
Preface to the Second Edition

The development of science and of creative activities of the spirit in general requires still another kind of freedom, which may be characterized as inward freedom. It is this freedom of the spirit which consists in the independence of thought from the restrictions of authoritarian and social prejudices as well as from unphilosophical routinizing and habit in general.

—Albert Einstein, *On Freedom*, 1940 [Ei]

A course of instruction will be the more successful the more its individual phases assume the character of experience.

—Hugo von Hofmannsthal, *Buch der Freunde* [H]

Out of all my books released prior to *Mathematical Coloring Book*, this one is my favorite. She lived with me for 20 years, since the idea was born in 1970, until her leaving me in 1990. Let us trace here highlights of the book's life after she went to live on her own. In this new edition, I am adding five chapters to the existing ten. In order to preserve the smooth flow of the original book, I add these new chapters as Part II. Part I remains unchanged, except, of course, for the correction of a number of typos.

Shortly after the book's release, the Hungarian mathematician Miklós Laczkovich informed me that the book inspired him to publish a paper on cutting triangles. For this edition, Miklós donated a note with an important new result—you will find it in Chapter 12.

At the eleventh hour, when this new edition was already in production, the winner of the 1990 and 1991 Colorado Mathematical Olympiads, and now post-doctoral fellow at Stanford University

Matthew Kahle submitted to *Geombinatorics* an essay [Kah] dedicated to my round-numbered birthday. In it, Matthew improves the result of the Five-Point Problem 7.2.1, poses further problems and formulates new conjectures. I dedicate Chapter 13 to his new work.

In 2005, the brilliant young freshman from Columbia University Mitya Karabash came to the University of Colorado at Colorado Springs, where I supervised his summer research. We looked at problems of tiling and covering, the chromatic number of the plane, and problems posed by Paul Erdős and me in this book.

To my delight, on June 4, 2007, Mitya delivered to me the most important result related to this book in the two decades that have passed since the release of the first edition. What exactly did Mitya prove? He did not solve the One-Hundred-Dollar Problem (8.6.6) that asks to classify all convex figures F for which $S(F) = 6$. But he settled conjecture 8.6.7 in the negative by constructing a figure F such that $S(F) = 6$, yet F cannot be mapped by an affine transformation into a however narrow frame formed by two concentric regular pentagons. The structure of the figures F with $S(F) = 6$ appears to be even harder than I imagined twenty years ago. Now we do not even have a working conjecture for the One-Hundred-Dollar Problem! Mitya's results have appeared as a sequence of two papers [Ka1] and [Ka2] in the quarterly *Geombinatorics*. Chapter 14 of this book is dedicated to these results.

In 2004 I posed two easy-to-formulate, hard-to-solve problems: Cover-Up and Cover-Up Squared. In Chapter 15 you will meet these problems and see how the progress was obtained during coffee hours at Princeton University by John H. Conway and me, and then by Mitya Karabash and me, and finally by Fan Chung and Ronald L. Graham. Plenty is left open for you to join in!

The book received kind notes from many journals and Paul Erdős, Leroy M. Kelly, Murray K. Klamkin, and others. The Russian geometer Vladimir G. Boltyanski in *SIAM Review* has described my goals perhaps better than I could:

We do not often have possibilities to look into a creative workshop of a mathematician. Usually mathematicians give an account of their results in a ground-out and logically

irreproachable form. But their creative pains, methods of investigation and means of obtaining results remain vague, especially for other mathematicians. And therefore every possibility to observe creative methods employed by a mathematician is interesting and useful. In particular, it is important for a beginner, that is, for a schoolboy or a schoolgirl who is interested in mathematics. First of all, it is important because pupils (and even first-year students in mathematical or technical colleges), as a rule, do not imagine what modern mathematics is, its scientific problems and its methods of investigation. School mathematics is far from authentic modern science with respect to its contents and methods.

... The beginner, who is interested in the book, not only comprehends a situation in a creative mathematical studio, not only is exposed to good mathematical taste, but also acquires elements of modern mathematical culture. And (not less important) the reader imagines the role and place of intuition and analogy in mathematical investigation; he or she fancies the meaning of generalization in modern mathematics and surprising connections between different parts of this science (that are, as one might think, far from each other) that unite them. . .

This makes the book alive, fresh, and easily readable. Alexander Soifer has produced a good gift for the young lover of mathematics. And not only for youngsters; the book should be interesting even to professional mathematicians.

Leroy M. Kelly observed the book's spirit in *The Mathematical Reviews*:

It is impossible to convey the spirit of the book by merely listing the problems considered or even a number of solutions. The manner of presentation and the gentle guidance toward a solution and hence to generalizations and new problems takes this elementary treatise out of the prosaic and into the stimulating realm of mathematical creativity. Not only young talented people but dedicated secondary teachers and even

a few mathematical sophisticates will find this reading both pleasant and profitable.

John Baylis wrote in *The Mathematical Gazette* (UK):

Alexander Soifer is a wonderful problem solver and inspiring teacher. His book will tell young mathematicians what mathematics should be like, and remind older ones who may be in danger of forgetting. This review has the simple aim of persuading as many people as possible to read it. . .

So why am I urging you to read this? Mainly because it is such a refreshing book. Professor Soifer makes the problems fascinating, the methods of attack even more fascinating, and the whole thing is enlivened by anecdotes about the history of the problems, some of their recent solvers, and the very human reactions of the author to some beautiful mathematics. Most of all, the book has charm, somehow enhanced by his slightly eccentric English, sufficient to carry the reader forward without minding being asked to do rather a lot of work. I am tempted to include several typical quotations but I'll restrain myself to two: From Chapter 8 "Here is an easy problem for your entertainment. Problem 8.1.2. Prove that for any parallelogram P , $\mathbf{S}(P) = 5$. Now we have a new problem, therefore we are alive! And the problem is this: what are all possible values of our newly introduced function $\mathbf{S}(F)$? Can the function $\mathbf{S}(F)$ help us to classify geometry figures?"

And from an introduction by Cecil Rousseau:

"There is a view, held by many, that mathematics books are dull. This view is not without support. It is reinforced by numerous examples at all levels, from elementary texts with page after page of mind-numbing drill to advanced books written in a relentless Theorem–Proof style.

"*How Does One Cut a Triangle?* is an entirely different matter. It reads like an adventure story. In fact, it is an adventure story, complete with interesting characters, moments of exhilaration, examples of serendipity, and unanswered questions. It conveys the spirit of mathematical discovery

and it celebrates the event as have mathematicians throughout history.”

And this isn’t just publishers going over the top—it’s all true!

I thank Colonel Dr. Robert Ewell for converting some of my designs into illustrations in the new chapters of the book.

I am so very grateful to the first readers of this new edition: Branko Grünbaum and Peter D. Johnson, Jr., for their comments and forewords.

I am deeply grateful to Ann Kostant for inviting this book’s new edition into the historic Springer.

I have good news for those who may be interested in my books: [S1], [S3] and [BS] will soon appear in new expanded editions in Springer. [S5] after 18 years in the writing came out in November of 2008. I hope my new books [ES] and [S6] will appear in Springer soon.

I welcome feedback—your problems, solutions, ideas.

Alexander Soifer
Colorado Springs, Colorado
May 8, 2008

Preface to the First Edition

If I see a really nice proof, I say it comes straight from the Book. . . God has a transfinite Book, which contains all theorems and their best proofs, and if He is well intentioned toward those, He shows them the Book for a moment. And you wouldn't even have to believe in God, but you must believe that the Book exists.

—Paul Erdős

How Does One Cut a Square?

—Isaac Yaglom

In 1970, I visited my professor, Isaac Moiseevich Yaglom, in his Moscow apartment. We talked about art and mathematics, two passions we share. Professor Yaglom complimented me on two problems about cutting triangles into similar and congruent triangles that I had just created and solved. A famous geometer, he was impressed with how algebra was used to obtain genuinely geometric results. “A spirit of the time,” he said and added, “Nobody would have thought of these solutions a few years ago.” Prof. Yaglom offered to publish these solutions in the proceedings of our Institute so that he could refer to them and add them as an appendix to an upcoming Springer-Verlag translation of his book, *How Does One Cut a Square?* Arrogantly, I turned this generous offer down. “Nobody reads these proceedings,” I said. Little did I know that Yaglom was their Editor-in-Chief! Yet, Prof. Yaglom modestly replied, “You are probably right.” I promised him, “I will write my own book, *How Does One Cut a Triangle?* with the title of your book, *How Does One Cut a Square?* as the epigraph.” Now, I am making good on my

promise, even after twenty years that took me from a university senior in Moscow to a mathematics professor in Colorado Springs.

Three years ago I lost my mother. She gave me life. She served as a role model of loyalty, integrity, enthusiasm, and love. A year and a half ago we all lost the famous Russian geometer and author of numerous great books, Isaac Moiseevich Yaglom. I grew up on his and Martin Gardner's books. At about the same time I met a legend, Paul Erdős. Our meetings and correspondence have inspired me to write this book and other works. This is why I am dedicating the book to these three great people.

This book is a research monograph on my newly created and solved problems of combinatorial geometry. Yet, it is not an ordinary research monograph, written for a narrow group of professionals. This book might be of interest to professional geometers, but I wrote it first of all for young, talented mathematicians who are still in high school or early college, and for their teachers, and for the leisure of all professional mathematicians. In a normal monograph, the main emphasis is on results obtained. I would not have written this book if the solutions were not beautiful and valuable in their own right.

I cannot help but quote I. M. Yaglom's preface from his book, *How Does One Cut a Square?* He so precisely describes my goals and aspirations:

Here we have a certain "fragment of mathematics" that characterizes mathematical trains of thought, techniques, and methods. The author was guided precisely by this thought: it is not the results that are of interest here so much as the reasoning leading to those results—not the "what" of the proofs that deserves attention so much as the "how."

My goal in this book is to show young, talented people what mathematics is and what mathematicians do. I cannot overemphasize the importance of this goal; our youth throughout the world too often graduate from high schools without the foggiest idea of what mathematics is. Many university and college graduates do not know either. Indeed, too often teachers do not know, so how on earth can we expect our students to find out?

Mathematics in school is reduced to a topic study, filled with “prove that A implies B by using the Pythagorean theorem.” *This book is a real, live “fragment of mathematics” with analytical proofs and constructions of counterexamples; with open and open-ended problems; with mathematical intuition leading research like a light at the end of a tunnel; with the synthesis of ideas from algebra, geometry, trigonometry, linear algebra, mathematical analysis; with beauty, elegance, and surprises.*

It is a special monograph in that I had to raise my pedagogical talent to the limit in an attempt to introduce to my young colleagues ideas from linear algebra and analysis normally studied later in college, and do it without sacrificing the rigor of mathematical reasoning. I kept some proofs out, but only those that would have led us too far from the main topic of our research or that would have made this book much thicker (I, too, hate the user-unfriendly look of calculus bricks!).

I tried to show that mathematics is alive—that every solved problem gives birth to myriad unsolved ones. This book is full of open problems. Many of them carry ten to fifty dollar prizes for their first solutions. Do not be afraid to try to be the first to solve a problem. As the *South Carolina Reflector* put it, “Think like a tea bag. You don’t know your strength until you get in hot water.”

This book was written in a dialogue with my colleagues, young and not so young. I thank them all. I am especially grateful to those whose solutions I included in this book: Dr. Semion Slobodnik from Moscow; Royce Peng, a high school student from California; Professor Cecil Rousseau from Memphis State University; Volodia Baranovski, a high school student from Siberia; and Boria Dubrov, a high school student from Minsk, USSR. I thank Professor Branko Grünbaum from the University of Washington for contributing an exciting open problem, Problem 8.7.10 and Conjecture 8.7.11.

Paul Erdős created problems 6.6 and 6.7, co-created all the problems of Chapter 9, and generously contributed a whole chapter of open problems in letters sent upon his reading my manuscript (Chapter 10). My hat goes off to him!

I am grateful to George Berzsenyi, Phillip Engel, Paul Erdős, Martin Gardner, Branko Grünbaum, and Cecil Rousseau for being

the first readers of the manuscript and providing me with most valuable feedback. I am honored that Philip Engel, Paul Erdős, Branko Grünbaum, and Cecil Rousseau have written forewords for this book.

I thank and applaud my dean, Dr. James A. Null, for fostering such a wonderful climate for creative work in the College of Letters, Arts and Sciences. And I thank my friend and secretary, Lynn Scott, for encoding my multilayered, hand-written manuscript. "It reads like a long letter," she said.

Well, here is my letter to you, my reader! Write back to me! I'm looking forward to your solutions and new problems.

Alexander Soifer
Colorado Springs, Colorado
January 1990

How Does One Cut a Triangle?

Soifer, A.

2009, XXX, 174 p. 83 illus., Softcover

ISBN: 978-0-387-74650-0