Preface

The theory of variational inequalities plays an important role in the study of both the qualitative and numerical analysis of nonlinear boundary value problems arising in mechanics, physics, and engineering science. For this reason, the mathematical literature dedicated to this field is extensive, and the progress made in the past four decades is impressive. A part of this progress was motivated by new models arising in contact mechanics. At the heart of this theory is the intrinsic inclusion of free boundaries in an elegant mathematical formulation.

Contact between deformable bodies abounds in industry and everyday life. Because of the industrial importance of the physical processes that take place during contact, a considerable effort has been made in their modeling, analysis, numerical analysis and numerical simulations, and, as a result, the mathematical theory of contact mechanics has made impressive progress recently. Owing to their inherent complexity, contact phenomena lead to mathematical models expressed in terms of strongly nonlinear evolutionary problems.

Antiplane shear deformations are one of the simplest classes of deformations that solids can undergo: in antiplane shear (or longitudinal shear) of a cylindrical body, the displacement is parallel to the generators of the cylinder and is independent of the axial coordinate. For this reason, the antiplane problems play a useful role as pilot problems, allowing for various aspects of solutions in solid mechanics to be examined in a particularly simple setting. In recent years, considerable attention has been paid to the analysis of such kinds of problems.

The purpose of this book is to introduce to the reader the theory of variational inequalities with emphasis on the study of contact mechanics and, more specifically, with emphasis on the study of antiplane frictional contact problems. The contents cover both abstract results in the study of variational inequalities as well as the study of specific antiplane frictional contact problems. This includes their modeling and variational analysis. Our intention is to illustrate the cross-fertilization between modeling and applications on the one hand, and nonlinear mathematical analysis on the other hand.

Thus, within the particular setting of antiplane shear, we show how new and nonstandard models in contact mechanics lead to new types of variational inequalities and, conversely, we show how the abstract results on variational inequalities can be applied to prove the unique solvability of the corresponding contact problems. In writing this book, our aim was also to draw the attention of the applied mathematics community to interesting two-dimensional models arising in solid mechanics, involving a single nonlinear partial differential equation that has the virtue of relative mathematical simplicity without loss of essential physical relevance.

Our book, divided into four parts with 11 chapters, is intended as a unified and readily accessible source for mathematicians, applied mathematicians, engineers, and scientists, as well as advanced graduate students. It is organized with two different aims, so that readers who are not interested in modeling and applications can skip Parts III and IV and will find an elementary introduction to the theory of variational inequalities in Part II of the book; alternatively, readers who are interested in modeling and applications will find in Parts III and IV the mechanical models that lead to the various classes of variational inequalities presented in Part II of the book.

A brief description of the parts of the book follows.

Part I is devoted to the basic notation and results that are fundamental to the developments later in this book. We review the background on functional analysis and function spaces that we need in the study of variational inequalities. The material presented is standard and can be found in many textbooks and monographs. For this reason, we present only very few details of the proofs.

Part II represents one of the main parts of the book and includes original results. We present various classes of variational inequalities for which we prove existence results and, for some of them, we prove uniqueness, regularity, and convergence results. To this end we use convexity, monotonicity, compactness, time discretization, regularization, and fixed point arguments. Most of the concepts and results presented in this part can be extended to more general variational inequalities involving nonlinear operators on reflexive Banach spaces or to hemivariational inequalities; however, since our aim is to provide an accessible presentation of the theory of variational inequalities with emphasis in the study of antiplane frictional contact problems, we restrict ourselves to the framework of Hilbert spaces, linear operators, and convex analysis, as is sufficient for later development.

The terminology we use in this part of book is the following: if the time derivative of the unknown function u appears in the formulation of a variational inequality (and, therefore, an initial condition for u is needed), we refer to it as an *evolutionary variational inequality.* Otherwise, we refer to it as an *elliptic variational inequality.* If the nondifferentiable convex functional j depends explicitly on u or on its time derivative $\dot{u}$, we refer to the corresponding variational inequality as a *quasivariational inequality.* If both the data and the solution of a variational inequality depend on the time variable that plays the role of a parameter, the corresponding variational inequality is called a *time-dependent variational inequality.* Finally, if an integral term containing the solution or its derivative appears in the formulation of a variational inequality, we refer to it as a *history-dependent variational inequality.* This classification is not strict and is intended to distinguish among the types of variational inequalities used in the mathematical theory of contact mechanics, as it is illustrated in Part IV.

Part III presents preliminary material of contact mechanics that is needed in the rest of the book. We summarize basic notions and equations of mechanics of continua, then we introduce the frictional contact conditions as well as the constitutive laws that are used in the rest of the book. We then specialize the equations and conditions in the context of the antiplane shear and, as an example, we study a displacement-traction problem involving linearly elastic materials. The material presented in this part provides the background for the modeling of the antiplane frictional contact problems studied in Part IV of the book.

Part IV represents the other main part of the book and is partially based on our original research. It deals with the study of static and quasistatic frictional antiplane contact problems. We model the material behavior with isotropic linearly elastic and viscoelastic constitutive laws and, in the case of viscoelastic materials, we consider both short and long memory. Friction is modeled with versions of Coulomb's law in which the friction bound is either a function that does not depend on the process variables or depends on the slip or slip rate. Particular attention is paid to history-dependent frictional problems in which the friction bound depends on the total slip or the total slip rate. For each one of the problems, we provide a variational formulation then we use the abstract results in Part II in order to establish existence and sometimes uniqueness, regularity, and convergence results.

Each of the four parts of the book is divided into several chapters. All the chapters are numbered consecutively. Mathematical relations (equalities, inequalities, and inclusions) are numbered by chapter and their order of occurrence. For example, (4.3) is the third numbered mathematical relation in Chapter 4. Definitions, problems, theorems, propositions, lemmas, and corollaries are numbered consecutively within each chapter. For example, in Chapter 9, Problem 9.5 is followed by Theorem 9.6.

Each part ends with a section in which we present bibliographical comments. We provide references for the principal results presented, as well as information on important topics related to but not included in the body of the text. The list of the references at the end of the book includes only papers or books that are closely related to the subjects treated in this monograph.

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