

*I bring here all: what have I lived thru,
And that what keeps my soul alive,
My rectitude and aspirations,
And what have seen my own eyes.*
– Boris Pasternak, *The Waves*

Contents

Foreword by Branko Grünbaum	ix
Foreword by Peter D. Johnson	xi
Foreword by Cecil Rousseau	xiii
Acknowledgments	xv
Greetings to the Reader	xxvi
Part I Merry-Go-Round	1
1 A Story of Colored Polygons and Arithmetic Progressions	3
1.1 The Story of Creation	3
1.2 The Problem of Colored Polygons	4
1.3 Translation into the Tongue of APs	6
1.4 Prehistory	7
1.5 Completing the Go-Round	8
Part II Colored Plane	11
2 Chromatic Number of the Plane: The Problem	13
3 Chromatic Number of the Plane: An Historical Essay	21
4 Polychromatic Number of the Plane and Results Near the Lower Bound	32
5 De Bruijn–Erdős Reduction to Finite Sets and Results Near the Lower Bound	39
6 Polychromatic Number of the Plane and Results Near the Upper Bound	43
6.1 Stechkin’s 6-Coloring	43
6.2 Best 6-Coloring of the Plane	44
6.3 The Age of Tiling	47
7 Continuum of 6-Colorings of the Plane	50
8 Chromatic Number of the Plane in Special Circumstances	57
9 Measurable Chromatic Number of the Plane	60
9.1 Definitions	60
9.2 Lower Bound for Measurable Chromatic Number of the Plane . . .	60
9.3 Kenneth J. Falconer	65
10 Coloring in Space	67
11 Rational Coloring	72
Part III Coloring Graphs	77
12 Chromatic Number of a Graph	79
12.1 The Basics	79
12.2 Chromatic Number and Girth.	82
12.3 Wormald’s Application	86
13 Dimension of a Graph	88
13.1 Dimension of a Graph	88
13.2 Euclidean Dimension of a Graph	93
14 Embedding 4-Chromatic Graphs in the Plane	99
14.1 A Brief Overture	99

14.2 Attaching a 3-Cycle to Foundation Points in 3 Balls	101
14.3 Attaching a k -Cycle to a Foundation Set of Type $(a_1, a_2, a_3, 0)_\delta$. .	102
14.4 Attaching a k -Cycle to a Foundation Set of Type $(a_1, a_2, a_3, 1)_\delta$. .	104
14.5 Attaching a k -Cycle to Foundation Sets of Types $(a_1, a_2, 0, 0)_\delta$ and $(a_1, 0, a_3, 0)_\delta$	104
14.6 Removing Coincidences	106
14.7 O'Donnell's Embeddings	107
14.8 Appendix	108
15 Embedding World Records	110
15.1 A 56-Vertex, Girth 4, 4-Chromatic Unit Distance Graph	111
15.2 A 47-Vertex, Girth 4, 4-Chromatic, Unit Distance Graph	116
15.3 A 40-Vertex, Girth 4, 4-Chromatic, Unit Distance Graph	117
15.4 A 23-Vertex, Girth 4, 4-Chromatic, Unit Distance Graph	121
15.5 A 45-Vertex, Girth 5, 4-Chromatic, Unit Distance Graph	124
16 Edge Chromatic Number of a Graph	127
16.1 Vizing's Edge Chromatic Number Theorem	127
16.2 Total Insanity around the Total Chromatic Number Conjecture . . .	135
17 Carsten Thomassen's 7-Color Theorem	140
Part IV Coloring Maps	145
18 How the Four-Color Conjecture Was Born	147
18.1 The Problem is Born	147
18.2 A Touch of Historiography	156
18.3 Creator of the 4 CC, Francis Guthrie	158
18.4 The Brother	161
19 Victorian Comedy of Errors and Colorful Progress	163
19.1 Victorian Comedy of Errors	163
19.2 2-Colorable Maps	165
19.3 3-Colorable Maps	168
19.4 The New Life of the Three-Color Problem.	173
20 Kempe–Heawood's Five-Color Theorem and Tait's Equivalence	176
20.1 Kempe's 1879 Attempted Proof	176
20.2 The Hole	180
20.3 The Counterexample	180
20.4 Kempe–Heawood's Five-Color Theorem	182
20.5 Tait's Equivalence	182
20.6 Frederick Guthrie's Three-Dimensional Generalization	185
21 The Four-Color Theorem	187
22 The Great Debate	195
22.1 Thirty Plus Years of Debate	195
22.2 Twenty Years Later, or Another Time – Another Proof	199
22.3 The Future that commenced 65 Years Ago: Hugo Hadwiger's Conjecture	205
23 How Does One Color Infinite Maps? A Bagatelle	207
24 Chromatic Number of the Plane Meets Map Coloring: Townsend–Woodall's 5-Color Theorem	209
24.1 On Stephen P. Townsend's 1979 Proof	209
24.2 Proof of Townsend–Woodall's 5-Color Theorem	211
Part V Colored Graphs	225
25 Paul Erdős	227
25.1 The First Encounter	228
25.2 Old Snapshots of the Young	230
26 De Bruijn–Erdős's Theorem and Its History	236
26.1 De Bruijn–Erdős's Compactness Theorem	236
26.2 Nicolaas Govert de Bruijn	239

27 Edge Colored Graphs: Ramsey and Folkman Numbers	242
27.1 Ramsey Numbers	242
27.2 Folkman Numbers	256
Part VI The Ramsey Principle	261
28 From Pigeonhole Principle to Ramsey Principle	263
28.1 Infinite Pigeonhole and Infinite Ramsey Principles	263
28.2 Pigeonhole and Finite Ramsey Principles	267
29 The Happy End Problem	268
29.1 The Problem	268
29.2 The Story Behind the Problem	272
29.3 Progress on the Happy End Problem	277
29.4 The Happy End Players Leave the Stage	
as Shakespearian Heroes	280
30 The Man behind the Theory: Frank Plumpton Ramsey	281
30.1 Frank Plumpton Ramsey and the Origin of the Term “Ramsey Theory”	281
30.2 Reflections on Ramsey and Economics, by Harold W. Kuhn	291
Part VII Colored Integers: Ramsey Theory Before Ramsey and Its AfterMath	297
31 Ramsey Theory Before Ramsey: Hilbert’s Theorem	299
32 Ramsey Theory Before Ramsey: Schur’s Coloring Solution of a Colored Problem and Its Generalizations	301
32.1 Schur’s Masterpiece	301
32.2 Generalized Schur	304
32.3 Non-linear Regular Equations	307
33 Ramsey Theory before Ramsey: Van derWaerden Tells the Story of Creation	309
34 Whose Conjecture Did Van der Waerden Prove? Two Lives Between Two Wars: Issai Schur and Pierre Joseph Henry Baudet	320
34.1 Prologue	320
34.2 Issai Schur	321
34.3 Argument for Schur’s Authorship of the Conjecture	330
34.4 Enters Henry Baudet II	334
34.5 Pierre Joseph Henry Baudet	336
34.6 Argument for Baudet’s Authorship of the Conjecture	340
34.7 Epilogue	346
35 Monochromatic Arithmetic Progressions: Life After Van der Waerden	347
35.1 Generalized Schur	347
35.2 Density and Arithmetic Progressions	348
35.3 Who and When Conjectured What Szemerédi Proved?	350
35.4 Paul Erdős’s Favorite Conjecture	353
35.5 Hillel Furstenberg	356
35.6 Bergelson’s AG Arrays	358
35.7 Van der Waerden’s Numbers	360
35.8 A Japanese Bagatelle	366
36 In Search of Van der Waerden: The Early Years	367
36.1 Prologue: Why I Had to Undertake the Search for Van derWaerden	367
36.2 The Family	369
36.3 Young Bartel	373
36.4 Van derWaerden at Hamburg	377
36.5 The Story of the Book	380
36.6 Theorem on Monochromatic Arithmetic Progressions	383

36.7 Göttingen and Groningen	385
36.8 Transformations of The Book	386
36.9 Algebraic Revolution That Produced Just One Book	387
36.10 Epilogue: On to Germany	392
37 In Search of Van der Waerden: The Nazi Leipzig, 1933–1945	393
37.1 Prologue	393
37.2 Before the German Occupation of Holland: 1931–1940	394
37.3 Years of the German Occupation of the Netherlands: 1940–1945	406
37.4 Epilogue: The War Ends	416
38 In Search of Van der Waerden: The Postwar Amsterdam, 1945	418
38.1 <i>Breidablik</i>	418
38.2 NewWorld or Old?	421
38.3 Defense	427
38.4 Van derWaerden and Van der Corput: Dialog in Letters.	434
38.5 A Rebellion in Brouwer’s Amsterdam	446
39 In Search of Van der Waerden: The Unsettling Years, 1946–1951	449
39.1 The <i>Het Parool</i> Affair	449
39.2 Job History 1945–1947	458
39.3 “America! America!”	462
39.4 Van der Waerden, Goudsmit and Heisenberg: A ‘Letteral Triangle’	465
39.5 Professorship at Amsterdam.	472
39.6 Escape to Neutrality	474
39.7 Epilogue: The Drama of Van der Waerden	480
Part VIII Colored Polygons: Euclidean Ramsey Theory	485
40 Monochromatic Polygons in a 2-Colored Plane	487
41 3-Colored Plane, 2-Colored Space, and Ramsey Sets	500
42 Gallai’s Theorem	505
42.1 Tibor Gallai and His Theorem	505
42.2 Double Induction	509
42.3 Proof of Gallai’s Theorem by Witt	509
42.4 Adriano Garsia.	514
42.5 An Application of Gallai	516
42.6 Hales-Jewett’s Tic-Tac-Toe	517
Part IX Colored Integers in Service of Chromatic Number of the Plane	519
43 Application of Baudet–Schur–Van der Waerden	521
44 Application of Bergelson–Leibman’s and Mordell–Faltings’ Theorems	525
45 Solution of an Erdős Problem: O’Donnell’s Theorem	529
45.1 O’Donnell’s Theorem	529
45.2 Paul O’Donnell	530
Part X Predicting the Future	533
46 What IfWe Had No Choice?	535
46.1 Prologue	535
46.2 The Axiom of Choice and its Relatives	537
46.3 The First Example	540
46.4 Examples in the plane	543
46.5 Examples in space	544
46.6 AfterMath & Shelah–Soifer Class of Graphs	546
46.7 An Unit Distance Shelah–Soifer Graph	549
47 A Glimpse into the Future: Chromatic Number of the Plane, Theorems and Conjectures	553

47.1 Conditional Chromatic Number of the Plane Theorem	553
47.2 Unconditional Chromatic Number of the Plane Theorem	554
47.3 The Conjecture	555
48 Imagining the Real, Realizing the Imaginary	557
48.1 What Do the Founding Set Theorists Think about the Foundations?	557
48.2 So, What Does It All Mean?	560
48.3 Imagining the Real vs. Realizing the Imaginary	562
Part XI Farewell to the Reader	565
49 Two Celebrated Problems	567
Bibliography	569
Name Index	595
Subject Index	603
Index of Notations	605

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