
Contents

Series Preface	ix
Preface	xiii
Overview	xv
1 Foundation Material	1
1.1 Results from Group Theory	1
1.2 Quadratic Congruences	10
1.3 Chebyshev Systems of Functions	15
2 Linear Algebra	23
2.1 Inner Product Spaces	23
2.2 Linear Functionals and Dual Spaces	26
2.3 A Special Class of Linear Operators	29
3 Characters of Finite Groups	33
3.1 Definition and Basic Properties of Characters	33
3.2 The Orthogonal Relations for Characters	40
4 The Fourier Transform	49
4.1 Definition and Some Properties	49
4.2 The Fourier Transform of Periodic Functions	54
4.2.1 Periodic functions on \mathbb{Z}_n	54
4.2.2 Periodic functions on \mathbb{Z}	57
4.3 The Inverse Fourier Transform	58

4.4	The Inversion Formula	59
4.5	Matrices of the Fourier Transform	61
4.6	Iterated Fourier Transforms	62
4.7	Is the Fourier Transform a Self-Adjoint Operator? . .	65
5	Convolution, Banach Algebras, and the Uncertainty Principle	67
5.1	The Convolution Operator	67
5.2	Banach Algebra	73
5.3	The Uncertainty Principle	76
6	A Reduction Theorem	81
6.1	The Tensor Decomposition of Vector Spaces	81
6.2	The Tensor Decomposition of the Fourier Transform	84
6.3	The Fourier Transform and Isometries	88
6.4	Reduction to Finite Cyclic Groups	89
7	Eigenvalues and Eigenvectors of the Fourier Transform	93
7.1	Symmetric and Antisymmetric Functions	93
7.2	Eigenvalues and Eigenvectors	98
7.3	Spectral Theorem	104
7.4	Ergodic Theorem	109
7.5	Multiplicities of Eigenvalues	112
8	The Quantum Fourier Transform	131
8.1	The Dirac Notation	131
8.2	The Fourier Transform in the Dirac Notation	134
9	Quadratic Gaussian Sums	141
9.1	The Number $G_n(1)$	141
9.2	Reduction Formulas	144
	References	153
	Index	155



<http://www.springer.com/978-0-8176-4915-9>

Fourier Analysis on Finite Abelian Groups

Luong, B.

2009, XVI, 159 p., Hardcover

ISBN: 978-0-8176-4915-9

A product of Birkhäuser Basel