Preface

In 1992 we published a book entitled *Fuzzy Measure Theory* (Plenum Press, New York), in which the term "fuzzy measure" was used for set functions obtained by replacing the additivity requirement of classical measures with weaker requirements of monotonicity with respect to set inclusion and continuity. That is, the book dealt with nonnegative set functions that were monotone, vanished at the empty set, and possessed appropriate continuity properties when defined on infinite sets.

It seems that *Fuzzy Measure Theory* was the only book available on the market at that time devoted to this emerging new mathematical theory. Some ten years after its publication we began to see that the subject had expanded so much that a second edition of the book, or even a new book on the subject, was needed. We eventually decided to write a new book because the new material we wished to include was too extensive for—and far beyond the usual scope—of a second edition. More importantly, we felt that some fundamental changes regarding this topic's scope and terminology would be desirable and timely.

As far as the scope of the new book, *Generalized Measure Theory*, is concerned, we felt, on the basis of recent developments in the literature, that the material should not be restricted to set functions that had to be nonnegative and monotone. Rather, it needed to capture a broader class of set functions; a function in this class would have only one requirement to qualify as a "measure": it would vanish at the empty set. Then, various special requirements could be introduced as needed to restrict this broad class of set functions to specialized subclasses. One of these subclasses would consist of nonnegative, monotone, and continuous set functions that vanish at the empty set—or fuzzy measures—the subject of our previous book.

Regarding terminology, it was obvious that we needed to revise it completely in view of the expanded scope of the book. First, we had to introduce a name for the most general measures. We did so by referring to nonnegative set functions that vanish at the empty set as *general measures* and referring to those that are not required to be nonnegative as *signed general measures*. Second, we needed to introduce appropriate names of the various subclasses of general measures or signed general measures. This we did in Chapters 3 and 4, where we followed, by and large, the terminology established in the literature. However, it should be emphasized that we made a deliberate decision to abandon the central term of our previous book, the term "fuzzy measure." We judge this term to be highly misleading. Indeed, the so-called fuzzy measures do not involve any fuzziness. They are just special set functions that are defined on specified classes of classical sets, not on classes of fuzzy sets. Since the primary characteristic of such functions is monotonicity, we deemed it reasonable to call these set functions *monotone measures* rather than fuzzy measures.

However, contrary to the concept of fuzzy measures in our previous book, monotone measures as understood in *Generalized Measure Theory* need not be continuous. If, in fact, they are continuous then they are here specifically referred to as *continuous monotone measures*. Moreover, if they are only semicontinuous from below or from above, then they are called, respectively, *lower-semicontinuous* or *upper-semicontinuous monotone measures*. Clearly, any continuous monotone measure is both lower-semicontinuous and upper-semicontinuous.

There is another reason why abandoning the term "fuzzy measure" is justified: It is certainly meaningful to fuzzify any class of measures, as we show in Chapter 14. A given class of measures is "fuzzified" when it is defined on fuzzy sets rather than on classical sets. However, the resulting term—"fuzzified fuzzy measures" we find awkward, not properly descriptive, and quite confusing. For all these reasons, we decided to replace the term "fuzzy measure" with "continuous monotone measure" and to use the term "monotone measure" when continuity or even semicontinuity is not required. When they *are* fuzzified we refer to these measures as "fuzzified monotone measures." When measures of any other type are defined on classes of fuzzy sets we refer to them as *fuzzified measures* of the respective type. We thus use names such as *fuzzified general measures, fuzzified monotone measures, fuzzified continuous monotone measures,* and the like.

We realize it is not likely that the confusing term "fuzzy measures" for "measures defined on classes of crisp sets" will soon disappear in the literature. However, we are confident that the time is ripe to stop using it. In a sense we have joined some major contributors to generalized measure theory who have already abandoned this ill-descriptive term.

We have made in this book a few additional terminological changes with respect to our previous book. However, all these changes affect special concepts, so we explain our rationale for making these changes as we introduce each concept.

Our previous book contains, in addition to its original material, six of our reprinted papers. In this book, no reprinted papers are included. Instead the original material is substantially expanded. Major expansions are in the area of integration, methods for constructing generalized measures, fuzzification of generalized measures, and applications of generalized measure theory.

Much like our previous book, this book is primarily a text for a one-semester graduate or upper division course. Such a course is suitable not only for programs in mathematics, where it might be offered at the junior or senior level, but also for programs in numerous other areas. These would include systems science, computer science, information science, and cognitive sciences, as well as artificial intelligence, quantitative management, mathematical social sciences, and virtually all areas of engineering and natural sciences. The book may also be useful for researchers in these areas.

Although a solid background in mathematical analysis is required for understanding the material presented, the book is otherwise self-contained. This is achieved by the inclusion of needed prerequisites regarding classical sets, classical measures, and fuzzy sets, as given in Chapter 2. In general, the book is written in the textbook style, characterized by generous use of examples and exercises. Each chapter concludes with notes containing relevant historical, bibliographical, and other remarks relating to the covered material, which are useful for further study of generalized measure theory and its applications. Compared with our previous book, the bibliography of *Generalized Measure Theory* is substantially expanded. Two glossaries are included for convenience of the reader, Glossary of Key Concepts (Appendix A) and Glossary of Symbols (Appendix B).

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