

5.4 Lösungen zu den Aufgaben

L 5.1

a) Methode 1: Multiplikation der Jones-Matrizen

$$\begin{aligned}\underline{J} &= \underline{J}_2 \underline{J}_1 = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \\ &= \begin{pmatrix} \cosh \left[\frac{a_1 + a_2 + j\omega(\tau_1 + \tau_2)}{2} \right] & -\sinh \left[\frac{a_1 + a_2 + j\omega(\tau_1 + \tau_2)}{2} \right] \\ -\sinh \left[\frac{a_1 + a_2 + j\omega(\tau_1 + \tau_2)}{2} \right] & \cosh \left[\frac{a_1 + a_2 + j\omega(\tau_1 + \tau_2)}{2} \right] \end{pmatrix}\end{aligned}\quad (\text{L5.1})$$

Eigenpolarisationen:

$$\chi'_{\text{e}} \cos \varphi = \frac{J_{21} + J_{22} \chi'_{\text{e}} \cos \varphi}{J_{11} + J_{12} \chi'_{\text{e}} \cos \varphi}$$

$$\rightarrow \chi'_{\text{e}_{12}} \cos \varphi = \frac{1}{2 J_{12}} \left[J_{22} - J_{11} \pm \sqrt{(J_{22} - J_{11})^2 + 4 J_{12} J_{21}} \right] \quad (\text{L5.2})$$

(L5.3)

$$\underline{\underline{\chi'_{\text{e}_{12}} \cos \varphi = \pm 1}}$$

$$T_z = \frac{J_{21}}{\chi'_{e} \cos \varphi} + J_{22}$$

$$T_{z\pm} = \cosh \left[\frac{a_1 + a_2 + j\omega(\tau_1 + \tau_2)}{2} \right] \pm \sinh \left[\frac{a_1 + a_2 + j\omega(\tau_1 + \tau_2)}{2} \right] \quad (\text{L5.4})$$

$$\rightarrow \underline{\underline{T_{z\pm} = \exp \left[\pm \frac{a_1 + a_2 + j\omega(\tau_1 + \tau_2)}{2} \right]}} \quad (\text{L5.5})$$

für $\chi'_{e_{2,1}} \cos \varphi = \mp 1$

b) Methode 2: Multiplikation der z-KÜF

$$T_{z1\pm} = \frac{J_{21}^1}{\chi'_{e_{2,1}} \cos \varphi} + J_{22}^1 = \cosh \left[\frac{a_1 + j\omega \tau_1}{2} \right] \pm \sinh \left[\frac{a_1 + j\omega \tau_1}{2} \right]$$

$$\underline{\underline{T_{z1\pm} = \exp \left[\pm \frac{a_1 + j\omega \tau_1}{2} \right]}} \quad (L5.6)$$

$$T_{z2\pm} = \frac{J_{21}^2}{\chi'_{out} \cos \varphi} + J_{22}^2$$

$$\begin{aligned}
 \chi'_{\text{out}} \cos \varphi &= \frac{J_{21}^1 + J_{22}^1 \chi'_{e_{2,1}} \cos \varphi}{J_{11}^1 + J_{12}^1 \chi'_{e_{2,1}} \cos \varphi} \\
 &= \frac{-\sinh \left[\frac{a_1 + j\omega \tau_1}{2} \right] \mp \cosh \left[\frac{a_1 + j\omega \tau_1}{2} \right]}{\cosh \left[\frac{a_1 + j\omega \tau_1}{2} \right] \pm \sinh \left[\frac{a_1 + j\omega \tau_1}{2} \right]} \\
 &= \mp 1 = \chi'_{e_{2,1}} \cos \varphi
 \end{aligned}
 \tag{L5.7}$$

$$\begin{aligned}
 T_{z2\pm} &= \cosh \left[\frac{a_2 + j\omega \tau_2}{2} \right] \pm \sinh \left[\frac{a_2 + j\omega \tau_2}{2} \right] \\
 \underline{\underline{T_{z2\pm} &= \exp \left[\pm \frac{a_2 + j\omega \tau_2}{2} \right]}}
 \end{aligned}
 \tag{L5.8}$$

$$\rightarrow \underline{\underline{T_{z\pm} = T_{z2\pm} T_{z1\pm} = \exp\left[\pm \frac{a_1 + a_2 + j\omega(\tau_1 + \tau_2)}{2}\right]}} \quad (\text{L5.9})$$

für $\chi'_{e_{2,1}} \cos \varphi = \mp 1$

L 5.2

a) Methode 1: Addition der Jones-Matrizen

$$\underline{\mathbf{J}} = \underline{\mathbf{J}}_1 + \underline{\mathbf{J}}_2 = \begin{pmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{pmatrix} = \left[\exp(-j\omega\tau_1) + \exp(-j\omega\tau_2) \right] \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (\text{L5.10})$$

$$\mathbf{T}_z = \frac{\mathbf{J}_{21}}{\chi'_{\text{in}} \cos \varphi} + \mathbf{J}_{22} = \mathbf{J}_{22}$$

$$\rightarrow \underline{\underline{\mathbf{T}_z = \exp(-j\omega\tau_1) + \exp(-j\omega\tau_2)}} \quad (\text{L5.11})$$

b) Methode 2: Addition der z-KÜF

$$T_{z1} = \frac{J_{21}^1}{\chi'_{\text{in}} \cos \varphi} + J_{22}^1 = \exp(-j\omega\tau_1) \quad (\text{L5.12})$$

$$T_{z2} = \frac{J_{21}^2}{\chi'_{\text{in}} \cos \varphi} + J_{22}^2 = \exp(-j\omega\tau_2) \quad (\text{L5.13})$$

$$\rightarrow \underline{\underline{T_z = T_{z1} + T_{z2} = \exp(-j\omega\tau_1) + \exp(-j\omega\tau_2)}} \quad (\text{L5.14})$$

L 5.3

a) Methode 1: Multiplikation der Jones-Matrizen

$$\underline{\underline{J}} = (\underline{\underline{E}} - \underline{\underline{J}}_2)^{-1} \underline{\underline{J}}_1 = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}$$

$$\underline{\underline{J}} = \frac{\exp(-j\omega\tau_1)}{1 - \exp(-j\omega\tau_2)} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (\text{L5.15})$$

$$T_z = \frac{J_{21}}{\chi'_{\text{in}} \cos \varphi} + J_{22} = J_{22}$$

$$\rightarrow \underline{\underline{T}}_z = \frac{\exp(-j\omega\tau_1)}{1 - \exp(-j\omega\tau_2)} \quad (\text{L5.16})$$

b) Methode 2: Rückkopplungsformel

$$T_{z1} = \frac{J_{21}^1}{\chi'_{\text{in}} \cos \varphi} + J_{22}^1 = \exp(-j\omega\tau_1) \quad (\text{L 5.17})$$

$$T_{z2} = \frac{J_{21}^2}{\chi'_{\text{out}} \cos \varphi} + J_{22}^2 = \exp(-j\omega\tau_2) \quad (\text{L 5.18})$$

mit $\chi'_{\text{out}} \cos \varphi \rightarrow \infty$

$$\rightarrow \underline{\underline{T_z = \frac{T_{z1}}{1 - T_{z2}} = \frac{\exp(-j\omega\tau_1)}{1 - \exp(-j\omega\tau_2)}}} \quad (\text{L 5.19})$$

L 5.4

a) Methode 1: Reihenschaltungsformel

$$T_z = T_{z2} T_{z1} = \exp \left[-\frac{a_1 + a_2 + j\omega(\tau_1 + \tau_2)}{2} \right] \quad (\text{L 5.20})$$

$$\underline{\underline{|T_z|^2 = \exp[-(a_1 + a_2)]}} \quad (\text{L 5.21})$$

b) Methode 2: Betragsformel

$$|T_z|^2 = |T_{z2}|^2 |T_{z1}|^2 \quad (\text{L 5.22})$$

$$|T_{z1}|^2 = \exp[-a_1] \quad (\text{L 5.23})$$

$$|T_{z2}|^2 = \exp[-a_2] \quad (\text{L 5.24})$$

$$\underline{\underline{|T_z|^2 = \exp[-(a_1 + a_2)]}} \quad (\text{L 5.25})$$

L 5.5

a) Methode 1: Parallelschaltungsformel

$$T_z = T_{z1} + T_{z2} = \exp[-j\omega\tau_1] + \exp[-j\omega\tau_2] \quad (\text{L5.27})$$

$$|T_z|^2 = [\cos(\omega\tau_1) + \cos(\omega\tau_2)]^2 + [\sin(\omega\tau_1) + \sin(\omega\tau_2)]^2 \quad (\text{L5.28})$$

$$= 2[1 + \cos(\omega\tau_1)\cos(\omega\tau_2) + \sin(\omega\tau_1)\sin(\omega\tau_2)] \quad (\text{L5.29})$$

$$\underline{\underline{|T_z|^2 = 2[1 + \cos[\omega(\tau_1 - \tau_2)]]}} \quad (\text{L5.30})$$

b) Methode 2: Betragsformel

$$|T_z|^2 = |T_{z1}|^2 + |T_{z2}|^2 + 2|T_{z1}||T_{z2}|\cos(\varphi_{z1} - \varphi_{z2}) \quad (\text{L5.31})$$

$$|T_{z1}| = |T_{z2}| = 1, \quad \varphi_{z1} = \omega\tau_1, \quad \varphi_{z2} = \omega\tau_2 \quad (\text{L5.32})$$

$$\rightarrow \underline{\underline{|T_z|^2 = 2[1 + \cos[\omega(\tau_1 - \tau_2)]]}} \quad (\text{L5.33})$$

L 5.6

a) Methode 1: Rückkopplungsformel

$$T_z = \frac{T_{z1}}{1 - T_{z2}} \quad (\text{L 5.34})$$

$$T_z = \frac{\exp[-j\omega\tau_1]}{1 - \exp[-j\omega\tau_2]} \quad (\text{L 5.35})$$

$$|T_z|^2 = \frac{1}{[1 - \cos(\omega\tau_2)]^2 + \sin^2(\omega\tau_2)} \quad (\text{L 5.36})$$

$$\underline{\underline{|T_z|^2 = \frac{1}{2[1 - \cos(\omega\tau_2)]}}} \quad (\text{L 5.37})$$

b) Methode 2: Betragsformel

$$|T_z|^2 = \frac{|T_{z1}|^2}{1 - 2|T_{z2}|\cos\varphi_{z2} + |T_{z2}|^2} \quad (\text{L 5.38})$$

$$|T_{z1}| = |T_{z2}| = 1, \quad \varphi_{z2} = \omega\tau_2 \quad (\text{L 5.39})$$

$$\rightarrow \underline{\underline{|T_z|^2 = \frac{1}{2[1 - \cos(\omega\tau_2)]}}} \quad (\text{L 5.40})$$

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