

SOBOLEV SPACES IN MATHEMATICS II

**APPLICATIONS IN ANALYSIS
AND PARTIAL DIFFERENTIAL
EQUATIONS**

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Sobolev Spaces in Mathematics II
Applications in Analysis and Partial Differential
Equations

Maz'ya, V. (Ed.)

2009, XXX, 388 p., Hardcover

ISBN: 978-0-387-85649-0