Preface

Differential forms have been widely studied and used in many fields, such as physics, general relativity, theory of elasticity, quasiconformal analysis, electromagnetism, and differential geometry. They can be used to describe various systems of partial differential equations and to express different geometrical structures on manifolds. Hence, differential forms have become invaluable tools for many fields. One of the purposes of this monograph is to present a series of estimates and inequalities for differential forms, particularly, for the forms satisfying the homogeneous A-harmonic equations, or the nonhomogeneous A-harmonic equations, or the conjugate A-harmonic equations in $\mathbf{R}^n$, $n \geq 2$. These estimates and inequalities are critical tools to investigate the properties of solutions to the nonlinear differential equations and to control oscillatory behavior in domains or on manifolds. These results can be further used to explore the global integrability of differential forms and to estimate the integrals of differential forms. Throughout this monograph we always keep in our mind that differential forms are the extensions of functions (functions are 0-forms). Hence, all results about differential forms presented in this monograph remain valid for functions defined in $\mathbf{R}^n$.

In Chapter 1, we study various versions of the Hardy–Littlewood inequalities for differential forms satisfying the conjugate A-harmonic equation. We first introduce some definitions and notation related to differential forms, which will be used in this monograph. Then, we discuss different versions of the A-harmonic equations and weight classes. From Sections 1.5, 1.6, and 1.7, we present the local and global Hardy–Littlewood inequalities with different weights in John domains and $L^s(\mu)$-averaging domains, respectively. We also give the best integrable exponents in Section 1.8. Finally, we investigate the Hardy–Littlewood inequalities with Orlicz norms.

In Chapter 2, we concentrate on the L^p-estimates for solutions of the nonhomogeneous A-harmonic equation. We also extend these estimates to the

$A_r(\Omega)$-weighted cases. We conclude Chapter 2 with the global norm comparison inequalities and some applications to the compositions of operators.

Chapters 3 and 4 treat the Poincaré inequalities and the Caccioppoli inequalities, respectively. Specifically, we present the Poincaré inequalities with L^p-norms and Orlicz norms for differential forms. We provide some estimates for Green's operator and the projection operator. As applications of the Poincaré inequalities, we also obtain some estimates for Jacobians of the Sobolev mappings. We develop both local and global Caccioppoli-type estimates with different weights in a domain or on a manifold in Chapter 4. Roughly speaking, these estimates provide upper bounds for the L^s-norm of ∇u (if u is a function) or du (if u is a form) in terms of the L^s-norm of differential form u. We also discuss Caccioppoli-type estimates with Orlicz norms.

Chapters 5 and 6 are concerned with the imbedding inequalities and the reverse Hölder inequalities, respectively. The imbedding inequalities for functions can be found in almost every book on partial differential equations; see Sections 7.7 and 7.8 in [63], for example. Hence, we only study the imbedding inequalities for differential forms in Chapter 5. We also explore the imbedding inequalities for some operators applied to differential forms and discuss various weighted cases. In Chapter 6, various versions of the reverse Hölder inequalities are established.

Chapter 7 is devoted to the integral estimates for some related operators, such as the homotopy operator, Laplace–Beltrami operator, and the gradient operator. We also develop some estimates for the compositions of operators, including the Hardy–Littlewood maximal operator and the sharp maximal operator. We know that the Jacobian of a quasiconformal mapping satisfies a stronger estimate, the reverse Hölder inequality. Then, what kind of estimates can we expect for the Jacobian of a mapping in a Sobolev class? We discuss the integrability of Jacobians in Chapter 8. Finally, in Chapter 9, we develop norm comparison theorems related to BMO-norms and Lipschitz norms. We also prove that the integrability exponents described in the Lipschitz norm comparison theorem are the best possible.

This monograph presents an up-to-date account of the advances made in the study of inequalities for differential forms and will hopefully stimulate further research in this area.

We would like to express our deep gratitude to our colleagues and friends who gave us various help and support during the preparation of this monograph. In particular, we are grateful for the valuable discussions with Professor Janet Mills, Professor Wynne Guy, and Professor John Carter. During the preparation of this monograph, Professor Yuming Xing, Professor Bing Liu, and Professor Peilin Shi generously devoted considerable time and effort in reading various versions of the manuscript and giving us many precious and thoughtful suggestions which led to substantial improvements in the text. We also want to thank Ms. Damle Vaishali at Springer Verlag, New York, for her support and cooperation.

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August, 2008

Springer

http://www.springer.com/978-0-387-36034-8

Inequalities for Differential Forms
Agarwal, R.P.; Ding, S.; Nolder, C.
2009, XVI, 388 p., Hardcover
ISBN: 978-0-387-36034-8