\omega\tau) d\tau$$

$$\underline{R}_{in}(\omega) = \frac{4}{\Delta\omega} \cdot \frac{\hat{D}_0^2}{1 + \left(\frac{\omega - \omega_0}{\Delta\omega/2}\right)^2} \begin{pmatrix} |e_x|^2 & |e_x||e_y|\exp[-j(\psi_x - \psi_y)] \\ |e_x||e_y|\exp[j(\psi_x - \psi_y)] & |e_y|^2 \end{pmatrix}$$

(L 4.6)

b) $\underline{R}_{\text{out}}(\omega) = \underline{I}(j\omega) \underline{R}_{\text{in}}(\omega) \underline{I}'^*(j\omega)$

$$\underline{R}_{\text{out}}(\omega) = \begin{pmatrix} \tilde{R}_{\text{out}}^{11}(\omega) & \tilde{R}_{\text{out}}^{12}(\omega) \\ \tilde{R}_{\text{out}}^{21}(\omega) & \tilde{R}_{\text{out}}^{22}(\omega) \end{pmatrix} \frac{4}{\Delta\omega} \cdot \frac{\hat{D}_0^2}{1 + \left(\frac{\omega - \omega_0}{\Delta\omega/2}\right)^2}$$

$$\begin{aligned} \tilde{R}_{\text{out}}^{11}(\omega) = & \left| e_x \right|^2 \cos^2\left(\frac{\omega\tau_0}{2}\right) \\ & + 2 \left| e_x \right| \left| e_y \right| \sin(\psi_x - \psi_y) \cos\left(\frac{\omega\tau_0}{2}\right) \sin\left(\frac{\omega\tau_0}{2}\right) \\ & + \left| e_y \right|^2 \sin^2\left(\frac{\omega\tau_0}{2}\right) \end{aligned}$$

$$\begin{aligned}
\tilde{R}_{\text{out}}^{12}(\omega) = & j \left[|e_x|^2 - |e_y|^2 \right] \cos\left(\frac{\omega\tau_o}{2}\right) \sin\left(\frac{\omega\tau_o}{2}\right) \\
& + |e_x| |e_y| \exp\left[j(\psi_x - \psi_y)\right] \sin^2\left(\frac{\omega\tau_o}{2}\right) \\
& + |e_x| |e_y| \exp\left[-j(\psi_x - \psi_y)\right] \cos^2\left(\frac{\omega\tau_o}{2}\right) \\
\tilde{R}_{\text{out}}^{21}(\omega) = & \tilde{R}_{\text{out}}^{12*}(\omega) \quad (\text{L 4.7})
\end{aligned}$$

$$\begin{aligned}
\tilde{R}_{\text{out}}^{22}(\omega) = & |e_x|^2 \sin^2\left(\frac{\omega\tau_o}{2}\right) \\
& - 2 |e_x| |e_y| \sin(\psi_x - \psi_y) \cos\left(\frac{\omega\tau_o}{2}\right) \sin\left(\frac{\omega\tau_o}{2}\right) \\
& + |e_y|^2 \cos^2\left(\frac{\omega\tau_o}{2}\right)
\end{aligned}$$

c)

$$\text{sp} \left[\underline{R}_{\text{out}}(\omega) \right] = \frac{4}{\Delta \omega} \cdot \frac{\hat{D}_0^2}{1 + \left(\frac{\omega - \omega_0}{\Delta \omega / 2} \right)^2} \underbrace{\left[\tilde{R}_{\text{out}}^{11}(\omega) + \tilde{R}_{\text{out}}^{22}(\omega) \right]}_{=1}$$

$$\rightarrow \underline{\underline{\text{sp} \left[\underline{R}_{\text{out}}(\omega) \right] = \frac{4}{\Delta \omega} \cdot \frac{\hat{D}_0^2}{1 + \left(\frac{\omega - \omega_0}{\Delta \omega / 2} \right)^2}}} \quad (\text{L 4.8})$$

d)

$$\langle I_{\text{out}} \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{sp} [R_{\text{out}}(\omega)] d\omega = \frac{\hat{D}_0^2}{2\pi} \cdot \underbrace{\frac{4}{\Delta\omega} \int_{-\infty}^{\infty} \frac{d\omega}{1 + \left(\frac{\omega - \omega_0}{\Delta\omega/2} \right)^2}}_{=2\pi}$$

$$\underline{\underline{\langle I_{\text{out}} \rangle = \hat{D}_0^2}} \quad (\text{L 4.9})$$

L 4.3

a)

$$D_{\text{zin}}(t) = D_{\text{yin}}(t) \tan \varphi \quad (\text{L 4.10})$$

$$\underline{\underline{D_{\text{zin}}(t) = \hat{D}_o |e_y| \tan \varphi \exp \left[j \left(\omega_o t - \Phi(t) - \psi_y \right) \right]}}$$

$$\begin{aligned}
 \text{b) } G_{\text{zin}}(t_1, t_2) &= \langle D_{\text{zin}}(t_1) D_{\text{zin}}^*(t_2) \rangle \\
 &= \hat{D}_0^2 |e_y|^2 \tan^2 \varphi \exp(j\omega_0 \tau) \langle \exp[-j\Delta\Phi] \rangle
 \end{aligned}$$

mit $\tau = t_1 - t_2$ und $\Delta\Phi = \Phi(t_1) - \Phi(t_2)$,

$$\langle \exp[-j\Delta\Phi] \rangle = \exp\left[-\frac{\Delta\omega |\tau|}{2}\right], \quad (\text{L 4.12})$$

$$G_{\text{zin}}(t_1, t_2) = R_{\text{zin}}(\tau) \quad (\text{L 4.13})$$

$$\underline{\underline{R_{\text{zin}}(\tau) = \hat{D}_0^2 |e_y|^2 \tan^2 \varphi \exp\left[j\omega_0 \tau - \frac{\Delta\omega |\tau|}{2}\right]}} \quad (\text{L 4.14})$$

c)

$$R_{\text{zin}}(\omega) = \int_{-\infty}^{\infty} R_{\text{zin}}(\tau) \exp(-j\omega\tau) d\tau$$

$$= \hat{D}_0^2 |e_y|^2 \tan^2 \varphi \int_{-\infty}^{\infty} \exp\left[-\frac{\Delta\omega |\tau|}{2}\right] \exp[-j(\omega - \omega_0)\tau] d\tau$$

$$R_{\text{zin}}(\omega) = \hat{D}_0^2 |e_y|^2 \tan^2 \varphi \frac{4}{\Delta\omega} \frac{1}{1 + \left(\frac{\omega - \omega_0}{\Delta\omega/2}\right)^2} \quad (\text{L 4.15})$$

d) $\underline{R}_{\text{out}}(\omega) = \underline{I}(j\omega) \underline{R}_{\text{in}}(\omega) \underline{I}'^*(j\omega),$

$$\underline{R}_{\text{in}}(\omega) = \frac{4}{\Delta\omega} \cdot \frac{\hat{D}_0^2}{1 + \left(\frac{\omega - \omega_0}{\Delta\omega/2}\right)^2} \cdot \begin{pmatrix} |e_x|^2 & |e_x| |e_y| \exp[-j(\psi_x - \psi_y)] \\ |e_x| |e_y| \exp[j(\psi_x - \psi_y)] & |e_y|^2 \end{pmatrix}$$

$$\underline{R}_{\text{out}}(\omega) = \frac{4}{\Delta\omega} \cdot \frac{\hat{D}_0^2}{1 + \left(\frac{\omega - \omega_0}{\Delta\omega/2}\right)^2} \cdot \begin{pmatrix} |e_x|^2 \exp(a_0) & |e_x| |e_y| \exp[j(\omega\tau_0 - \psi_x + \psi_y)] \\ |e_x| |e_y| \exp[-j(\omega\tau_0 - \psi_x + \psi_y)] & |e_y|^2 \exp(-a_0) \end{pmatrix}$$

$$R_{\text{zout}}(\omega) = R_{\text{out}}^{22}(\omega) \tan^2 \varphi \quad (\text{L 4.16})$$

$$R_{\text{zout}}(\omega) = \hat{D}_0^2 \exp(-a_0) |e_y|^2 \tan^2 \varphi \frac{4}{\Delta \omega} \cdot \frac{1}{1 + \left(\frac{\omega - \omega_0}{\Delta \omega / 2}\right)^2} \quad (\text{L 4.17})$$

e)

$$R_{\text{zout}}(\omega) = |T_z(j\omega)|^2 R_{\text{zin}}(\omega),$$

$$T_z(j\omega) = \exp \left[- \left(\frac{a_0}{2} + j \frac{\omega \tau_0}{2} \right) \right],$$

$$|T_z(j\omega)|^2 = \exp(-a_0),$$

$$R_{\text{zin}}(\omega) = \hat{D}_0^2 |e_y|^2 \tan^2 \varphi \frac{4}{\Delta \omega} \cdot \frac{1}{1 + \left(\frac{\omega - \omega_0}{\Delta \omega / 2} \right)^2},$$

$$R_{\text{zout}}(\omega) = \hat{D}_0^2 \exp(-a_0) |e_y|^2 \tan^2 \varphi \frac{4}{\Delta \omega} \cdot \frac{1}{1 + \left(\frac{\omega - \omega_0}{\Delta \omega / 2} \right)^2} \quad (\text{L 4.18})$$

f)

$$\begin{aligned}\langle I_{\text{zout}} \rangle &= \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{\text{zout}}(\omega) d\omega \\ &= \frac{\hat{D}_0^2 \exp(-a_0) |e_y|^2 \tan^2 \varphi}{2\pi} \cdot \underbrace{\frac{4}{\Delta \omega} \int_{-\infty}^{\infty} \frac{d\omega}{1 + \left(\frac{\omega - \omega_0}{\Delta \omega / 2} \right)^2}}_{=2\pi}\end{aligned}$$

$$\rightarrow \underline{\underline{\langle I_{\text{zout}} \rangle = \hat{D}_0^2 \exp(-a_0) |e_y|^2 \tan^2 \varphi}} \quad (\text{L 4.19})$$

L 4.4

a)

$R_{\text{out}}^{\text{ew}} =$

$$= \begin{pmatrix} T_x |e_x|^2 T_x^* & T_x |e_x| |e_y| \exp(j\psi) T_y^* & T_x |e_x| |e_y| \tan\varphi \exp(j\psi) T_z^* \\ T_y |e_x| |e_y| \exp(-j\psi) T_x^* & T_y |e_y|^2 T_y^* & T_y |e_y|^2 \tan\varphi T_z^* \\ T_z |e_x| |e_y| \tan\varphi \exp(-j\psi) T_x^* & T_z |e_y|^2 \tan\varphi T_y^* & T_z |e_y|^2 \tan^2\varphi T_z^* \end{pmatrix}$$

(L 4.20)

b)

$$\underline{R}_{\text{outo}}^{\text{erw}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \cos^2 \varphi & \cos \varphi \sin \varphi \\ 0 & \cos \varphi \sin \varphi & \sin^2 \varphi \end{pmatrix} = \underline{R}_{\text{ino}}^{\text{erw}} \quad (\text{L 4.21})$$

$$\begin{aligned}
 \text{c) } & \begin{vmatrix} -\lambda & 0 & 0 \\ 0 & \cos^2 \varphi - \lambda & \cos \varphi \sin \varphi \\ 0 & \cos \varphi \sin \varphi & \sin^2 \varphi - \lambda \end{vmatrix} = 0 \\
 & \lambda \begin{vmatrix} \cos^2 \varphi - \lambda & \cos \varphi \sin \varphi \\ \cos \varphi \sin \varphi & \sin^2 \varphi - \lambda \end{vmatrix} = 0 \\
 & \lambda \left[(\cos^2 \varphi - \lambda)(\sin^2 \varphi - \lambda) - \cos^2 \varphi \sin^2 \varphi \right] = 0 \\
 & \lambda \left[\lambda^2 - \underbrace{(\cos^2 \varphi + \sin^2 \varphi)}_{=1} \lambda \right] = 0 \\
 & \lambda^2 (\lambda - 1) = 0 \qquad \qquad \qquad (\text{L 4.22})
 \end{aligned}$$

Eigenwerte:

$$\rightarrow \underline{\underline{\lambda_1 = 0}}, \underline{\underline{\lambda_2 = 0}}, \underline{\underline{\lambda_3 = 1}} \qquad \qquad \qquad (\text{L 4.23})$$

$$d) \quad \lambda_{1,2} = 0 : \text{Rang } \underline{R}_{\text{outo}}^{\text{erw}} = 1$$

$$\begin{pmatrix} 0 & \cos^2 \varphi & \cos \varphi \sin \varphi \end{pmatrix} \begin{pmatrix} x_{1,2} \\ y_{1,2} \\ z_{1,2} \end{pmatrix} = 0 \quad (\text{L 4.24})$$

Eigenvektoren:

$$\underline{\underline{\vec{b}_1 = \begin{pmatrix} x_1 \\ -z_1 \tan \varphi \\ z_1 \end{pmatrix}}}, \quad \underline{\underline{\vec{b}_2 = \begin{pmatrix} x_2 \\ -z_2 \tan \varphi \\ z_2 \end{pmatrix}}} \quad (\text{L 4.25})$$

$$\lambda_3 = 1: \quad \text{Rang}(\underline{R}_{\text{outo}}^{\text{erw}} - \underline{E}) = 2$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\sin^2 \varphi & \cos \varphi \sin \varphi \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (\text{L 4.26})$$

Eigenvektor:

$$\underline{\underline{\vec{b}_3 = z_3 \begin{pmatrix} 0 \\ \cot \varphi \\ 1 \end{pmatrix}}} \quad (\text{L 4.27})$$

e)

Unitäre Eigenvektoren:

$$\vec{n}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad \vec{n}_2 = \begin{pmatrix} 0 \\ -\sin \varphi \\ \cos \varphi \end{pmatrix}; \quad \vec{n}_3 = \begin{pmatrix} 0 \\ \cos \varphi \\ \sin \varphi \end{pmatrix} \quad (\text{L 4.28})$$

Transformationsmatrix:

$$\underline{\underline{B}}_{\text{out}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\sin \varphi & \cos \varphi \\ 0 & \cos \varphi & \sin \varphi \end{pmatrix} = \underline{\underline{B}}_{\text{out}}'^* = \underline{\underline{B}}_{\text{out}}' \quad (\text{L 4.29})$$

f)

$$\underline{R}_{\text{outo}}^{\text{derw}} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underline{R}_{\text{out}}^{\text{derw}}(\omega) = \frac{4}{\Delta\omega} \cdot \frac{\hat{D}_0^2}{1 + \left(\frac{\omega - \omega_0}{\Delta\omega/2}\right)^2} \underline{R}_{\text{outo}}^{\text{derw}}$$

$$\underline{R}_{\text{out}}^{\text{derw}}(\omega) = \frac{4}{\Delta\omega} \cdot \frac{\hat{D}_0^2}{1 + \left(\frac{\omega - \omega_0}{\Delta\omega/2}\right)^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\text{L 4.31})$$

g)

$$\underline{\underline{R_{\text{zout}}^{\text{d}}(\omega) = \frac{4}{\Delta\omega} \cdot \frac{\hat{D}_0^2}{1 + \left(\frac{\omega - \omega_0}{\Delta\omega/2}\right)^2}}} \quad (\text{L 4.32})$$

$$\vec{D}_{\text{out}}^{\text{d}}(j\omega) = \underline{B}_{\text{out}}' \vec{D}_{\text{out}}(j\omega), \quad (\text{L 4.33})$$

$$\varepsilon_1 = \varepsilon_3 : D_{\text{yout}}(j\omega) = D_{\text{zout}}(j\omega) \cot \varphi \quad (\text{L 4.34})$$

$$\begin{pmatrix} D_{\text{xout}}^{\text{d}}(j\omega) \\ D_{\text{yout}}^{\text{d}}(j\omega) \\ D_{\text{zout}}^{\text{d}}(j\omega) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{D_{\text{zout}}(j\omega)}{\sin \varphi} \end{pmatrix} \quad (\text{L 4.35})$$

$$\rightarrow \underline{\underline{D_{\text{zout}}(j\omega) = D_{\text{zout}}^{\text{d}}(j\omega) \sin \varphi}} \quad (\text{L 4.36})$$

$$R_{\text{zout}}(\omega) = \langle D_{\text{zout}}(j\omega) D_{\text{zout}}^*(j\omega) \rangle$$

$$= \langle D_{\text{zout}}^{\text{d}}(j\omega) D_{\text{zout}}^{*\text{d}}(j\omega) \rangle \sin^2 \varphi$$

$$\rightarrow \underline{\underline{R_{\text{zout}}(\omega) = R_{\text{zout}}^{\text{d}} \sin^2 \varphi}} \quad (\text{L 4.37})$$

$$\underline{\underline{R_{\text{zout}} = \frac{4}{\Delta\omega} \cdot \frac{\hat{D}_0^2 \sin^2 \varphi}{1 + \left(\frac{\omega - \omega_0}{\Delta\omega/2}\right)^2}}} \quad (\text{L 4.38})$$

Gemäß L 4.36 kann mit $D_{\text{zout}}^{\text{d}}(j\omega) = D_{y''\text{out}}(j\omega)$ festgestellt werden, dass die Diagonalisierung der ausgangsseitigen Kohärenzmatrix bei horizontaler Eingangspolarisation schon bei Verwendung der schrägen Komponente $D_{y''\text{out}}(j\omega)$ im x'', y'', z'' -Koordinatensystem am Ausgang gegeben ist.

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