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# Preface

While we were busy putting together the present collection of articles celebrating the twentieth birthday of our journal, *Discrete & Computational Geometry*, and, in a way, of the field that has become known under the same name, two more years have elapsed. There is no doubt that *DCG* has crossed the line between childhood and adulthood.

By the mid-1980s it became evident that the solution of many algorithmic questions in the then newly emerging field of computational geometry required classical methods and results from discrete and combinatorial geometry. For instance, visibility and ray shooting problems arising in computer graphics often reduce to Helly-type questions for line transversals; the complexity (hardness) of a variety of geometric algorithms depends on McMullen's upper bound theorem on convex polytopes or on the maximum number of "halving lines" determined by  $2n$  points in the plane, that is, the number of different ways a set of points can be cut by a straight line into two parts of the same size; proximity questions stemming from several application areas turn out to be intimately related to Erdős's classical questions on the distribution of distances determined by  $n$  points in the plane or in space.

On the other hand, the algorithmic point of view has fertilized several fields of convexity and of discrete geometry which had lain fallow for some years, and has opened new research directions. Computing the convex hull or the diameter of a point set, or estimating the volume of a convex body or the maximum density of a packing of translates of a given convex body, has motivated a wide range of exciting new questions concerning classical concepts in discrete geometry. Motion planning problems have triggered the systematic study of the "combinatorial complexity" of the boundary of the union of geometric objects, and hence the development of Davenport-Schinz theory, the use of epsilon-nets, Vapnik-Chervonenkis dimension, and probabilistic techniques. Similar methods have been needed for range searching, and this has also led to a renaissance of geometric discrepancy theory.

In the last two decades, *DCG* has provided a common platform for mathematicians working in the theory of packing and covering, and in convexity and combinatorial geometry, as well as for computer scientists interested in computational geometry, computational topology, geometric optimization, graph drawing, motion planning, and so on. In fact, exceeding all the expectations of its editors, the journal has served

as an effective catalyst in the creation of a new generation of researchers working on the common borderline between mathematics and computer science.

The present selection of 28 exceptionally strong articles, many of which solve longstanding open problems, reflects the current state of our subject, its many different facets, and its strong links to other important disciplines.

Nevo and Barvinok–Novik study problems related to Barnette’s Lower Bound and McMullen’s Upper Bound Theorem, respectively. Nagel gives a proof of the Kalai–Kleinschmidt–Lee conjecture for the maximum number of empty simplices in a simplicial polytope. Miller–Pak and Damian–Flatland–O’Rourke prove the existence of nonoverlapping unfoldings of certain manifolds. Billera–Hsiao–Provan construct nearly polytopal CW spheres with special properties. Khachiyan–Boros–Borys–Elbassioni–Gurvich show that generating all vertices of a polyhedron is a hard problem. Braun establishes improved estimates for the roots of Ehrhart polynomials of lattice polytopes. Przesławski–Yost give new conditions for the decomposability of polytopes as a Minkowski sum, while Richardson–Vu–Wu describe the asymptotic behavior of certain random polytopes.

Schreiber–Sharir design optimal shortest path algorithms on polytopes. Niyogi–Smale–Weinberger show how to find the homology of the underlying submanifold of a probability distribution with high confidence. Basu–Zell establish new bounds on Betti numbers of projections of semialgebraic sets. Efficient algorithms for snap rounding in pixel geometry and for computing optimal embeddings of paths, trees, and cycles in two and three dimensions are presented by Hersherberger and by Agarwal–Klein–Knauer–Langerman–Morin–Sharir–Soss, respectively. Agarwal–Har–Peled–Yu apply coresets to design approximation algorithms to shape fitting. Shewchuk generalizes constrained Delaunay triangulations to higher dimensions, while Boissonat–Cohen–Steiner–Vegter find the first provably correct implicit surface meshing algorithm, where the mesh is isotopic to the surface.

Ackerman–Buchin–Knauer–Pinchasi–Rote and Gerken solve Murty’s and Erdős’s many-decade-old problems for finite point configurations. Pfender proves that every finite graph can be obtained as the visibility graph of a rational point set in the plane, while Pelsmayer–Schaefer–Štefankovič construct the first examples showing that the crossing number of a graph is not necessarily the same as its odd-crossing number. Aliev uses convex geometry to make progress on an old Erdős–Moser problem in additive number theory. Lee–Solomyak use dynamical systems to answer a question of Lagarias on Delone sets. Gronchi–Longinetti solve an extremal problem for polygons that plays a role in X-ray tomography. Bárány–Hubard–Jerónimo, Borcea–Goaoc–Petitjean, and Cheong–Goaoc–Holmsen–Petitjean solve various hyperplane- and line-transversal problems in Euclidean spaces.

*Discrete & Computational Geometry* saw the light of day in 1986, and this volume celebrates its majority. By now, the field, which has become inseparable from the journal, has acquired its own characteristics, its own methodology and toolbox. Deep connections have been discovered between its basic problems and many other fields of mathematics and computer science, such as additive combinatorics, topology, real algebraic geometry, randomized algorithms, and data structures. The field has its annual conferences: the ACM Symposia, the Fall Workshops, and the European Workshops on Computational Geometry, and a biennial meeting in Schloss Dagstuhl. Established research institutes such as DIMACS, MSRI, and IPAM regularly run special semester

programs dedicated to the subject, and Oberwolfach sponsors a meeting every few years. We have several excellent textbooks for teaching discrete and computational geometry, not to mention two comprehensive handbooks. The “genie” has been let out of the bottle. Its movements and actions are now largely independent of the original intentions of its “creators,” who include the founding editors of *DCG*. It has been a tremendous pleasure and honor to edit the journal, to watch it grow alongside the field proper, and to serve the community built around it. Our everlasting gratitude must also go to the late Walter Kaufmann-Bühler, who had the foresight to accept our original invitation to Springer-Verlag to publish a journal in this new field.

We dedicate the present volume to the members of the very active and gifted community of researchers who have taken part in the development of the field during the past more-than-two decades; many of them are represented in its pages.

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