

# Preface

A unified theory of superconductivity in elements, compound and cuprates is presented. Superconductivity is the most striking phenomenon in solid state physics. The electrical resistance due to impurities and phonons in a metal suddenly drops to zero below a critical temperature  $T_c$ . Not all elemental metals show superconductivity, which suggests that the phenomenon depends on the lattice structure and the Fermi surface. In a historic 1957 paper Bardeen, Cooper and Schrieffer (BCS) start with a reduced Hamiltonian  $H_0$  containing “electron” and “hole” kinetic energies and a pairing interaction, calculate the ground-state energy  $E_0$  to obtain  $E_0 = N_0 w_0$ , where  $N_0$  is the number of the Cooper pairs (pairons) and  $w_0$  the ground-state energy of a pairon, using the energy minimum principle. They also obtain the zero temperature energy gap equation (18.32), the solution of which yields the ground-state energy  $w_0$  and the excitation energy  $E_k = \sqrt{\epsilon_k^2 + \Delta_0^2}$ , where  $\epsilon_k$  is the “electron” (or “hole”) energy with the momentum  $k$ , and  $\Delta_0$  the energy gap. The reduced Hamiltonian  $H_0$  and the trial ground-state vector can be written in terms of the pairon variables only while only quasi-electron variables appear in the gap equation. Hence it is impossible to guess even the existence of a gap in the quasi-electron energy spectrum. We recalculate the ground-state energy and the quasi-electron energy, using the standard quantum statistical methods. Our calculations are lengthy, but we have a major advantage: no need of guessing of the trial ground-state vector. BCS extended their theory to a finite temperature. They obtained a temperature-dependent energy gap equation, the solution of which yields the famous connection between the critical temperature  $T_c$  and the zero-temperature gap  $\Delta_0$ :  $k_B T_c = (1.77)^{-1} \Delta_0$ .

The cause of superconductivity is well established by the BCS theory. It is the phonon exchange attraction, which binds a Cooper pair composed of two electrons with opposite spin directions. The Center-of-Mass (CM) motion of a composite is bosonic (fermionic) according to whether the composite contains an even (odd) number of elementary fermions. The Cooper pairs, each having two electrons, move as bosons. In the ground state there can be no currents super or normal. We must consider moving pairons with finite CM momenta for the supercurrents. Cooper found that Cooper pairs move with a *linear dispersion relation*:  $\epsilon = w_0 + (1/2)v_F p$ , where  $v_F$  is the Fermi velocity (magnitude). This relation was obtained for a three-

dimensional (3D) system. For a 2D system, we obtain  $\epsilon = w_0 + (2/\pi)v_F p$ . Flux quantization experiments indicate that the charge carriers in the supercurrent have the charge (magnitude)  $2e$  in agreement with the BCS theory. Josephson interference in a Superconducting Quantum Interference Device (SQUID) show that the pairons move as *bosons* with a *linear dispersion relation* just as the photons in a laser. The systems of free pairons, moving with the linear dispersion relation, undergo a Bose–Einstein Condensation (BEC) in 3D (2D). The critical temperature  $T_c$  is given by  $k_B T_c = 1.01 \hbar v_F n_0^{1/3}$  ( $1.24 \hbar v_F n_0^{1/2}$ ) where  $n_0 \equiv N_0/V$  ( $N_0/A$ ) is the pairon density, and  $V$  ( $A$ ) the sample volume (area). The interpairon distance  $r_0 \equiv n_0^{-1/3}$  ( $n_0^{-1/2}$ ) is several times greater than the BCS pairon size  $\xi_0 = 0.181 \hbar v_F (k_B T_c)^{-1}$ . Hence, the BEC occurs without the pairon overlap, which justifies the free-pairon model. The superconducting transition will be regarded as a BEC transition. The electronic heat capacity in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  has a maximum at  $T_c$  with a shoulder above  $T_c$ , which can only be explained naturally in terms of a model in which many pairons participate in the phase transition. (No feature above  $T_c$  is predicted by the BCS theory.) Above  $T_c$  pairons move independently in all allowed directions, and they contribute to the resistive conduction. Below  $T_c$  condensed pairons move without resistance. Non-condensed pairons, unpaired electrons and vortices contribute to the resistive conduction.

In 1986 Bednorz and Müller reported a discovery of high-temperature (high- $T_c$ ) cuprate superconductors ( $\text{BaLaCuO}$ ,  $T_c \sim 30$  K). Since then many investigations have been carried out on high- $T_c$  cuprates. These cuprates possess all basic superconducting properties: zero resistance, sharp phase change, Meissner effect, flux quantization, Josephson interference, and gaps in elementary excitation energy spectra. In addition these cuprate superconductors exhibit 2D conduction, short coherence length  $\xi_0$  ( $\sim 10$  Å), high critical temperature  $T_c$  ( $\sim 100$  K), two energy gaps, *d*-wave pairon, unusual transport and magnetization behaviors above  $T_c$ , and a dome-shaped doping dependence of  $T_c$ .

Because the supercondensate can be described in terms of free moving pairons, all of the properties of a superconductor can be computed without mathematical complexities. This simplicity is in great contrast to the far more complicated rigorous treatment required for a ferromagnet phase transition. The authors believe that everything essential about superconductivity can be presented to second-year graduate students. Students are assumed to be familiar with basic differential, integral and vector calculus, and partial differentiation. Knowledge of mechanics, electromagnetism, solid state, and statistical physics at the junior-senior level and quantum theory at the first-year graduate level are prerequisite. A substantial part of the difficulty that students face in learning the theory of superconductivity lies in the fact that they should have not only a good background in many branches of physics but also be familiar with a number of advanced physical concepts such as bosons, fermions, Fermi surface, “electrons,” “holes,” phonons, density of states, phase transitions. The reader may find it useful to refer to the companion book, *Quantum Theory of Conducting Matter* by Fujita and Ito. Second quantization may or may not be covered in the first-year quantum course. But this theory is indispensable in

the microscopic theory of superconductivity. It is fully reviewed in Appendix A. The book is written in a self-contained manner. Thus, non-physics majors who want to learn the microscopic theory of superconductivity step by step with no particular hurry may find it useful as a self-study reference. Many fresh, and some provocative, views are presented. Experimental and theoretical researchers in the field are also invited to examine the text. Problems at the end of a section are usually of the straightforward exercise type, directly connected with the material presented in that section. By doing these problems one by one, the reader may grasp the meanings of the newly introduced subjects more firmly. The book is based on the materials taught by S. Fujita for several courses in quantum theory of solids, advanced topics in modern physics, and quantum statistical mechanics at the University at Buffalo.

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