Chapter 2
The Added Masses of Planar Contours Moving in an Ideal Unlimited Fluid

For calculation of added masses of various elongated bodies the planar sections technique (see Chap. 3) is widely applied. Application of this method requires a knowledge of the added masses of corresponding cross sections in a planar flow. Theory of functions of a complex variable allows us to describe explicitly the planar flow around most planar contours that correspond to real ship structures. In this chapter we discuss added masses of various planar contours moving in an infinite two-dimensional fluid. We give the main formulas for computation of added masses using techniques due to Sedov [206].

2.1 Sedov's Technique

Using Sedov's technique one can efficiently determine the added masses of planar contours moving in an infinite fluid [206]. Consider the function of a complex variable

$$z = f(\zeta) = \frac{k}{\zeta} + k_0 + k_1\zeta + k_2\zeta^2 + \cdots, \tag{2.1}$$

which defines the conformal map of the unit disc in ζ-plane to exterior (filled with fluid) of a given contour C in z-plane (the "uniformization map"). Then the added masses of the contour C moving in the fluid are given by the formulas:

$$\lambda_x = -\rho\big[S - 2\pi k\bar{k} + \pi(kk_1 + \bar{k}\bar{k}_1)\big]; \tag{2.2}$$

$$\lambda_y = -\rho\big[S - 2\pi k\bar{k} - \pi(kk_1 + \bar{k}\bar{k}_1)\big]; \tag{2.3}$$

$$\lambda_{xy} = i\rho\pi(kk_1 - \bar{k}\bar{k}_1); \tag{2.4}$$

$$\lambda_{x\omega} = \rho\big[Sy_* - \pi(kc_1 + \bar{k}\bar{c}_1)\big]; \tag{2.5}$$

$$\lambda_{y\omega} = \rho\big[-Sx_* + \pi i(kc_1 - \bar{k}\bar{c}_1)\big]; \tag{2.6}$$

$$\lambda_\omega = \frac{i\rho}{2}\oint_l \bar{w}_3\left(\frac{1}{\zeta}\right)\frac{dw_3}{d\zeta}d\zeta. \tag{2.7}$$

In formulas (2.2)–(2.7) we use the following notations: contour of integration l is the unit circle $|\zeta|=1$ taken in positive (counterclockwise) direction; $f(\zeta)$ is the "uniformization" function (2.1). Let us introduce the function $\bar{f}$ defined by its Laurent series at $\zeta=\infty$:

$$\bar{f}\left(\frac{1}{\zeta}\right) := \bar{k}\zeta + \bar{k}_0 + \frac{\bar{k}_1}{\zeta} + \frac{k_2}{\zeta^2} + \cdots,$$

where k, k_j $(j=0,1,2,\ldots)$ are the coefficients in the Laurent expansion (2.1). The function w_3 is defined as follows:

$$w_3(\zeta) = -\frac{1}{4\pi}\oint_l f(\eta)\bar{f}\left(\frac{1}{\eta}\right)\frac{\eta+\zeta}{\eta-\zeta}\frac{d\eta}{\eta};$$
$$\frac{dw_3}{d\zeta} = -\frac{1}{2\pi}\oint_l f(\eta)\bar{f}\left(\frac{1}{\eta}\right)\frac{d\eta}{(\eta-\zeta)^2};$$
$$\left(\frac{dw_3}{d\zeta}\right)_{\zeta=0} = c_1 = -\frac{1}{2\pi}\oint_l f(\eta)\bar{f}\left(\frac{1}{\eta}\right)\frac{d\eta}{\eta^2}.$$

Similarly we define the analytic function $\bar{w}_3$: if the function w_3 has the Taylor series $w_3(\zeta)=c_1\zeta+c_2\zeta^2+\cdots$, then

$$\bar{w}_3\left(\frac{1}{\zeta}\right) := \frac{\bar{c}_1}{\zeta} + \frac{\bar{c}_2}{\zeta^2} + \cdots.$$

The integral

$$S = -\frac{i}{2}\oint_l \overline{f(\zeta)}\frac{df}{d\zeta}d\zeta$$

gives the area of the interior of the contour C (notice that, in contrast to the function $\bar{f}(\zeta^{-1})$ which is holomorphic, the function $\overline{f(\zeta)}$ is an *antiholomorphic* function);

$$z_* \equiv x_* + iy_* := -\frac{i}{2S}\oint_l f(\zeta)\overline{f(\zeta)}\frac{df}{d\zeta}d\zeta$$

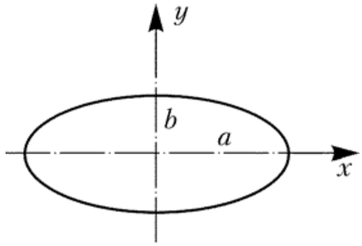
is the complex coordinate of the centroid of the figure bounded by the contour C. The overline here denotes ordinary complex conjugation.

These formulas show that the added masses of the contour C can be found if the conformal map (2.1) is known. Then the integrals over contour l can be computed by the residue theorem.

Below we use the formulas (2.2)–(2.7) for calculation of added masses of various contours. Due to the Cauchy theorem, the function $w_3(\zeta)$ can be found as follows [206]. Let us write $(i/2)f(\zeta)\bar{f}(1/\zeta)$ as $f_1(\zeta)+f_2(\zeta)$, where $f_1(\zeta)$ is regular for $|\zeta|<1$, and $f_2(\zeta)$ is regular for $|\zeta|>1$. Then $w_3(\zeta)=2f_1(\zeta)$.

Below we use this techniques to find added masses of various simple contours.

Fig. 2.1 Elliptic contour



2.2 The Added Masses of Simple Contours

2.2.1 Elliptic Contour, Circular Contour and Interval (Plate)

The map of the exterior of the ellipse with half-axes a and b (Fig. 2.1) to the interior of the unit circle in the ζ-plane is given by the function

$$z = f(\zeta) = -\frac{1}{2}\left[(a-b)\zeta + (a+b)\frac{1}{\zeta}\right].$$

Using the general formulas (2.2)–(2.7), we obtain

$$\begin{aligned} \lambda_{11} = \rho\pi b^2; \qquad \lambda_{22} = \rho\pi a^2;& \\ \lambda_{66} = \frac{\rho\pi}{8}\left(a^2 - b^2\right)^2;& \\ \lambda_{12} = \lambda_{16} = \lambda_{26} = 0.& \end{aligned} \tag{2.8}$$

Circle. The added masses of the circular contour are given by the formulas (2.8) assuming $a = b = r$. Then $\lambda_{11} = \lambda_{22} = \rho\pi r^2$; $\lambda_{12} = \lambda_{16} = \lambda_{26} = \lambda_{66} = 0$.

Interval (plate). The added masses of the interval (plate) of length $2a$ are given by the formulas (2.8) assuming $b = 0$. Then $\lambda_{22} = \rho\pi a^2$; $\lambda_{66} = \rho\pi a^4/8$; $\lambda_{11} = \lambda_{12} = \lambda_{16} = \lambda_{26} = 0$.

2.2.2 Elliptic Contour with One Rib, T-shape Contour

The conformal map of the exterior of an ellipse with one rib (Fig. 2.2) in the z-plane to the unit disc in the ζ-plane is given by

$$\begin{aligned} z = f(\zeta) = \frac{ci}{2}\Bigg\{ & \frac{(a+b)(1+m)}{4c}\left(\zeta + \frac{1}{\zeta}\right) + \frac{m-1}{2}\frac{a+b}{c} \\ & + \frac{a+b}{c}\left[\left(\frac{1+m}{4}\frac{1}{\zeta} + \frac{1+m}{4}\zeta + \frac{m-1}{2}\right)^2 - 1\right]^{1/2} \\ & + \frac{c/(a+b)}{\frac{m-1}{2} + \frac{1+m}{4}(\zeta + \frac{1}{\zeta}) + [(\frac{1+m}{4}\zeta + \frac{1+m}{4}\frac{1}{\zeta} + \frac{m-1}{2})^2 - 1]^{1/2}}\Bigg\}, \end{aligned} \tag{2.9}$$

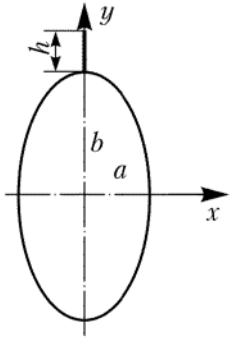


Fig. 2.2 Elliptic contour with one rib

where a, b are the half-axes of the ellipse; $c = \sqrt{b^2 - a^2}$; h is the height of the rib; $m = (b+h)/(a+b) + a/(b+h+\sqrt{a^2+h^2+2bh})$; $i = \sqrt{-1}$. By expanding the right-hand side of (2.9) in powers of ζ, we obtain

$$k = i\frac{(a+b)(1+m)}{4}; \qquad k_0 = i\frac{(a+b)(m-1)}{2};$$
$$k_1 = i\frac{(a+b)^2(m^2+2m-3)+4(b^2-a^2)}{4(a+b)(m+1)}; \qquad k_2 = i\frac{4a(m-1)}{(m+1)^2};$$
$$k_3 = -i\frac{2a(3m^2-10m+7)}{(1+m)^3}; \qquad c_1 = -\frac{i}{4}\left(m^2-1\right)(a+b)^2.$$

Using the general Sedov formulas we obtain the following expressions for the added masses:

$$\lambda_{11} = \frac{\rho\pi b^2}{4}\left[(m+1)^2\left(1+\frac{a}{b}\right)^2 - 4\frac{a}{b}\left(2+\frac{a}{b}\right)\right];$$
$$\lambda_{22} = \pi\rho a^2; \qquad \lambda_{16} = -\frac{\pi\rho b^3}{8}\left(1+\frac{a}{b}\right)^3\left(m^2-1\right)(m+1);$$
$$\lambda_{66} = \rho\pi\frac{(a+b)^2}{2^7}\left[(a+b)^2\left(9m^4+4m^3-10m^2+4m-7\right)+16(b-a)^2\right];$$
$$\lambda_{12} = \lambda_{26} = 0. \tag{2.10}$$

Dependence of coefficients

$$k_{11} = \frac{\lambda_{11}}{\pi\rho b^2}; \qquad k_{16} = \frac{8\lambda_{16}}{\pi\rho b^3};$$
$$k_{66} = \frac{128\lambda_{66}}{\pi\rho b^4}$$

on parameters h/b and a/b is shown in Figs. 2.4–2.6.

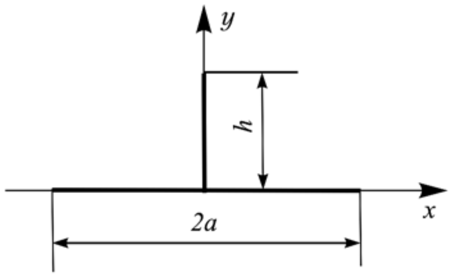


Fig. 2.3 T-shape profile

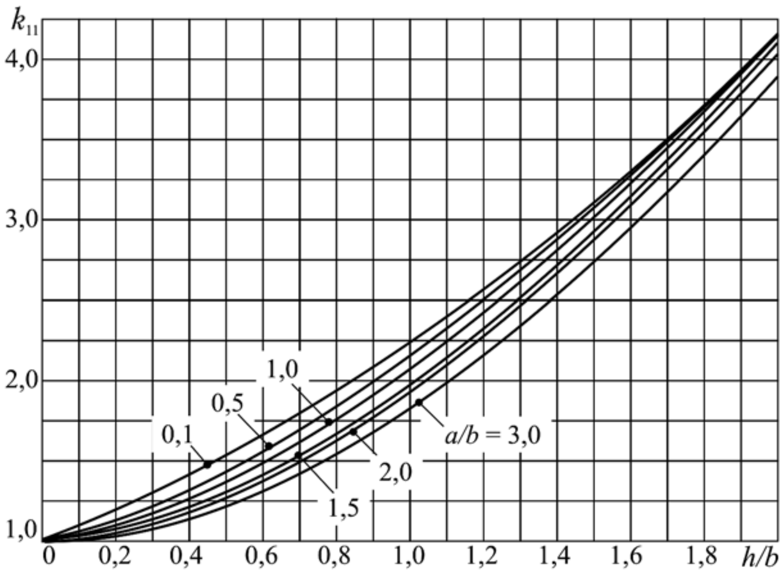


Fig. 2.4 Coefficient k_{11} of added masses of an ellipse with one rib

T-shape. The added masses of the T-shape (Fig. 2.3) can be obtained from formulas (2.10) assuming that $b = 0$:

$$m = \frac{h}{a} + \frac{a}{h + \sqrt{a^2 + h^2}};$$
$$\lambda_{11} = \frac{\pi}{4}\rho a^2\left[(m+1)^2 - 4\right]; \qquad \lambda_{22} = \pi\rho a^2;$$
$$\lambda_{16} = -\frac{\pi}{8}\rho a^3\left(m^2 - 1\right)(m+1);$$
$$\lambda_{66} = \frac{\pi}{2^7}\rho a^4\left(9m^4 + 4m^3 - 10m^2 + 4m + 9\right); \qquad \lambda_{12} = \lambda_{26} = 0.$$

The coefficients $k_{11} = \lambda_{11}/(\rho\pi a^2)$; $k_{16} = (8\lambda_{16})/(\rho\pi a^3)$; $k_{66} = (128\lambda_{66})/(\pi\rho a^4)$ are presented in Table 2.1 for $0.1 \le h/a \le 5$.

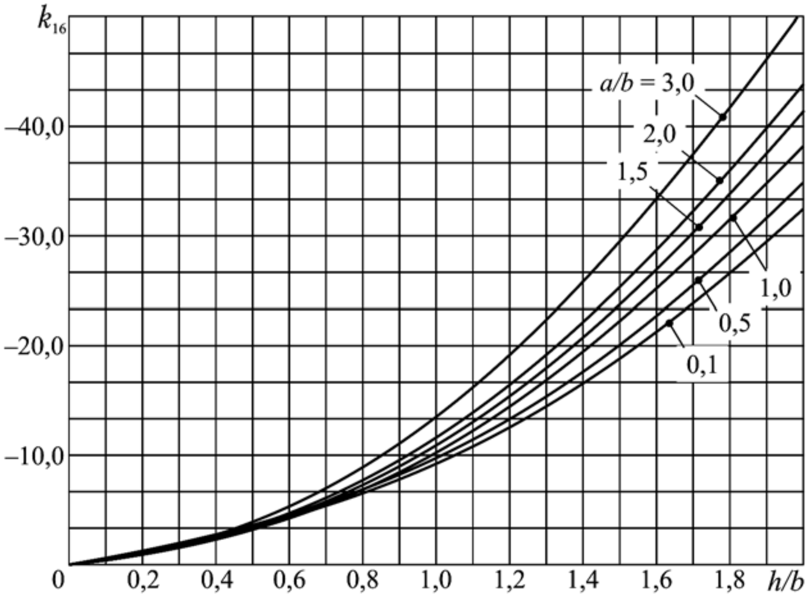


Fig. 2.5 Coefficient k_{16} of added masses of an ellipse with one rib

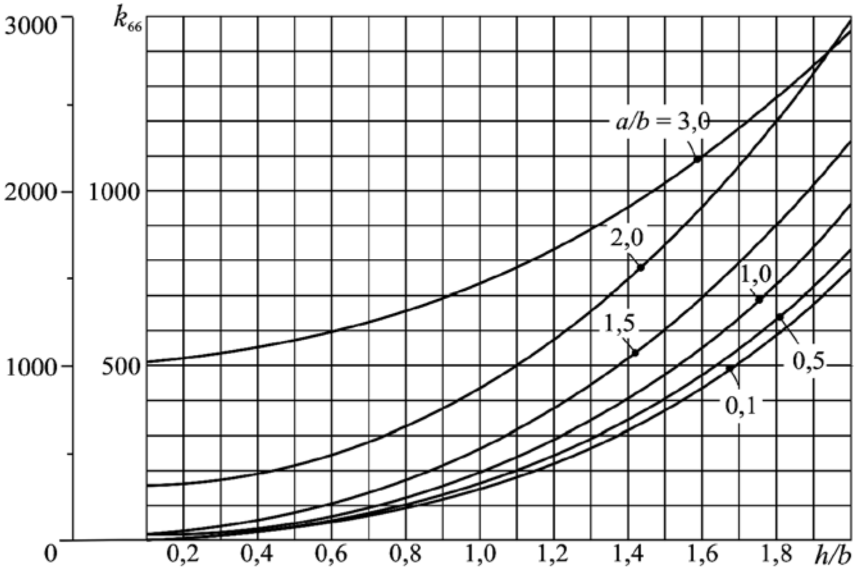


Fig. 2.6 Coefficients k_{66} of added masses of an ellipse with one rib. The *left vertical axis* corresponds to the curve $a/b = 3.0$; the *right vertical axis* corresponds to other values of a/b

Table 2.1 Coefficients of added masses of T-shape profile

h/a	k_{11}	k_{16}	k_{66}
0.1	0.005	−0.020	16.2
0.2	0.020	−0.082	16.7
0.3	0.044	−0.184	17.5
0.4	0.079	−0.332	18.8
0.5	0.122	−0.529	20.6
0.7	0.233	−1.090	24.0
1.0	0.457	−2.41	42.0
1.5	0.964	−6.31	102.0
2.0	1.62	−12.9	238
3.0	3.33	−37.4	853
4.0	5.56	−82.0	2690
5.0	8.30	−152	6380

2.2.3 Elliptic Contour with Two Symmetric Ribs

The exterior of the contour in the z-plane (Fig. 2.7) is mapped to the unit disc in the ζ-plane by function

$$z = f(\zeta) = \frac{c}{2}\left[\frac{m(a+b)}{2c}\left(\zeta + \frac{1}{\zeta}\right) + \frac{a+b}{c}\sqrt{\frac{m^2}{4}\left(\zeta + \frac{1}{\zeta}\right)^2 - 1}\right.$$
$$\left. + \frac{c}{a+b}\frac{1}{\frac{m}{2}(\zeta + \frac{1}{\zeta}) + \sqrt{\frac{m^2}{4}(\zeta + \frac{1}{\zeta})^2 - 1}}\right],$$

where

$$c = \sqrt{a^2 - b^2}; \qquad m = \frac{a+h}{a+b} + \frac{b}{a+h+\sqrt{b^2+h^2+2ah}};$$

h is the height of the ribs.

Expanding the function $f(\zeta)$ in powers of ζ, we obtain the coefficients

$$k = \frac{1}{2}(a+b)m; \qquad k_0 = 0; \qquad k_1 = \frac{1}{2m}\left[(a+b)\left(m^2 - 1\right) + a - b\right];$$
$$k_2 = 0; \qquad k_3 = \frac{(m^2-1)b}{m^3}.$$

Then it is easy to find the added masses

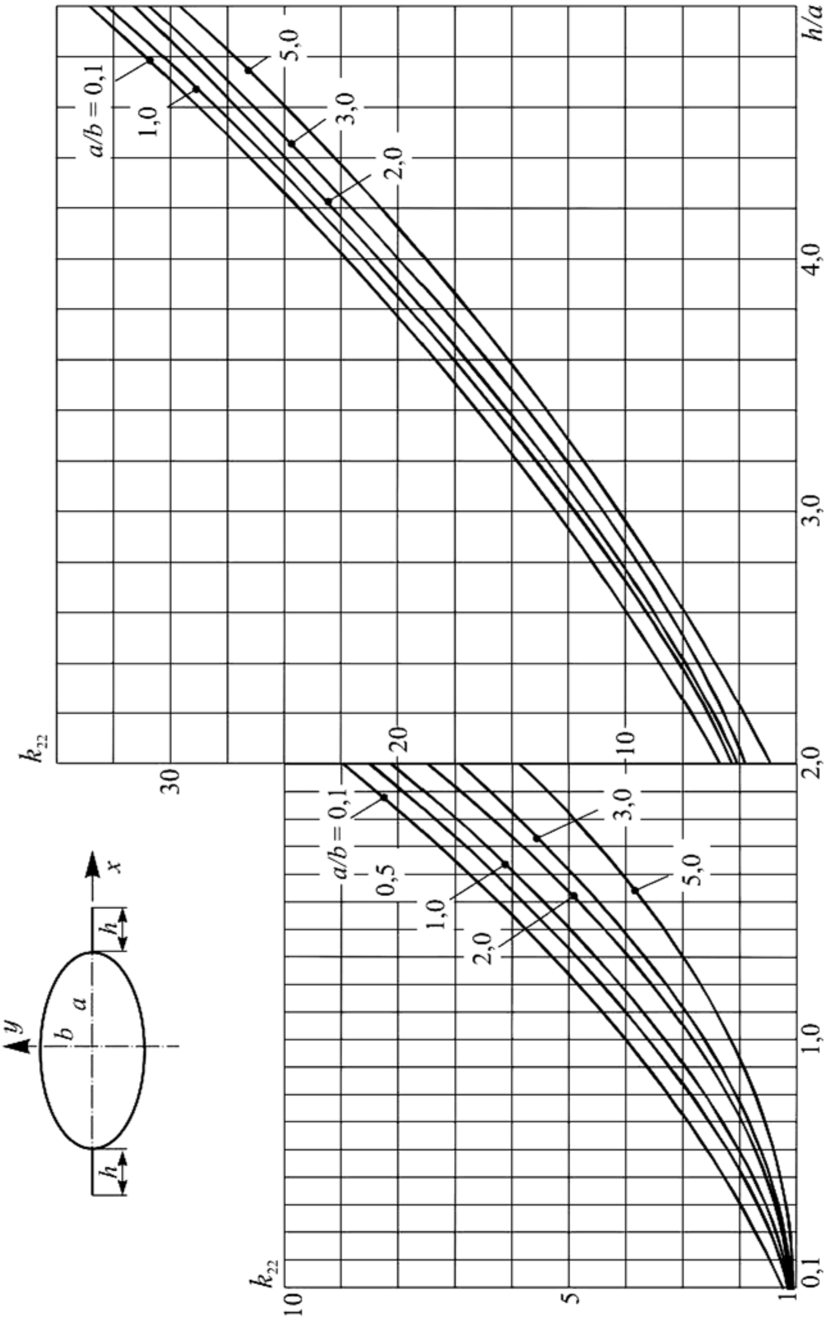


Fig. 2.7 Coefficient k_{22} for elliptic contour with two ribs

$$\lambda_{11} = \rho\pi b^2; \qquad \lambda_{22} = \rho\pi a^2\left[m^2\left(1+\frac{b}{a}\right)^2 - 2\frac{b}{a} - \frac{b^2}{a^2}\right];$$

$$\lambda_{66} = \frac{\pi}{8}\rho(a+b)^2\left[(a+b)^2\left(m^4-1\right)+(a-b)^2\right];$$

$$\lambda_{12} = \lambda_{16} = \lambda_{26} = 0. \tag{2.11}$$

The values of coefficients $k_{22} = \lambda_{22}/(\pi\rho a^2)$ and $k_{66} = (8\lambda_{66})/(\pi\rho a^4)$ are given in Figs. 2.7–2.9.

Choosing in (2.11) $b = a$, we obtain the expressions for the added masses of a circle with two ribs.

Alternative expressions for the added masses of the circle of radius a with two symmetric ribs of height h are given in the book [158]:

$$\lambda_{22} = \pi\rho s^2\left(1-\frac{a^2}{s^2}+\frac{a^4}{s^4}\right); \qquad s = a+h;$$

$$\lambda_{66} = \frac{\pi\rho s^4}{8}\left\{\left[\left(1+R^2\right)^2 \arctan\frac{1}{R}\right]^2 + 2R\left(1-R^2\right)\left(R^4-6R^2+1\right)\arctan\frac{1}{R} - \pi^2R^4 + R^2\left(1-R^2\right)^2\right\}$$

where $R = a/(a+h)$.

2.2.4 Elliptic Contour with Horizontal and Vertical Ribs

The added masses of an elliptic contour with two horizontal ribs of the same height and with two vertical ribs of different height (Fig. 2.10) are defined by the formulas [158]

$$\lambda_{22} = \pi\rho\left(4c^2 - k^2 - 2ab - b^2\right),$$

$$\lambda_{33} = \pi\rho\frac{s^2(a^2+b^2)+2ab^2(a-b)-2abs(s^2-a^2+b^2)^{1/2}}{(a-b)^2}$$

where the parameters c and k can be found from the following equations:

$$k = \frac{as-b(s^2+a^2-b^2)^{1/2}}{a-b}; \qquad c = \frac{f_1+f_2}{4};$$

$$f_1^2 = k^2 + \left[\tau_1 + \frac{(a+b)^2}{4\tau_1}\right]^2; \qquad f_2^2 = k^2 + \left[\tau_2 + \frac{(a+b)^2}{4\tau_2}\right]^2;$$

$$\tau_1 = \frac{1}{2}\left[t_1 + \left(t_1 - a^2 + b^2\right)^{1/2}\right]; \qquad \tau_2 = \frac{1}{2}\left[t_2 + \left(t_2 - a^2 + b^2\right)^{1/2}\right].$$

The values a, b, s, t_1, t_2 are defined as shown in Fig. 2.10.

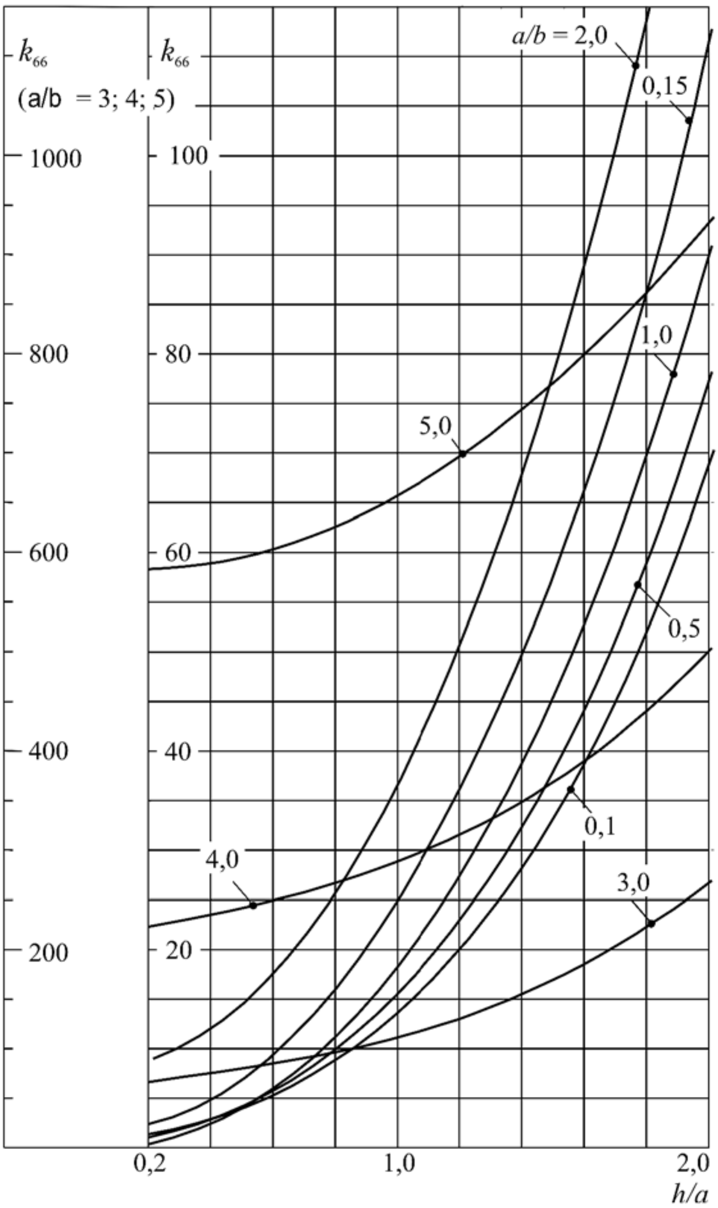


Fig. 2.8 Coefficient k_{66} for an elliptic contour with two ribs. The *left vertical axis* corresponds to values $a/b = 3, 4, 5$; the *right vertical axis* corresponds to all other values of a/b

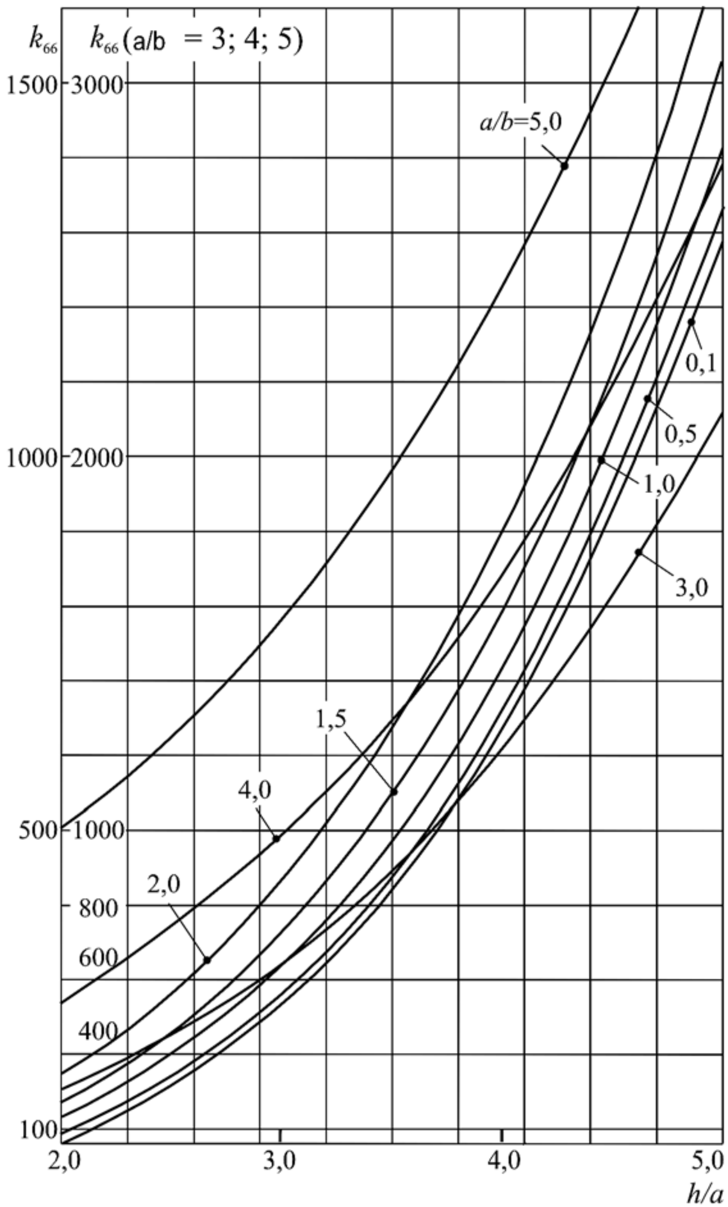
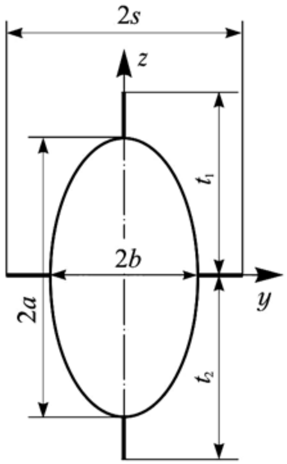


Fig. 2.9 Coefficient k_{66} for elliptic contour with two ribs. The *left vertical axis* corresponds to the values of a/b equal to 3, 4, 5; the *right vertical axis* corresponds to all other values of a/b

Fig. 2.10 Elliptic contour with four ribs



2.2.5 *Symmetrical Profile Made up of Two Intersecting Intervals (Plates)*

For this type of profile (Fig. 2.11) the added masses can be obtained as a partial case of formulas (2.11) assuming that $a = 0$:

$$m = \frac{h}{b} + \frac{b}{h + \sqrt{b^2 + h^2}};$$
$$\lambda_{11} = \pi\rho b^2; \qquad \lambda_{22} = \rho\pi h^2;$$
$$\lambda_{66} = \pi\rho b^4 \frac{m^4}{8}; \qquad \lambda_{12} = \lambda_{16} = \lambda_{26} = 0.$$

The values of the coefficient $k_{66} = (8\lambda_{66})/(\pi\rho b^4)$ are given in Table 2.2.

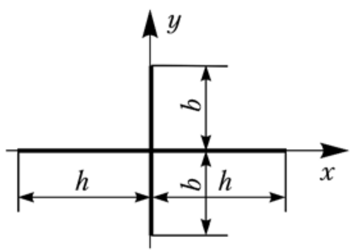


Fig. 2.11 Symmetric profile consisting of two plates intersecting at a right angle

Table 2.2 Coefficient k_{66} for some values of h/b

h/b	0.1	0.2	0.3	0.4	0.5	0.7	1.0	1.5	2.0	3.0	4.0	5.0
k_{66}	1.02	1.08	1.19	1.34	1.56	2.22	4.0	10.6	25.0	100.0	289.0	676.0

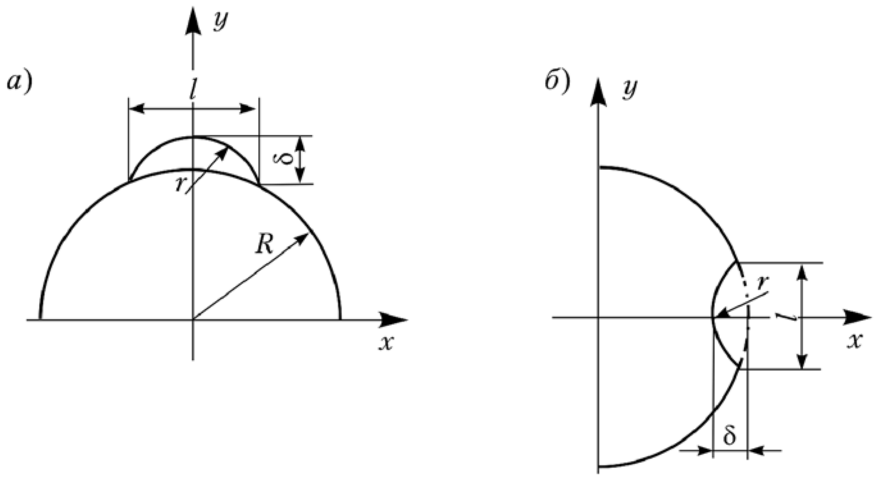


Fig. 2.12 Circle with two external **(a)** and two internal **(b)** hitches

2.2.6 Circle with Two Hitches

The added masses of the circular contour with two external hitches (Fig. 2.12a) are determined by the method of electro-hydrodynamic analogy (see Chap. 9) in [48]. The parameters of the hitch are: $\delta/r = 0.077$; $l/r = 0.76$; $s/(\pi R^2) = 0.027$ where δ is the height of the hitch, r is the radius of curvature of the hitch, l is the length of the hitch, s is the area of the hitch, R is the radius of the circle. The coefficients of added masses for the circle with external hitches are given by: $k_{11} = \lambda_{11}/\pi\rho R^2 = 1.09$; $k_{22} = \lambda_{22}/\pi\rho R^2 = 0.954$. If there are two internal hitches (Fig. 2.12b) with parameters $\delta/r = 0.115$; $l/r = 0.55$; $s/(\pi R^2) = 0.03$, then $k_{11} = 1.06$.

For other positions of hitches on the circle and other contours the added masses were also found in [48].

2.2.7 Circle with Two Side Ribs

The added masses of the circle with two plates located at the angle of 45 degrees to the diameter are found in [201]. The experimental results can be presented as the following graphs:

$$k_{22} = \frac{\lambda_{22}}{\rho\pi R^2} = f_1\left(\frac{h}{2R}\right); \qquad k_{33} = \frac{\lambda_{33}}{\rho\pi R^2} = f_2\left(\frac{h}{2R}\right),$$

where R is the radius of the main circle; h is the height of the ribs. These curves are shown in Fig. 2.13. In the same figure we show the dependence of dimensionless coordinate $l/2R$ of the point of application of inertial forces on $h/2R$. Knowing l one can compute the added mass $\lambda_{24} = l\lambda_{22}$.

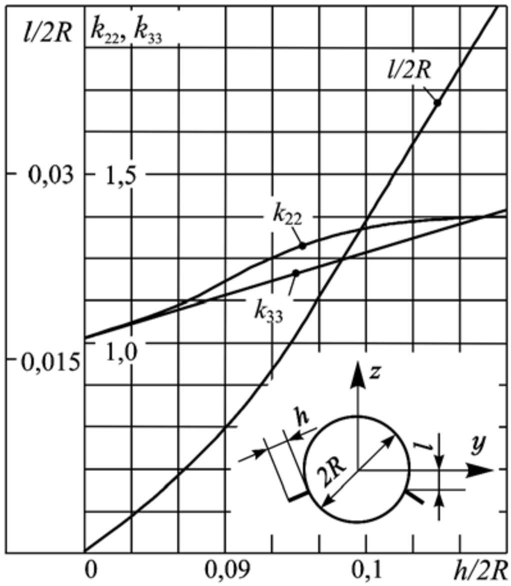


Fig. 2.13 Coefficient of added masses of circle with two side ribs

2.2.8 Circle with Cross-like Positioned Ribs

The formulas for the added masses of the circle with cross-like positioned ribs of the same height are as follows [158]:

$$\lambda_{22} = \lambda_{33} = \pi \rho s^2 \left(1 - \frac{a^2}{s^2} + \frac{a^4}{s^4}\right),$$

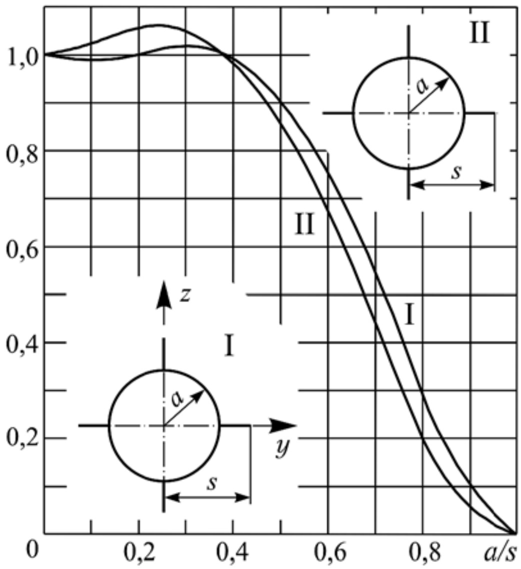
where a is the radius of the circle; $s = a + h$; h is the height of the ribs (Fig. 2.14). The added mass $\lambda_{66} = 2\rho s^4 k_{66} a/(\pi s)$, where the coefficient k_{66} can be found from Fig. 2.14 (the curve I).

For comparison in the same Fig. 2.14 we draw the curve II which shows the dependence of the coefficient $k_{66} = (8\lambda_{66})/(\pi \rho s^4)$ on a/s for the circle with *two* symmetric ribs (the angle between the ribs is equal to π).

If the heights of vertical ribs on the circle differ from the heights of the horizontal ribs, then the added masses of the contour are as follows:

$$\lambda_{22} = \frac{\pi \rho s^2}{4} \left\{ \frac{b^2}{s^2}\left(1 + \frac{a^4}{b^4}\right) + \frac{c^2}{s^2}\left(1 + \frac{a^4}{c^4}\right) - 2\left(1 + \frac{a^2}{s^2}\right)^2 \right.$$
$$\left. + 2\left[\left(1 + \frac{a^4}{s^2 b^2}\right)\left(1 + \frac{b^2}{s^2}\right)\left(1 + \frac{a^4}{s^2 c^2}\right)\left(1 + \frac{c^2}{s^2}\right)\right]^{1/2} \right\};$$
$$\lambda_{33} = \pi \rho s^2 \left(1 - \frac{a^2}{s^2} + \frac{a^4}{s^4}\right).$$

Fig. 2.14 Coefficient of added masses of a circle with four cross-like positioned ribs



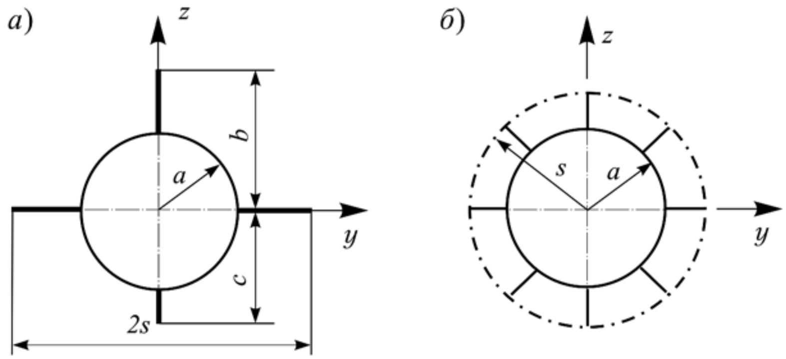


Fig. 2.15 Circle with asymmetric **(a)** and symmetric **(b)** lateral ribs

If on the circle there are three or more equidistant ribs (see Fig. 2.15b), then

$$\lambda_{22} = \lambda_{33} = 2\pi\rho s^2 \left\{ \left[\frac{1 + (a^2/s^2)^{n/2}}{2} \right]^{4/n} - \frac{1}{2}\left(\frac{a}{s}\right)^2 \right\};$$

$$\lambda_{66} = 0.533\rho s^4, \quad \text{if } n = 3,\ a = 0;$$
$$\lambda_{66} = \frac{2}{\pi}\rho s^4, \quad \text{if } n = 4,\ a = 0;$$
$$\lambda_{66} = \frac{\pi}{2}\rho s^4, \quad \text{if } n = \infty,\ a = 0.$$

2.2.9 Circle with Two Tangent Horizontal Ribs

If two horizontal ribs of span $2s$ are tangent to circle of radius a (Fig. 2.16) and also there are two vertical ribs of different heights, then the added masses are given by [158]:

$$\lambda_{22} = 2\pi\rho\left\{c^2 - \frac{a^2}{2} + \frac{4c^2\sin\lambda\cos^2(\lambda/2)}{3(\lambda+\sin\lambda)}\left[\sin^2\frac{\lambda}{2} - \frac{3\lambda\cos^2(\lambda/2)}{\lambda+\sin\lambda}\right] + 2\left(r^2 - c^2\right)\right\},$$

$$\lambda_{33} = 2\pi\rho\left\{c^2 - \frac{a^2}{2} - \frac{4c^2\sin\lambda\cos^2(\lambda/2)}{3(\lambda+\sin\lambda)}\left[\sin^2\frac{\lambda}{2} - \frac{3\lambda\cos^2(\lambda/2)}{\lambda+\sin\lambda}\right]\right\},$$

where the parameter λ is defined from the equation

$$\frac{a}{s} = \frac{1}{\pi}\left\{\operatorname{arcsh}\left(\frac{\lambda}{2}\tan\frac{\lambda}{2}\right)^{1/2} + \left[\frac{\lambda}{2}\tan\frac{\lambda}{2} + \left(\frac{\lambda}{2}\right)^2\tan^2\frac{\lambda}{2}\right]^{1/2}\right\}.$$

Then one finds the values $c = a\pi/(\lambda+\sin\lambda)$; variable h is determined by the equation

$$2 + \frac{b}{2} = \pi\left(\frac{\lambda}{h/c+1} + \arctan\frac{\sin\lambda}{h/c-\cos\lambda}\right)^{-1};$$

the parameter f is determined by the equation

$$\frac{d}{a} = \pi\left(\frac{\lambda}{f/c-1} + \arctan\frac{\sin\lambda}{f/c+\cos\lambda}\right)^{-1}.$$

Then we compute

$$r = \frac{1}{4}\left(h + \frac{c^2}{h} + f + \frac{c^2}{f}\right).$$

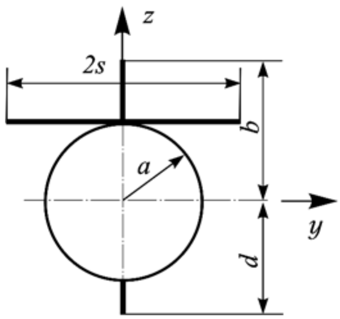


Fig. 2.16 Circle with horizontal ribs located in a tangent plane

2.2.10 Regular Inscribed Polygon

The values for the added masses of the regular polygon inscribed in the circle with the radius a depend on the number of its sides n and are defined by the following formulas [158]:

$$\begin{aligned}
\lambda_{22} &= \lambda_{33} = 0.654\pi\rho a^2, \quad \text{if } n = 3,\\
\lambda_{22} &= \lambda_{33} = 0.787\pi\rho a^2, \quad \text{if } n = 4,\\
\lambda_{22} &= \lambda_{33} = 0.823\pi\rho a^2, \quad \text{if } n = 5,\\
\lambda_{22} &= \lambda_{33} = 0.867\pi\rho a^2, \quad \text{if } n = 6.
\end{aligned}$$

2.2.11 Zhukowskiy's Foil Profile

The expressions for the added masses of the Zhukowskiy foil profile (Fig. 2.17) were derived by L. Sedov [206]:

$$\lambda_{11} = \frac{\pi\rho a^2}{4}\left(r^2 + R^2 - 2\cos 2\alpha\right); \qquad \lambda_{22} = \frac{\pi\rho a^2}{4}\left(r^2 + R^2 + 2\cos 2\alpha\right);$$

$$\lambda_{12} = \frac{\pi\rho a^2}{2}\sin 2\alpha; \qquad \lambda_{16} = \frac{\pi\rho a^3}{8}\left[r^2 + R^2 + 4(r+R)\cos\alpha\right]\sin\alpha;$$

$$\lambda_{26} = \frac{\rho\pi a^3}{8}\left[r^3 + R^3 + \left(r^2 + R^2\right)\cos\alpha + 2(r+R)\cos 2\alpha\right];$$

$$\lambda_{66} = \frac{\rho\pi a^4}{8}\left(8r^2R^2\cos^4\alpha - 2rR\sin^2 2\alpha + \cos 4\alpha\right)r^2R^2, \tag{2.12}$$

where the parameters a, α, R, r of the formulas can be approximately expressed via the geometrical characteristics of the given profile [110]: the value of the chord c,

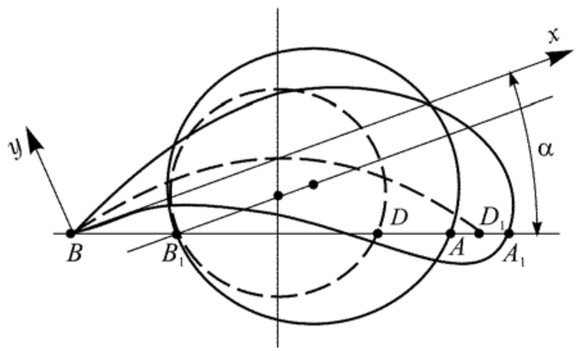


Fig. 2.17 Foil profile of Zhukowskiy

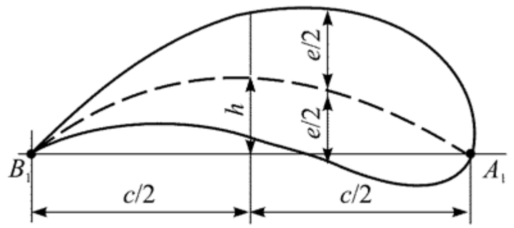


Fig. 2.18 Characteristic dimensions of foil profile of N.E. Zhukowskiy

the maximal thickness of the profile e_m and the height of the arch h (Fig. 2.18):

$$\mu = 0.77\frac{e_m}{c - 0.6e_m}; \qquad a = \frac{c}{2(1+\mu^2)};$$
$$R = \frac{1+\mu}{\cos\alpha}; \qquad r = \frac{1+\mu}{\cos\alpha(1+2\mu)}; \qquad \tan\alpha = \frac{2h}{c}\left(1+\mu^2\right).$$

The chord c of the profile is determined by the length of the interval A_1B_1 connecting the profile back edge with the frontal point A_1 posed at maximal distance from the back edge. The local thickness of the profile e, the skeleton line position (dotted line on the figure), and the arch height h are defined by the scheme shown in Fig. 2.18. For convenience we show in Figs. 2.19–2.23 the graphs obtained by using

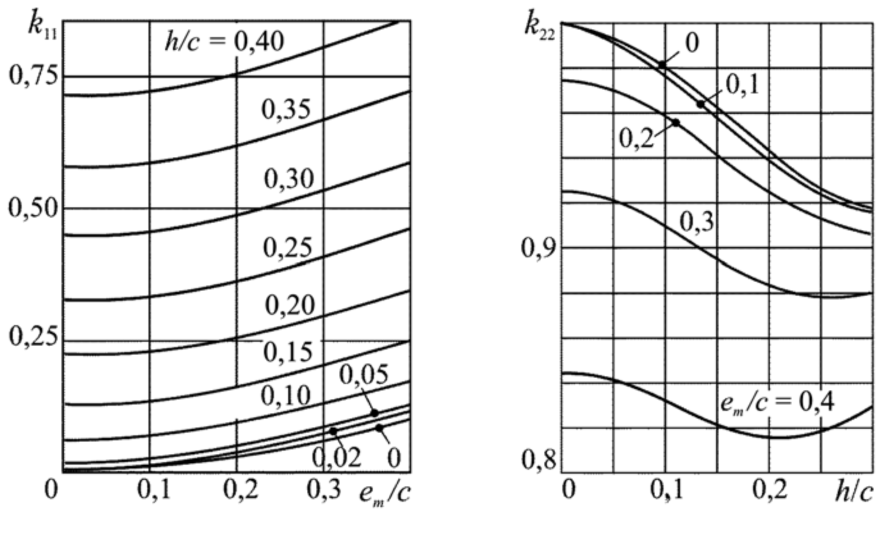


Fig. 2.19 Coefficients of added masses k_{11} and k_{22} of foil profile of Zhukowskiy

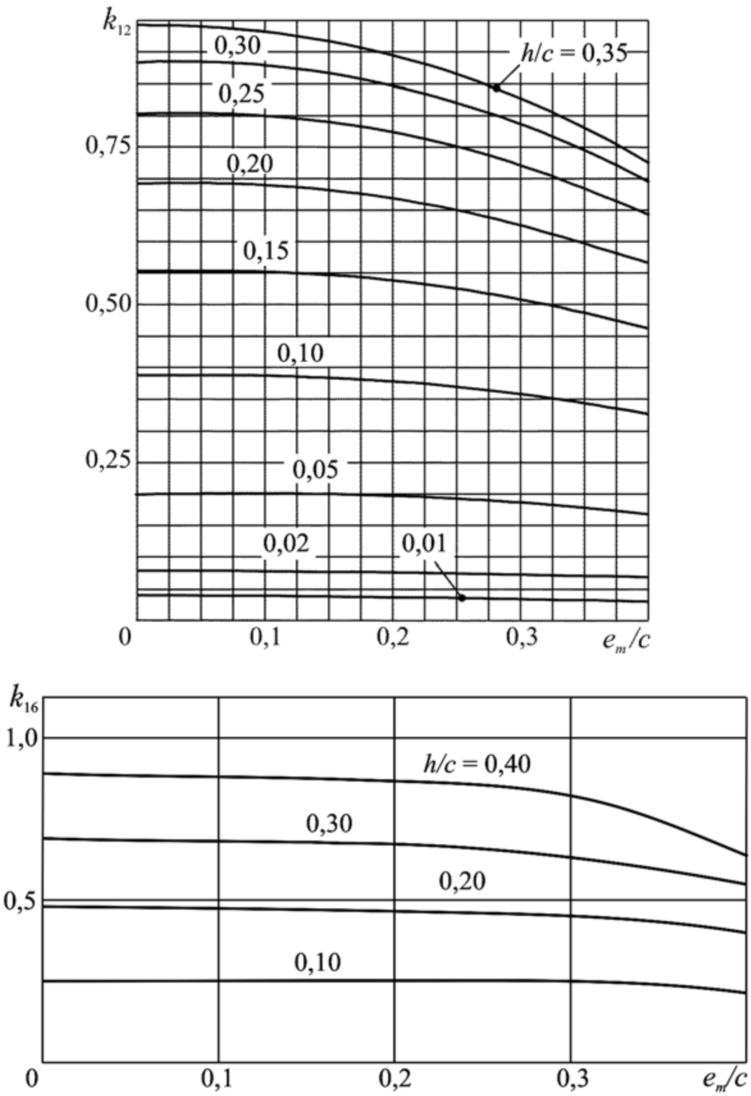


Fig. 2.20 Coefficients of added masses k_{12} (*above*) and k_{16} of Zhukowskiy's foil profile

the expressions (2.12).

$$k_{11}\left(\frac{h}{c},\frac{e_m}{c}\right) := \frac{4\lambda_{11}}{\rho\pi c^2}; \qquad k_{22}\left(\frac{h}{c},\frac{e_m}{c}\right) := \frac{4\lambda_{22}}{\rho\pi c^2}; \qquad k_{12}\left(\frac{h}{c},\frac{e_m}{c}\right) := \frac{8\lambda_{12}}{\rho\pi c^2};$$

$$k_{16}\left(\frac{h}{c},\frac{e_m}{c}\right) := \frac{8\lambda_{16}}{\rho\pi c^3}; \qquad k_{26}\left(\frac{h}{c},\frac{e_m}{c}\right) := \frac{8\lambda_{26}}{\rho\pi c^3}; \qquad k_{66}\left(\frac{h}{c},\frac{e_m}{c}\right) := \frac{16\lambda_{66}}{\rho\pi c^4};$$

$$\tan\alpha = \tan\alpha\big[(h/c, e_m/c)\big]; \qquad \mu = \mu\left(\frac{e_m}{c}\right).$$

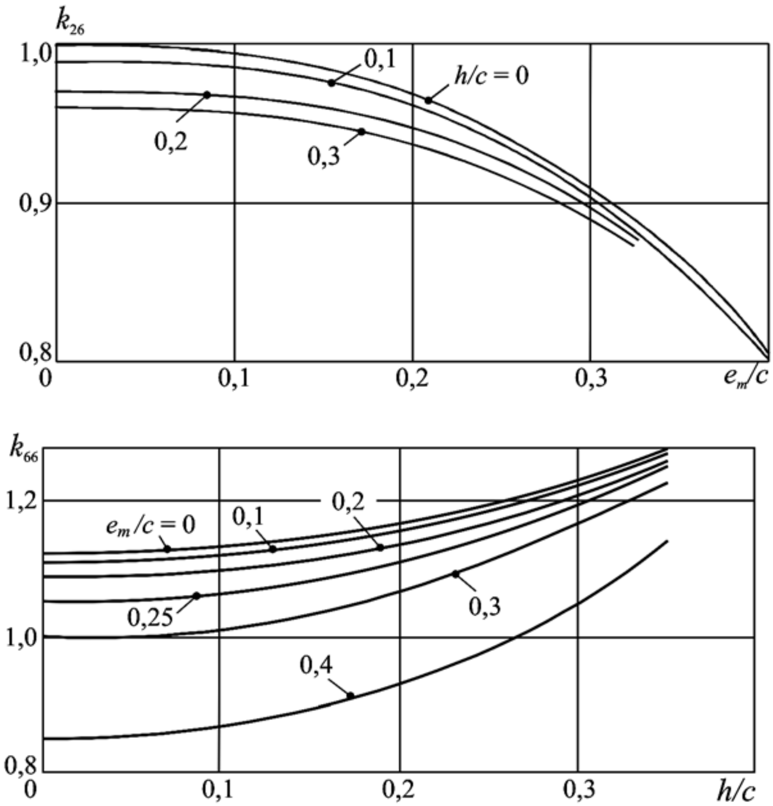


Fig. 2.21 Coefficients of added masses k_{26} (*above*) and k_{66} of Zhukowskiy's foil profile

We stress here that the coordinate system xy has its origin at the back edge of the profile, and the axis x has the angle α with the chord of the profile A_1B_1 (Fig. 2.17).

2.2.12 Arch of the Circle under Different Positions of Coordinate Axes

The added masses of the arch of a circle under various choices of coordinate system (Fig. 2.24) are given by the formulas [206]:

$$\lambda_{11} = \frac{\rho\pi a^2}{2}\tan^2\alpha; \qquad \lambda_{22} = \frac{\rho\pi a^2}{2}\left(1+\frac{1}{\cos^2\alpha}\right); \qquad \lambda_{12} = \lambda_{26} = 0.$$

If the origin is located at the middle of the arch as shown in Fig. 2.24a, then

$$\lambda_{16} = \frac{\rho\pi a^3}{4}\frac{\sin\alpha}{\cos^3\alpha}, \qquad \lambda_{66} = \frac{\rho\pi a^4}{8}\frac{1}{\cos^4\alpha}.$$

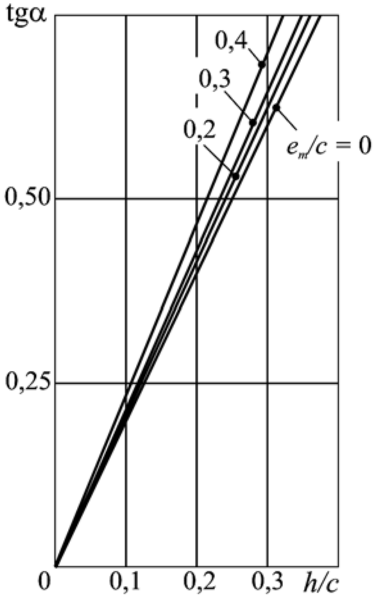


Fig. 2.22 Relation between the angle of (zero) lifting force with thickness and height of the arch of a Zhukowskiy profile

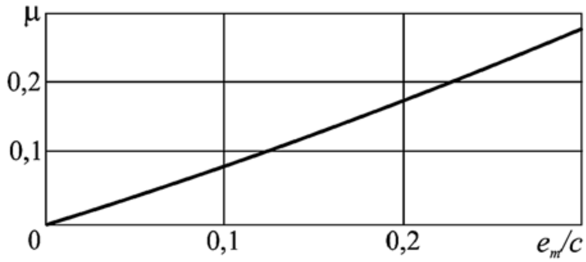


Fig. 2.23 Relation between the parameter μ and the relative width of Zhukowskiy's foil profile

If the origin coincides with the center of the circle as shown in Fig. 2.24b, then $\lambda_{16} = \lambda_{66} = 0$.

2.2.13 Lense Formed by Two Circular Arches

The added masses of the lens formed by two circular arches of radius R are given by [183]:

$$\lambda_{11} = \rho R^2 \left[\sin 2\beta - \frac{2\beta}{180}\pi + 2\pi \sin^2 \beta \frac{\frac{\beta}{180}(2 - \frac{\beta}{180})}{3(1 - \frac{\beta}{180})^2} \right];$$

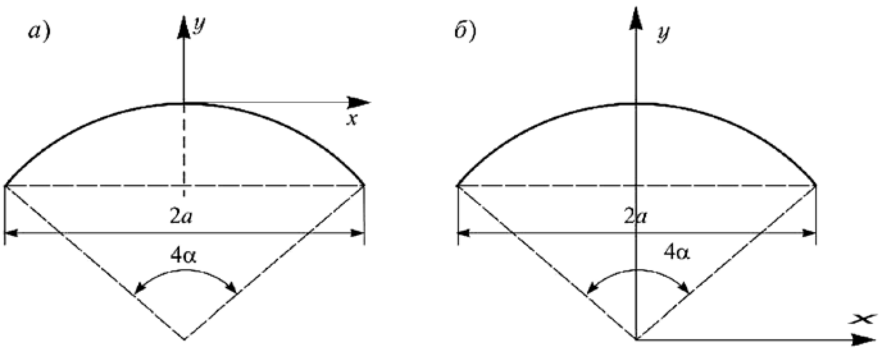


Fig. 2.24 Choice of coordinate axes for the arch of the circle

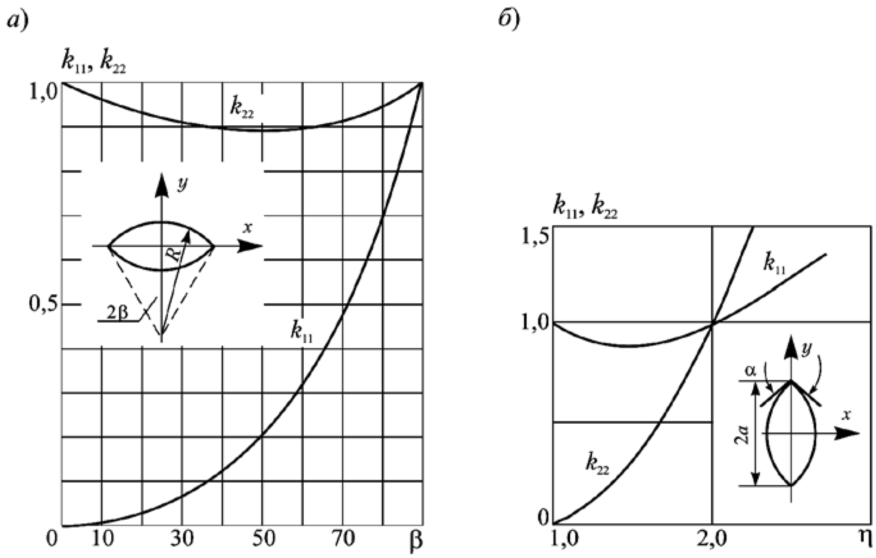


Fig. 2.25 Added masses of the lens formed by two circular arches

$$\lambda_{22} = \rho R^2 \left[\sin 2\beta - \frac{2\beta}{180}\pi + 2\pi \sin^2 \beta \frac{3 - 4\frac{\beta}{180} + 2(\frac{\beta}{180})^2}{3(1 - \frac{\beta}{180})^2} \right],$$

where 2β is the central angle of the arches thus formed (in degrees). The coefficients $k_{11} = \lambda_{11}/(\rho\pi R^2 \sin^2 \beta)$, $k_{22} = \lambda_{22}/(\rho\pi\ R^2 \sin^2 \beta)$ can be found from Fig. 2.25a.

These results are generalized in the work [80]. The dependence of coefficients k_{11} and k_{22} on η is shown in Fig. 2.25b (where $k_{11} = \lambda_{11}/\pi\rho a^2$; $k_{22} = \lambda_{22}/\pi\rho a^2$; $\alpha = 2\pi/\eta$).

2.2.14 Hexagon, Rectangle, Rhomb, Octagon, Square with Four Ribs

The formulas for the added masses of hexagon (derived by Sokolov), rhomb and rectangle (Fig. 2.26) are presented in the works [183, 206].

The graphs for coefficient $k_{11} = \lambda_{11}/(\rho\pi b^2)$ as a function of d/b for the cases of a hexagon (for various angles β), a rectangular (curve I) and a rhomb (curve II) are shown in Fig. 2.26.

Let us consider the flow around two rhombs located next to each other in such a way that they touch each other at a corner and their orientation is the same. Then in the flow orthogonal to the line connecting centers of the rhombs, the values λ_{11} for the added mass of each rhomb in this system is 1.55 times higher in comparison with its added mass in an infinite fluid [177]. The added moments of inertia of a rectangle are computed in the work [246]. Dependence of coefficient $k_{66} = 8\lambda_{66}/\rho\pi b^4$ on the ratio a/b of the sides of the rectangle under rotation of the rectangle around the central point are shown in Fig. 2.27.

Under the rotation of a regular hexagon around the central point, its added moment of inertia is defined by approximate formula [246] $\lambda_{66} = 0.055\pi\rho a^4$, where $2a$ is the distance between the parallel opposite edges of the hexagon. The values for the added mass $\lambda_{11} = k_{11}\pi\rho a^2$ and the added moment of inertia $\lambda_{66} = k_{66}(\pi/8)\rho a^4$ of the square with the side $2a$ and four ribs of length d are presented in Fig. 2.28 as functions of the ratio d/a. The square is assumed to rotate around its central point.

2.2.15 Plate with Flap

The added masses of a plate with a flap are of particular practical interest, since such contour gives a good approximation to a flow around thin wing profiles with flaps of various relative length.

This scheme is also applied to determine the hydrodynamic characteristics of a system of two ships moving along a curved trajectory [177].

The added masses of the plate L_1 with flap of length L_2 located at an angle $\delta = \pi/2k$ to the main plate are calculated in [177] using Sedov's method. The exterior of the plate with a flap is mapped on the unit disc by a Christoffel–Schwarz integral. The values for the added masses are then calculated by formulas (2.2)–(2.7).

The coefficients of added masses $k_{11} = 4\lambda_{11}/(\pi\rho L_1^2)$; $k_{22} = 4\lambda_{22}/(\pi\rho L_1^2)$; $k_{12} = 4\lambda_{12}/(\pi\rho L_1^2)$; $k_{66} = 16\lambda_{66}/(\pi\rho L_1^4)$; $k_{16} = 8\lambda_{16}/(\pi\rho L_1^3)$; $k_{26} = 8\lambda_{26}/(\pi\rho L_1^3)$ as functions of the ratio L_1/L_2 are shown in Figs. 2.29–2.31. Parameter k shown in these figures is related to the angle between the plate and the flap by $\delta = \pi/2k$. The value $k = \infty$ corresponds to $\delta = 0$.

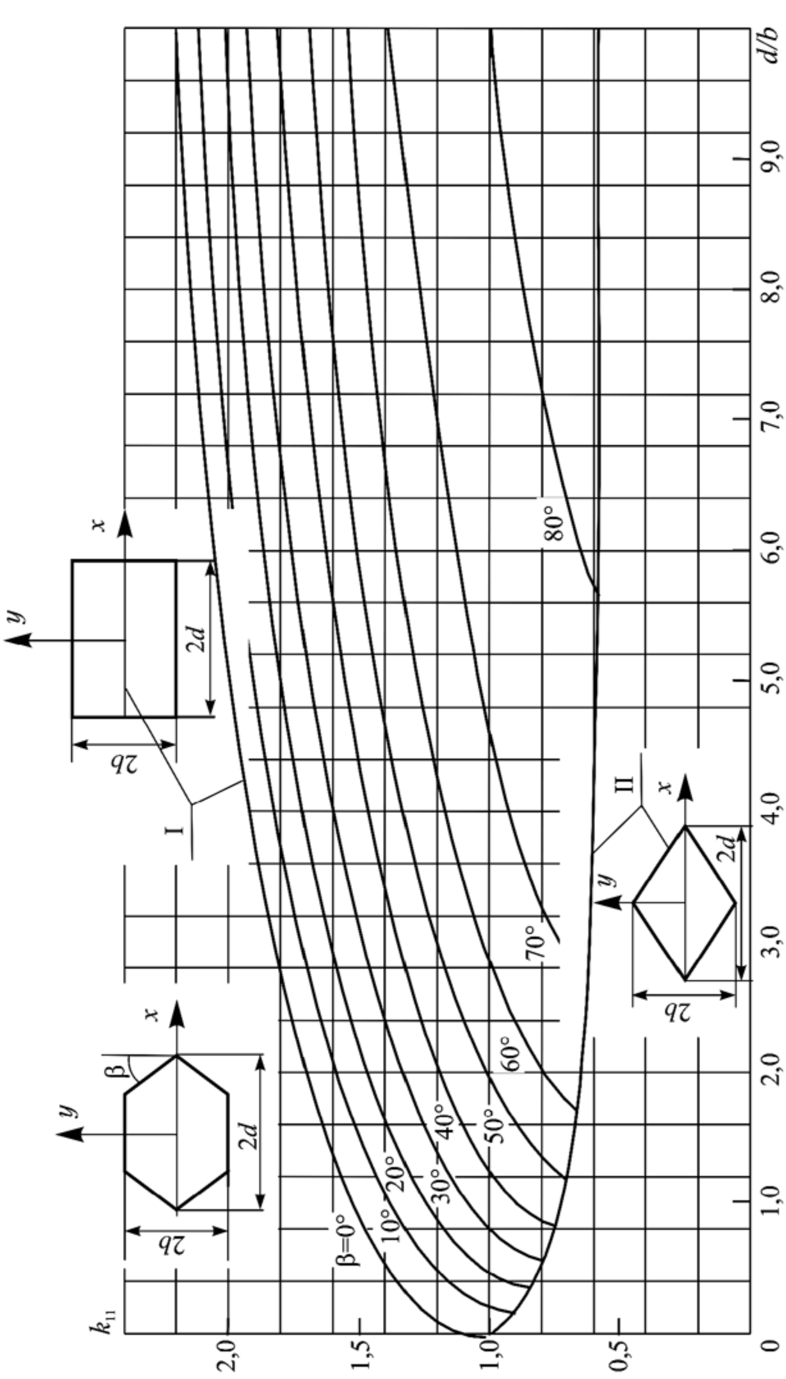


Fig. 2.26 Coefficients of added masses of *hexagon*, *rectangle* and *rhombus*

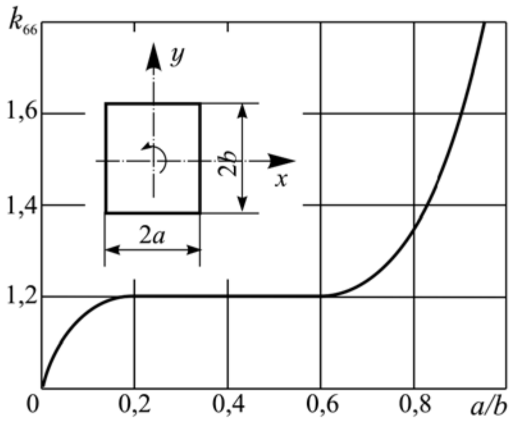


Fig. 2.27 Added moment of inertia of a *rectangle*

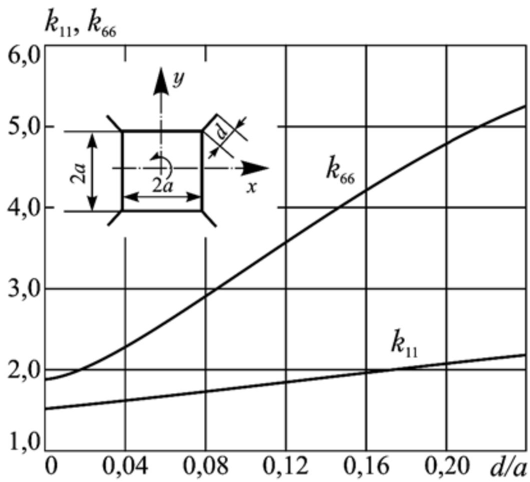


Fig. 2.28 Coefficients of added masses of *square* with ribs

2.3 Added Masses of Lattices

2.3.1 Two Plates Located on One Line

Formulas for the added masses of two intervals (plates) of lengths l_1 and l_2 located on the same line at distance d (Fig. 2.32) have the following form [183, 206]:

$$\lambda_{22} = \frac{\rho\pi}{4}\left(l_1^2 + l_2^2\right)\mu(p, q);$$

$$\lambda_{26} = \frac{\rho\pi}{16}(2p + q + 1)\left(q^2 - 1\right)l_1^3;$$

$$\lambda_{66} = \frac{\rho\pi}{64}\left[\frac{1}{2}\left(q^2 - 1\right)^2 + (2p + q + 1)^2\left(q^2 + 1\right)\right]l_1^4,$$

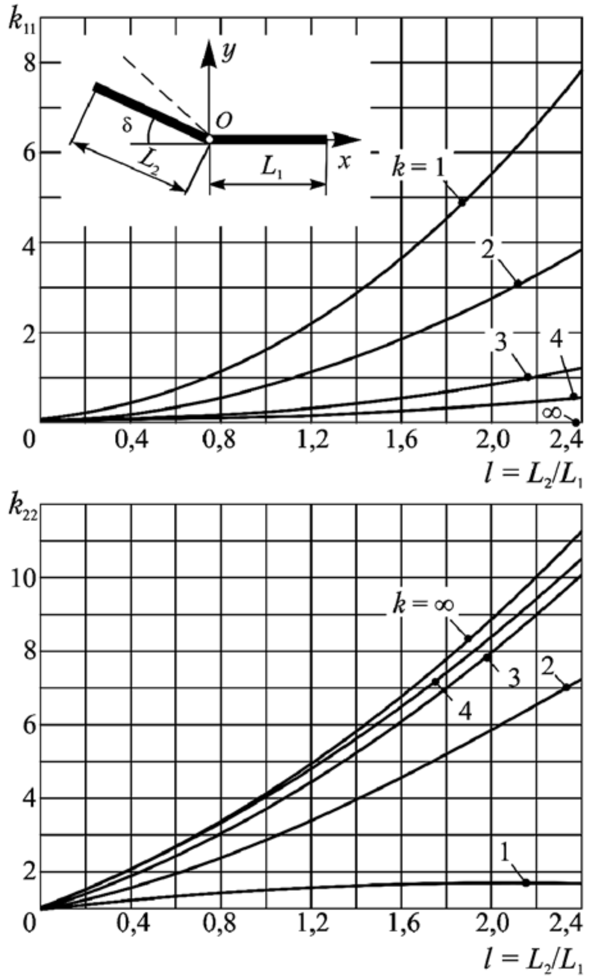


Fig. 2.29 Coefficients of added masses of a plate with flap

where $p = d/l_1$, $q = l_2/l_1$; the values for the coefficient $\mu(p, q) = \mu(p/q, 1/q)$ shown in Fig. 2.32 are defined via complete elliptic integrals of the first and second kind $F(k)$, $E(k)$:

$$\mu(p,q) = \frac{(1+2p+q)^2 - 4\frac{E(k)}{F(k)}(1+p)(p+q)}{1+q^2};$$

$$k^2 = \frac{q}{(1+p)(p+q)}.$$

In the computation of λ_{26}, λ_{66} we assumed that the origin is chosen to lie at an equal distance from the centers of the plates.

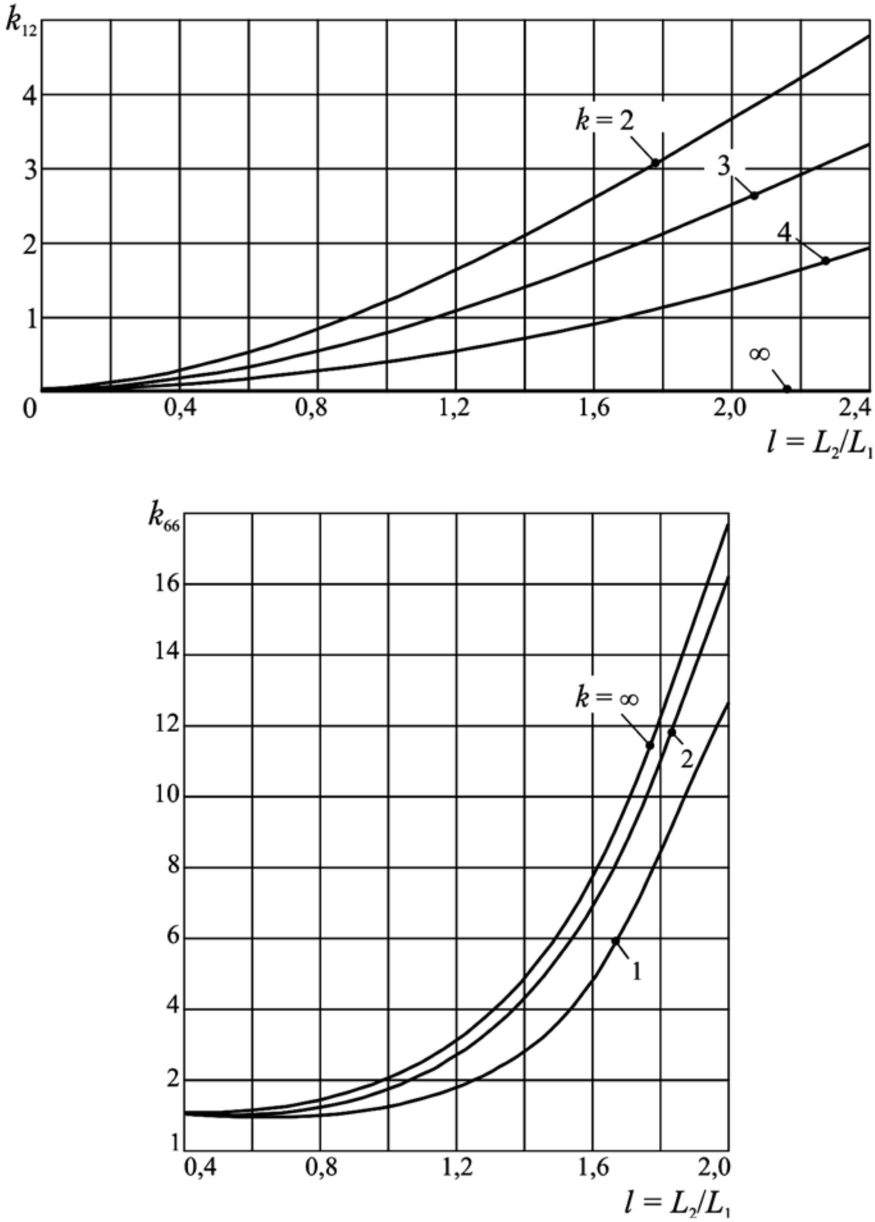


Fig. 2.30 Coefficients of added masses of a plate with flap

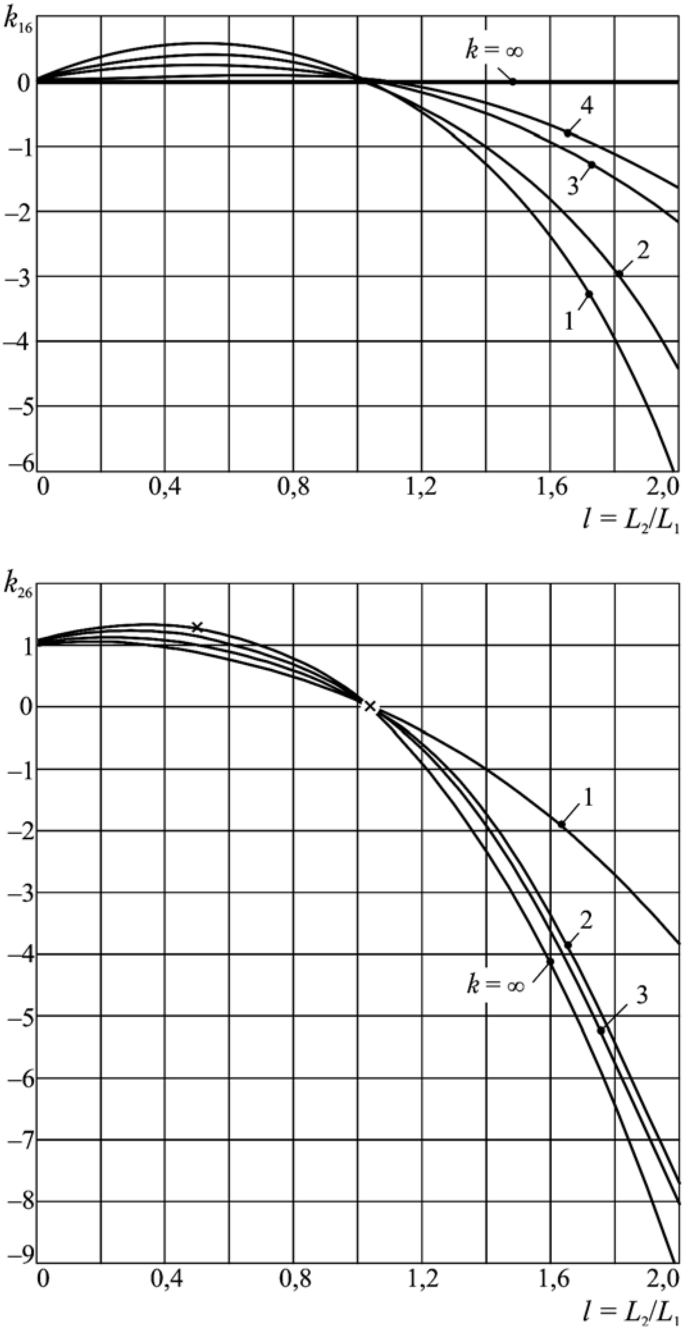


Fig. 2.31 Coefficients of added masses of a plate with flap

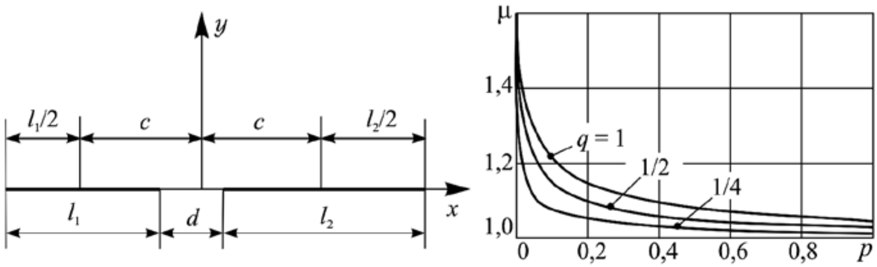


Fig. 2.32 Values of functions $\mu(p, q)$: $\mu(0, 1) = 2$; $\mu(0, 1/2) = 1.8$; $\mu(0, 1/4) = 1.46$

2.3.2 Three Plates Located on One Line

Formulas for the added masses of three plates symmetrically located on one line (Fig. 2.33) look as follows[1]:

$$\lambda_{22} = 2\pi\rho\left[\frac{1}{2}\left(c^2 + b^2 - a^2\right) - \left(c^2 - a^2\right)\frac{E(k)}{F(k)}\right];$$
$$\lambda_{66} = \frac{\pi\rho}{8}\left[\left(c^2 + b^2 - a^2\right)^2 - 4b^2\left(c^2 - a^2\right)\frac{E(k_1)}{F(k_1)}\right].$$

Here E, F are complete elliptic integrals of the first and second kind;

$$k^2 = \frac{c^2 - b^2}{c^2 - a^2}, \qquad k_1^2 = \frac{a^2(c^2 - b^2)}{b^2(c^2 - a^2)}.$$

The lengths of the plates are given by: $l_1 = 2a$, $l_2 = l_3 = c - b$. The gap between

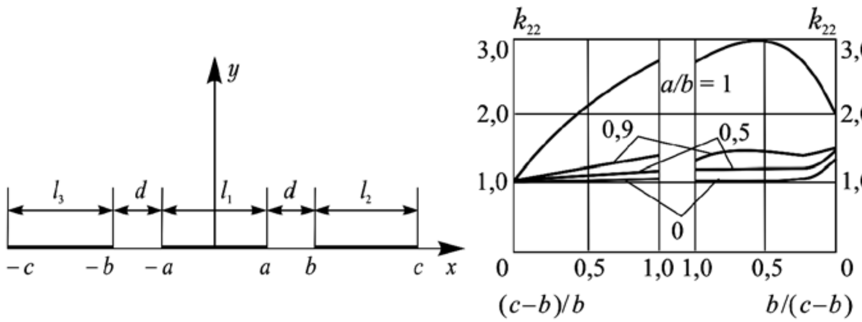


Fig. 2.33 Coefficients of added masses of three plates

[1]These formulas were derived by V.F. Shushpalov, see [183].

the plates equals $d = b - a$. The expressions for λ_{22} looks as follows:

$$\lambda_{22} = k_{22}\rho\pi\left[a^2 + \frac{(c-b)^2}{2}\right],$$

where dependence of k_{22} on a/b, $(c-b)/b$ for $(c-b)/b < 1$, or $b/(c-b)$ for $(c-b)/b > 1$ are shown in Fig. 2.33.

2.3.3 Lattice of Plates

Consider the lattice of parallel plates of width d (from a two-dimensional perspective these are intervals of length d). The axis of the lattice is assumed to have an inclination β to the plane of the plates. The interval between the plates calculated along the axis of the lattice is denoted by l (Fig. 2.34).

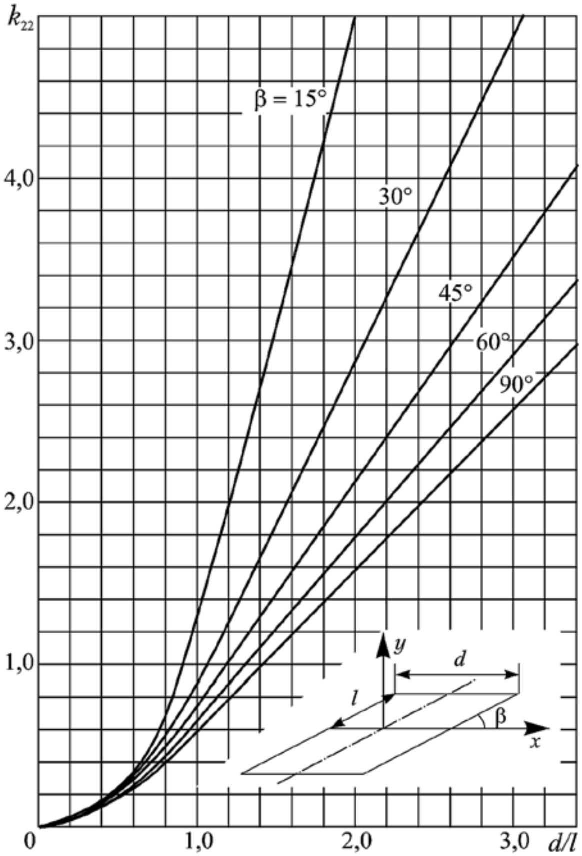


Fig. 2.34 Coefficients of added masses of a plate lattice

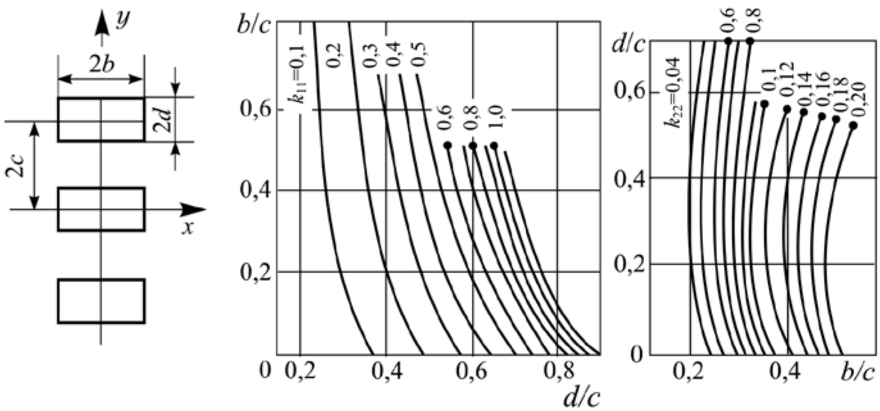


Fig. 2.35 Coefficients of added masses of a lattice of *rectangles*

Dependence of coefficient $k_{22} = \lambda_{22}/pl^2$ on parameters β and d/l is presented in Fig. 2.34. If the lattice consists of intervals lying on one line (lattice of horizontal plates), then $\beta = 0$ and

$$k_{22} = -\frac{2}{\pi} \ln \cos \frac{\pi d}{2l}.$$

If $\beta = \pi/2$ (vertical lattice of parallel plates) then

$$k_{22} = \frac{2}{\pi} \ln \cosh \frac{\pi d}{2l}.$$

2.3.4 Lattice of Rectangles

Consider the lattice with interval $2c$ of rectangles of width $2b$ and height $2d$ (Fig. 2.35). The added masses of each rectangle were computed in [91]. The values for coefficients $k_{11} = \lambda_{11}/(4\rho c^2)$, $k_{22} = \lambda_{22}/(4\rho c^2)$ as functions of b/c and d/c are shown in Fig. 2.35.

2.4 Added Masses of a Duplicated Shipframe Contour Moving in Unlimited Fluid

Let us briefly describe the method of computing of the added masses in this case. The description of motion of a two-dimensional contour in an ideal incompressible two-dimensional fluid reduces to computation of the complex potential of the planar flow $w(\tau) = \varphi(y, z) + i\psi(y, z)$ [116]. Knowing the potential $w(\tau)$ one can find the components of velocity v_y, v_z in the whole plane of $\tau = y + iz$. Then, using the

Cauchy–Lagrange formula one can determine the pressure at any point, including the points of the contour. To find the function $w(\tau)$ it is sufficient to find the current function $\psi(y,z)$; the function φ can be found from the Cauchy–Riemann equations $\partial\varphi/\partial y = \partial\psi/\partial z$; $\partial\varphi/\partial z = -\partial\psi/\partial y$.

The vortex-free condition leads to Laplace equation $\Delta\psi = 0$. The boundary conditions for this equation can be formulated as follows. If at infinity the fluid does not move, then $v_y = \partial\psi/\partial z = 0$ and $v_z = -\partial\psi/\partial y = 0$. On the contour we have the water-tightness condition

$$v_n = u_n, \tag{2.13}$$

where v_n is the normal component of velocity of fluid at the contour and u_n is the normal component of the velocity of the same point of the contour.

For v_n we have

$$\begin{aligned} v_n &= v_y \cos(n,y) + v_z \cos(n,z) = v_y \sin\alpha - v_z \cos\alpha \\ &= v_y \frac{dz}{ds} - v_z \frac{dy}{ds} = \frac{\partial\psi}{\partial z}\frac{dz}{ds} + \frac{\partial\psi}{\partial y}\frac{dy}{ds} = \frac{d\psi}{ds}, \end{aligned}$$

where α is the angle between the element ds of the current line and the axis $0y$ (Fig. 2.36). For u_n we get

$$u_n = u_y \sin\alpha - u_z \cos\alpha = (U_y - \omega z)\frac{dz}{ds} - (U_z + \omega y)\frac{dy}{ds},$$

where U_y, U_z are components of the velocity vector of the origin of the moving coordinate system yOz attached to the contour onto the axes Oy, Oz (we assume that for the moment of observation the stationary and non-stationary coordinate systems coincide); ω is the angular velocity of the contour rotation. Now condition (2.13) can be written down in a more detailed form:

$$\frac{d\psi}{ds} = (U_y - \omega z)\frac{dz}{ds} - (U_z + \omega y)\frac{dy}{ds}. \tag{2.14}$$

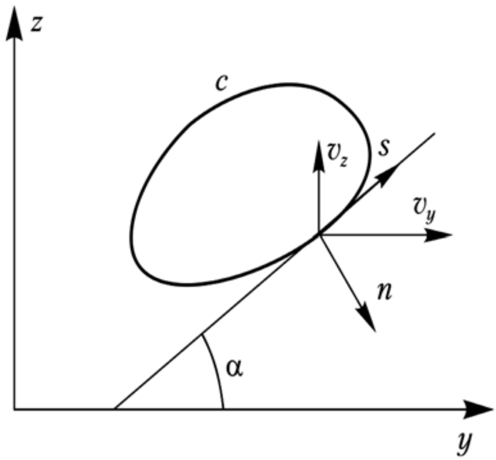


Fig. 2.36 System of coordinates

Using the formula (2.14) we get (up to an arbitrary additive constant) the condition

$$\psi = U_y z - U_z y - \frac{\omega}{2}\left(y^2 + z^2\right), \tag{2.15}$$

to be fulfilled on the contour. Taking into account (2.15) one represents the function $w(\tau)$ in the form

$$w(\tau) = U_y w_2(\tau) + U_z w_3(\tau) + \omega w_4(\tau),$$

where the functions $w_2(\tau)$, $w_3(\tau)$ and $w_4(\tau)$ are determined by geometric properties (i.e. the shape) of the contour only; these functions characterize the perturbed potential flow of the fluid under the motion of the contour with unit velocities along the axes Oy, Oz and under rotation, respectively. The functions $w_k(\tau) = \varphi_k(\tau) + i\psi_k(\tau)$ are regular outside of the contour and vanish at infinity. On the contour C their imaginary parts, according to (2.15), satisfy the conditions

$$\psi_2|_C = z; \qquad \psi_3|_C = -y; \qquad \psi_4|_C = -\frac{1}{2}\left(y^2 + z^2\right). \tag{2.16}$$

To find the potential of the fluid around contour C in the τ-plane it is sufficient to find the function

$$\tau = y + iz = f(\zeta), \tag{2.17}$$

which conformally maps the exterior of the contour to the exterior of the unit circle in the plane of $\zeta = \xi + i\eta$ [116, 127, 129, 130, 206], since the potential of the fluid flow around the circle is known.

The function $f(\zeta)$ can be in general represented as the following series:

$$f(\zeta) = k\zeta + k_0 + \frac{k_1}{\zeta} + \frac{k_2}{\zeta^2} + \cdots. \tag{2.18}$$

If the contour C (Fig. 2.37) is symmetric with respect to the z-axis, then the expansion (2.18) contains only terms of odd order.

Consider the following form of the function f [100]:

$$f(\zeta) \equiv y + iz := -\frac{iT}{1+p+q}\left(\zeta + p\zeta^{-1} + q\zeta^{-3}\right), \tag{2.19}$$

where the coefficients k, k_1, k_3 are replaced by the combinations of the value T (the waterdraft of the frame) and the parameters p, q. On the unit circle, $\zeta = e^{i\theta}$. By substituting this value into the formula (2.19) and separating the real and imaginary parts, we obtain the parametrical description of the contour C:

$$\begin{cases} y = T\dfrac{(1-p)\sin\theta - q\sin 3\theta}{1+p+q}; \\ z = -T\dfrac{(1+p)\cos\theta + q\cos 3\theta}{1+p+q}. \end{cases} \tag{2.20}$$

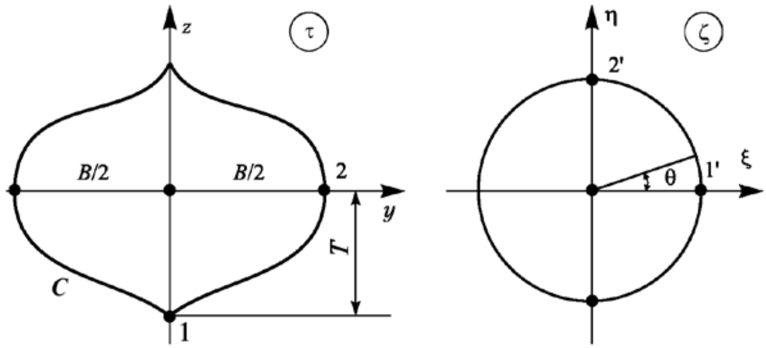


Fig. 2.37 Map of a duplicated shipframe to the unit circle

The chosen form of the function $f(\zeta)$ gives $z = -T$, $y = 0$ when $\theta = 0$. The second condition, $y = B/2$, $z = 0$ when $\theta = \pi/2$, gives one relation between the parameters p and q:

$$\frac{1+p+q}{1-p+q} = 2\frac{T}{B}.$$

By calculating the area bounded by the contour C: $S = 2\int_0^{B/2} z\,dy$, taking into account Eqs. (2.20) and using the usual notation $\beta = S/BT$ for the coefficient of the plumpness of the shipframe, we find the second relation between the parameters p and q:

$$\beta = \frac{\pi}{4}\frac{1-p^2-3q^2}{(1+p+q)^2}\frac{2T}{B}.$$

Therefore, for each pair of values β and $2T/B$ one can find corresponding values p and q, and draw a contour C in the τ-plane, using (2.20).

The tables for values p, q for $0.5 \le \beta \le 1$ and $0.2 \le 2T/B \le 10$ are given in the monograph by Huskind [100].

Let us turn to determining of the characteristic functions $w_k(\tau) = \varphi_k(\tau) + i\psi_k(\tau)$ $(k = 2, 3, 4)$. Imaginary parts of these functions have to satisfy conditions (2.16) on the contour C; these conditions, taking into account (2.17), can be rewritten as

$$\Im w_2 = \Im\tau; \qquad \Im w_3 = -\Re\tau; \qquad \Im w_4 = -\frac{\tau\bar{\tau}}{2}. \tag{2.21}$$

On the basis of (2.19), (2.21) we can find the following relation for the function w_2:

$$\Im w_2 = \Im\left[-\frac{iT}{1+p+q}\left(\zeta + p\zeta^{-1} + q\zeta^{-3}\right)\right].$$

Taking into account that $\Im(i\zeta) = \Re\zeta$ and on the unit circle $\Re\zeta = \Re\zeta^{-1}$, we can rewrite the previous equation as

$$\Im w_2 = \Im\left\{-\frac{iT}{1+p+q}\left[(1+p)\zeta^{-1} + q\zeta^{-3}\right]\right\}.$$

Therefore, we have found the function ψ_2 which satisfies the water-tightness conditions on the contour and also the stationarity condition at infinity (when $\zeta \to \infty$). Now the function w_2 can be taken in the form

$$w_2 = -\frac{iT}{1+p+q}\left[(1+p)\zeta^{-1} + q\zeta^{-3}\right]. \tag{2.22}$$

Similar arguments lead (using the relation $\Im\zeta = -\Im\zeta^{-1}$ valid on the unit circle) to the formulas for w_3, w_4:

$$w_3 = -\frac{T}{1+p+q}\left[(p-1)\zeta^{-1} + q\zeta^{-3}\right], \tag{2.23}$$

$$w_4 = -\frac{iT^2}{(1+p+q)^2}\left[p(1+q)\zeta^{-2} + q\zeta^{-4}\right]. \tag{2.24}$$

In the right-hand side of (2.24) we omit an unessential constant. By separating in the expressions (2.22), (2.23), (2.24) the real and imaginary parts, we obtain φ_k, ψ_k ($k = 2, 3, 4$). By using the formulas for the added masses

$$\lambda_{ik} = -\rho \int_{C_1} \varphi_i \, d\psi_k,$$

where the integration is performed only over one half of the duplicated shipframe contour (in the plane of the fluid), which corresponds to integration from $-\pi/2$ to $\pi/2$ over θ in the ζ-plane, it is easy to find the dependence of the added masses on parameters determining the profile of the shipframe:

$$\begin{aligned}
\lambda_{22} &= \rho\frac{\pi T^2}{2}\frac{(1+p)^2 + 3q^2}{(1+p+q)^2} = \rho\frac{\pi T^2}{2}k_{22};\\
\lambda_{33} &= \rho\frac{\pi B^2}{8}\frac{(1-p)^2 + 3q^2}{(1-p+q)^2} = \rho\frac{\pi B^2}{8}k_{33};\\
\lambda_{24} &= \frac{\rho T^3}{2}\frac{1}{(1+p+q)^2}\left\{\frac{8}{3}p(1+p) + \frac{16}{35}q^2(20+7p)\right.\\
&\quad \left. + q\left[\frac{4}{3}(1+p)^2 - \frac{4}{5}(1-p)(7-5p)\right]\right\} = \frac{\rho T^3}{2}k_{24};\\
\lambda_{44} &= \rho\frac{\pi B^4}{256}\frac{16[p^2(1+q)^2 + 2q^2]}{(1-p+q)^4} = \rho\frac{\pi B^4}{256}k_{44}.
\end{aligned}$$

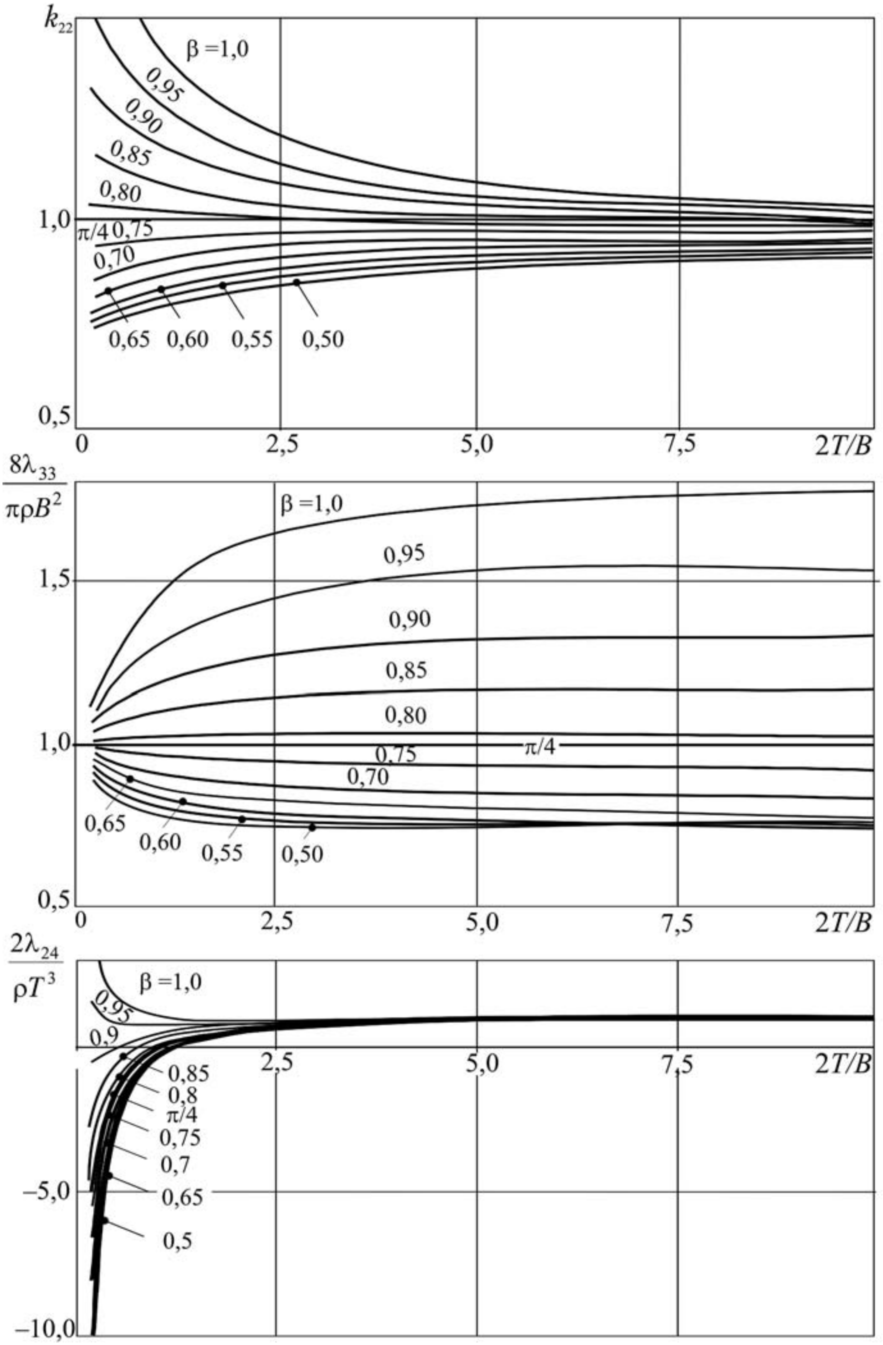


Fig. 2.38 Coefficients of added masses of shipframes determined via the method of a duplicated contour

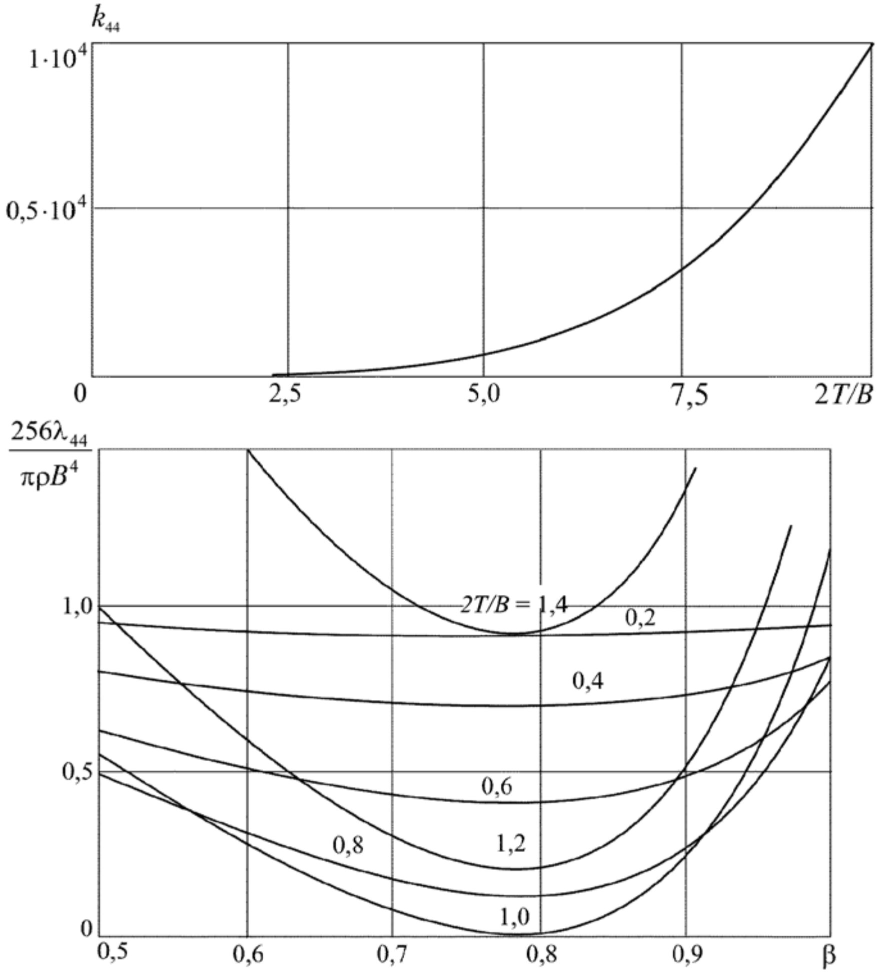


Fig. 2.39 Coefficients of added masses of shipframes determined via the method of a duplicated contour

The graphs of dependencies $k_{ij}(\beta, 2T/b)$ are presented in Figs. 2.38, 2.39.[2]

The profiles of shipframe corresponding to functions $f(\zeta)$ having three terms in their Laurent series are shown in Figs. 2.40–2.43 (these profiles were drawn by Dorofeuk [50]; for $B/2T = 0.2$; 0.4; 0.6; 0.8; 1; 1.2; 1.5; 2 these profiles were obtained by Lewis in [131]). Notice that dependence of Lewis shipframes on only two parameters $B/2T$ and β imposes certain restrictions on their applicability. In Fig. 2.44 we present a diagram showing the range of parameters where the Lewis form of the shipframe can be used in practice. In Table 2.3 for each profile we

[2]Similar graphs for the range $T/B \leq 1$ were found in the work [151].

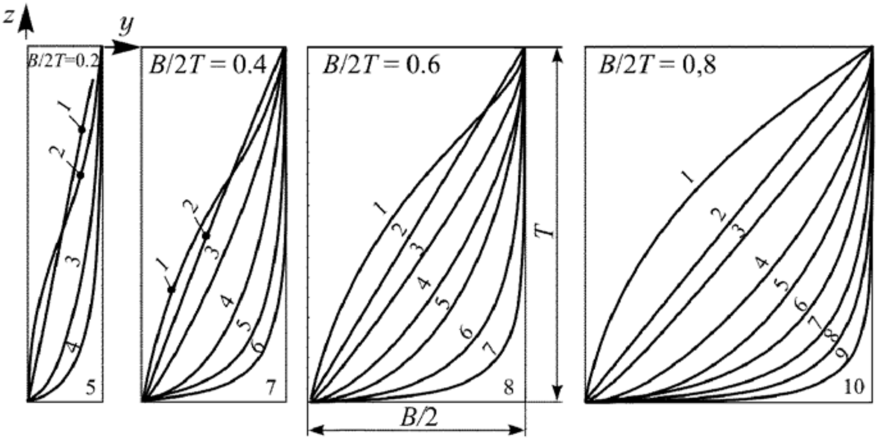


Fig. 2.40 Shape of a Lewis shipframe for different $B/2T$ and β

present corresponding values of β, $k_{22} = 2\lambda_{22}/(\pi\rho T^2)$ and $k_{33} = 8\lambda_{33}/(\pi\rho B^2)$. The last coefficient in the calculations of the ship oscillations is usually denoted by c_v. Indexes v and h correspond to the added masses in vertical and horizontal directions, respectively.

Dorofeuk calculated also several shipframe profiles and their inertial characteristics keeping six terms in the Laurent series of function $f(\zeta)$ (Fig. 2.45, Fig. 2.46, Table 2.4). In Tables 2.3 and 2.4 the values of k_{240} were computed taking into account the presence of free surface; in computation of other coefficients the presence of free surface is non-essential.

Besides the added masses of the "analytical" ships frames obtained on the basis of a function of the type (2.18), one is also interested in the added masses of the real shipframes. For two ships whose shipframes are shown in Fig. 2.47 and Fig. 2.48 the characteristics of these shipframes are given in Table 2.5 (they were found by Pavlov using the method of electro-hydrodynamic analogy (EHDA), see Chap. 9). The added masses of shipframes in the vertical direction can be found from the formula

$$\lambda_{33} = \frac{\pi\rho B^2}{8} k_{33}. \tag{2.25}$$

The added masses of shipframes in the horizontal direction (which are computed taking into account the presence of the free surface) can be found from the formula

$$\lambda_{220} = \frac{2\rho T^2}{\pi} k_{220}. \tag{2.26}$$

In the formulas (2.25), (2.26) B is the frame width on the water-line, T is the waterdraft of the frame. We notice the difference of the constant coefficients in the formulas (2.25) and (2.26).

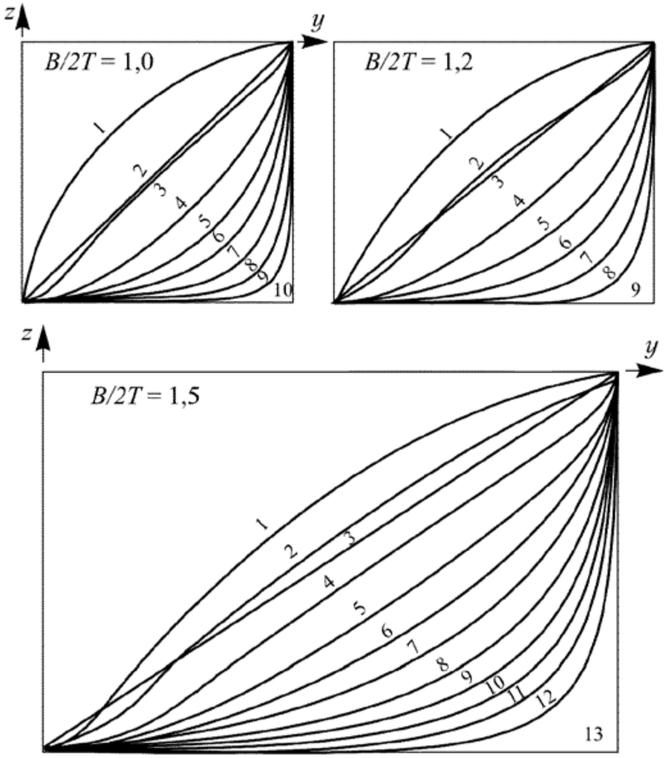


Fig. 2.41 Shape of a Lewis shipframe for different $B/2T$ and β

In practice it is convenient to use an approximate analytical representation of the coefficients

$$k_{33} = c_v = \frac{\lambda_{33}}{(\pi/2)\rho(B/2)^2} \quad \text{and} \quad k_{220} = c_h = \frac{\lambda_{220}}{(2/\pi)\rho T^2}$$

(see Fig. 2.49) in terms of characteristic dimensions of the frames. Possible inaccuracies arising from the use of this approximation can be corrected by use of computer simulation.

Now we present the formulas for k_{33} derived by Ivanuta and Boyanovsky [35]. Similar formulas for k_{220} are given in Chap. 5.

One can use the relation obtained in the work [128]

$$k_{33} = \frac{(1 + k_1)^2 + \sum_{n=3}^{\infty} n k_n^2}{b_k^2},$$

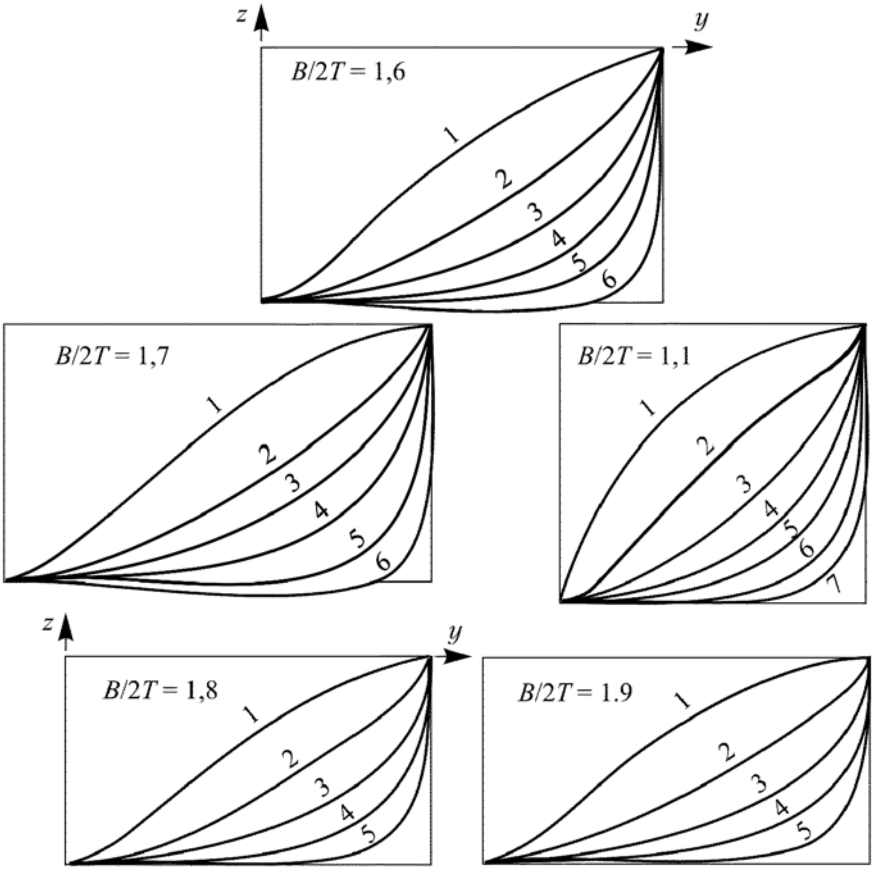


Fig. 2.42 Shape of a Lewis shipframe for different $B/2T$ and β

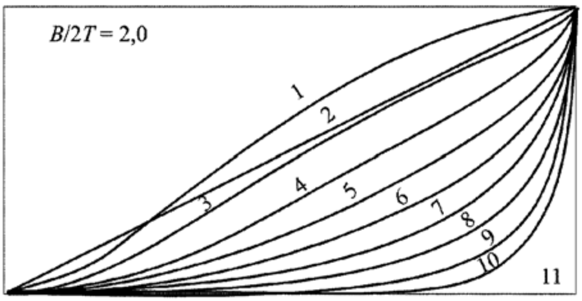


Fig. 2.43 Shape of a Lewis shipframe for different $B/2T$ and β

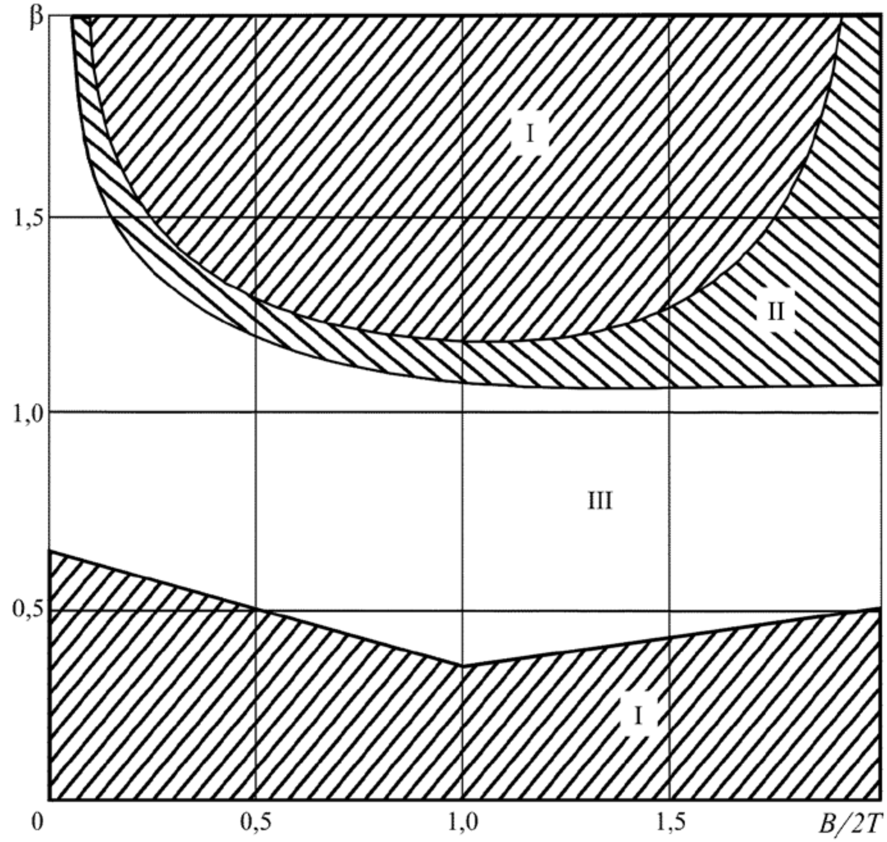


Fig. 2.44 Domains of applicability of a Lewis shipframe: *I*—Lewis shape of shipframe is not used; *II*—Lewis shape of shipframe is not recommended; *III*—Lewis shape of shipframe is recommended

Table 2.3 Coefficients of added masses of Lewis shipframes

$B/2T$	Cont. No.	β	k_{22}	$k_{220} = c_h$	$k_{33} = c_v$	$k_{44} = c_{\text{tor}}$	$k_{240} = c_{\text{incl}}$
0.2	1	0.500	–	–	0.611	–	–
	2	0.535	0.906	0.98	0.75	1.02	1.01
	3	0.785	1.00	1.00	1.00	1.00	1.00
	4	0.920	1.06	0.995	1.40	1.01	1.01
	5	1.000	–	–	1.98	–	–
0.3	1	0.505	0.87	1.03	0.75	1.08	1.02
	2	0.785	1.00	1.00	1.00	1.00	1.00
	3	0.925	1.10	1.11	1.40	1.04	1.04

(*continued on next page*)

Table 2.3 (continued)

$B/2T$	Cont. No.	β	k_{22}	$k_{220} = c_h$	$k_{33} = c_v$	$k_{44} = c_{\text{tor}}$	$k_{240} = c_{\text{incl}}$
0.4	1	0.470	0.84	1.05	0.75	1.16	1.06
	2	0.500	–	–	0.65	–	–
	3	0.630	0.91	1.01	0.80	1.05	1.00
	4	0.785	1.00	1.00	1.00	1.00	1.00
	5	–	–	–	1.20	–	–
	6	0.940	1.12	1.02	1.40	2.15	1.11
	7	1.00	–	–	1.76	–	–
0.5	1	0.440	0.80	1.08	0.75	1.35	1.09
	2	0.610	0.87	1.00	0.80	1.13	1.00
	3	0.785	1.00	1.00	1.00	1.00	1.00
	4	0.880	1.07	1.00	1.20	1.06	1.06
	5	0.945	1.10	1.04	1.40	1.23	1.15
0.6	1	0.410	0.79	1.12	0.75	1.69	1.15
	2	0.500	–	–	0.69	–	–
	3	0.615	0.87	1.03	0.80	1.24	1.0
	4	0.710	0.90	1.02	0.90	1.05	0.98
	5	0.785	1.00	1.00	1.00	1.00	1.00
	6	0.885	1.10	1.02	1.20	1.14	1.11
	7	0.955	1.2	1.05	1.40	1.53	1.27
	8	1.000	–	–	1.64	–	–
0.7	1	0.495	0.79	1.09	0.75	2.21	1.10
	2	0.595	0.84	1.04	0.80	1.63	1.02
	3	0.705	0.91	1.01	0.90	1.13	0.97
	4	0.785	1.00	1.00	1.00	1.00	1.00
	5	0.910	1.16	1.03	1.20	1.57	1.28
	6	0.960	1.25	1.06	1.40	2.24	1.50
0.8	1	0.350	0.76	1.22	0.75	5.85	1.47
	2	0.500	–	–	0.73	–	–
	3	0.565	0.82	1.07	0.80	2.95	1.02
	4	0.700	0.92	1.02	0.90	1.36	0.93
	5	0.785	1.00	1.00	1.00	1.00	1.00
	6	0.850	1.08	1.01	1.10	1.31	1.19
	7	0.895	1.14	1.02	1.20	1.98	1.37
	8	0.935	1.23	1.05	1.30	3.31	1.67
	9	0.970	1.25	1.07	1.40	4.80	1.94
	10	1.00	–	–	1.57	–	–

(*continued on next page*)

Table 2.3 (continued)

$B/2T$	Cont. No.	β	k_{22}	$k_{220} = c_h$	$k_{33} = c_v$	$k_{44} = c_{\text{tor}}$	$k_{240} = c_{\text{incl}}$
0.9	1	0.325	0.74	1.27	0.75	24.55	2.10
	2	0.550	0.79	1.08	0.80	10.24	1.04
	3	0.700	0.91	1.01	0.90	2.74	0.80
	4	0.785	1.00	1.00	1.00	1.00	1.00
	5	0.860	1.05	1.01	1.10	2.94	1.53
	6	0.900	1.15	1.03	1.20	6.28	1.96
	7	0.945	1.26	1.06	1.30	12.9	2.68
	8	0.980	1.36	1.10	1.40	21.0	3.45
1.0	1	0.295	0.74	1.33	0.75	0.035	−0.011
	2	0.500	–	–	0.76	–	–
	3	0.540	0.80	1.10	0.80	0.014	−0.0002
	4	0.690	0.90	1.02	0.90	0.003	0.002
	5	0.785	1.00	1.00	1.00	0	0
	6	0.850	1.08	1.01	1.10	0.002	−0.004
	7	0.910	1.20	1.04	1.20	0.005	−0.010
	8	0.950	1.29	1.08	1.30	−0.008	−0.017
	9	0.990	1.38	1.12	1.40	0.011	−0.026
	10	1.000	–	–	1.51	–	–
1.1	1	0.265	0.74	1.40	0.75	34.00	−0.475
	2	0.520	0.76	1.12	0.80	14.70	0.98
	3	0.680	0.84	1.02	0.90	3.79	1.30
	4	0.785	1.00	1.00	1.00	1.00	1.00
	5	0.850	1.06	1.01	1.10	3.12	0.40
	6	0.910	1.18	1.05	1.20	9.60	−0.49
	7	0.955	1.26	1.09	1.30	18.20	−1.41
1.2	1	–	–	–	0.76	–	–
	2	0.510	0.64	1.14	0.80	4.98	1.00
	3	0.500	–	–	0.78	–	–
	4	0.680	0.92	1.03	0.90	1.83	1.16
	5	0.785	1.00	1.00	1.00	1.00	1.00
	6	0.860	1.16	1.01	1.10	1.64	0.64
	7	0.920	1.31	1.05	1.20	3.50	0.13
	8	0.960	1.43	1.11	1.30	6.23	-0.45
	9	1.00	–	–	1.47	–	–
1.3	1	0.500	0.80	1.17	0.80	3.03	1.00
	2	0.675	0.88	1.03	0.90	1.43	1.14
	3	0.785	1.00	1.00	1.00	1.00	1.00

(*continued on next page*)

Table 2.3 (continued)

$B/2T$	Cont. No.	β	k_{22}	$k_{220}=c_h$	$k_{33}=c_v$	$k_{44}=c_{\mathrm{tor}}$	$k_{240}=c_{\mathrm{incl}}$
	4	0.860	1.12	1.02	1.10	1.35	0.72
	5	0.920	1.27	1.07	1.20	2.24	0.34
	6	0.970	1.42	1.13	1.30	3.80	-0.17
1.4	1	0.480	0.75	1.20	0.80	2.32	1.02
	2	0.675	0.87	1.03	0.90	1.27	1.12
	3	0.785	1.00	1.00	1.00	1.00	1.00
	4	0.870	1.14	1.02	1.10	1.23	0.77
	5	0.930	1.29	1.07	1.20	1.87	0.41
	6	0.975	1.43	1.15	1.30	2.88	-0.01
1.5	1	–	–	–	0.78	–	–
	2	0.470	0.78	1.24	0.80	1.96	1.02
	3	0.500	–	–	0.81	–	–
	4	–	–	–	0.85	–	–
	5	0.665	0.87	1.07	0.90	1.21	1.11
	6	–	–	–	0.95	–	–
	7	0.785	1.00	1.00	1.00	1.00	1.00
	8	–	–	–	1.05	–	–
	9	0.875	1.17	1.03	1.10	1.18	0.77
	10	–	–	–	1.15	–	–
	11	0.930	1.28	1.08	1.20	1.62	0.47
	12	0.985	1.49	1.19	1.25	2.40	0.07
	13	1.000	–	–	1.42	–	–
1.6	1	0.455	0.74	1.26	0.80	1.75	1.02
	2	0.660	0.84	1.04	0.90	1.16	1.11
	3	0.785	1.00	1.00	1.00	1.00	1.00
	4	0.870	1.14	1.03	1.10	1.12	0.89
	5	0.940	1.34	1.09	1.20	1.49	0.50
	6	0.990	1.50	1.18	1.30	2.04	0.03
1.7	1	0.440	0.73	1.30	0.80	1.62	1.02
	2	0.655	0.81	1.05	0.90	1.13	1.10
	3	0.785	1.00	1.00	1.00	1.00	1.00
	4	0.880	1.14	1.04	1.10	1.12	0.79
	5	0.940	1.32	1.10	1.20	1.39	0.53
	6	1.01	1.59	1.23	1.30	1.96	0.11

(*continued on next page*)

Table 2.3 (continued)

$B/2T$	Cont. No.	β	k_{22}	$k_{220} = c_h$	$k_{33} = c_v$	$k_{44} = c_{\text{tor}}$	$k_{240} = c_{\text{incl}}$
1.8	1	0.425	0.75	1.33	0.80	1.53	1.02
	2	0.650	0.86	1.05	0.90	1.11	1.10
	3	0.785	1.00	1.00	1.00	1.00	1.00
	4	0.875	1.20	1.03	1.10	1.08	0.80
	5	0.955	1.43	1.14	1.20	1.41	0.47
1.9	1	0.410	0.84	1.35	0.80	1.45	1.05
	2	0.645	0.93	1.06	0.90	1.10	1.11
	3	0.785	1.00	1.00	1.00	1.00	1.00
	4	0.885	1.35	1.04	1.10	1.08	0.80
	5	0.965	1.58	1.15	1.20	1.37	0.50
2.0	1	0.400	0.63	1.41	0.81	1.48	1.03
	2	0.500	–	–	0.84	–	–
	3	–	–	–	0.85	–	–
	4	0.640	0.85	1.06	0.90	1.09	1.11
	5	–	–	–	0.95	–	–
	6	0.785	1.00	1.00	1.00	1.00	1.00
	7	–	–	–	1.05	–	–
	8	0.885	1.24	1.04	1.10	1.07	0.81
	9	–	–	–	1.15	–	–
	10	0.960	1.45	1.15	1.20	1.27	0.54
	11	1.000	–	–	1.36	–	–

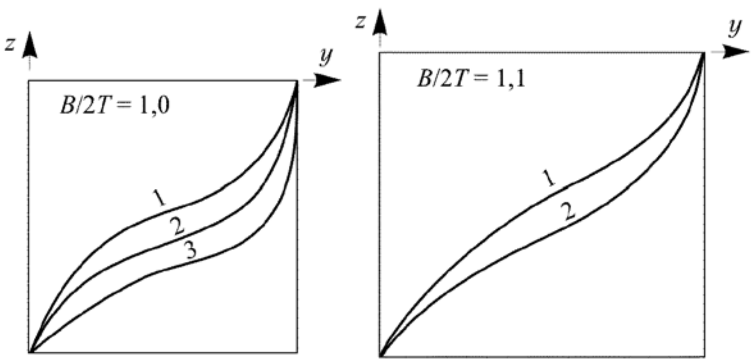


Fig. 2.45 Shape of a Lewis shipframe if six terms of function f are non-trivial

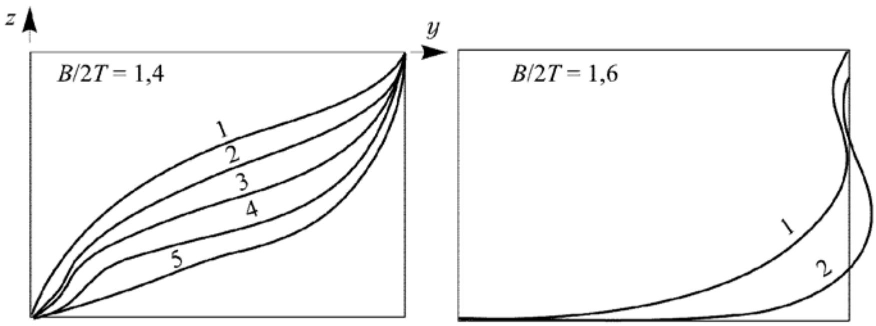


Fig. 2.46 Shape of a Lewis shipframe if six terms of function f are non-trivial

Table 2.4 Inertial characteristics of shipframe profiles

$B/2T$	Cont. No.	β	$k_{220} = c_h$	$k_{33} = c_v$	$k_{44} = c_{\mathrm{tor}}$	$k_{240} = c_{\mathrm{incl}}$
1	1	0.505	0.94	0.925	0.020	0.430
	2	0.610	0.83	1.02	0.012	0.474
	3	0.695	0.80	1.09	0.008	0.522
1.1	1	0.510	0.99	0.875	16.5	1.11
	2	0.605	0.90	0.915	8.97	1.43
1.4	1	0.400	1.07	0.88	3.35	0.87
	2	0.490	0.97	0.91	2.16	0.09
	3	0.575	0.86	0.95	2.15	0.10
	4	0.665	0.85	0.99	1.67	1.06
	5	0.755	0.89	1.01	1.34	1.04
1.6	1	–	1.06	1.14	1.41	0.76
	2	–	1.06	1.42	2.69	0.12

where $b_k^2 = (\pi/8\beta)(B/T)(1 - \sum_{n=1}^{\infty} nk_n^2)$; β is the coefficient of the plumpness of the shipframe, k_n are the coefficients in the expansion

$$z = f(\zeta) = \zeta + \frac{k_1}{\zeta} + \frac{k_3}{\zeta^3} + \frac{k_5}{\zeta^5} + \cdots.$$

If only values for k_1 and k_3 are assumed to be non-vanishing, we get the following formulas:

$$k_{33} = c_v = \left(\frac{2T}{B} - a\right)\left(\frac{2T}{B} - a + 1\right) + 1, \tag{2.27}$$

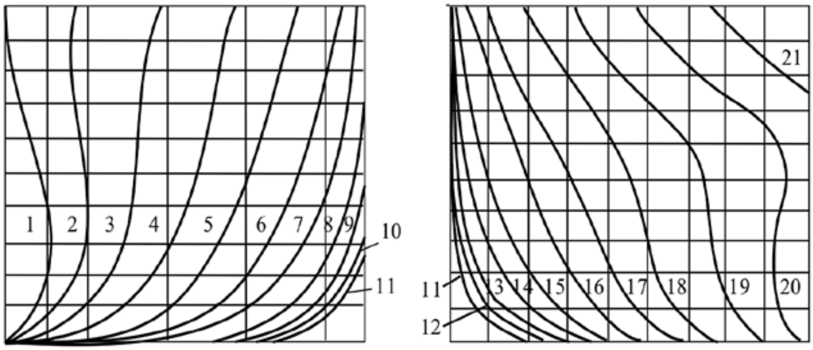


Fig. 2.47 Shipframe shapes for ship A

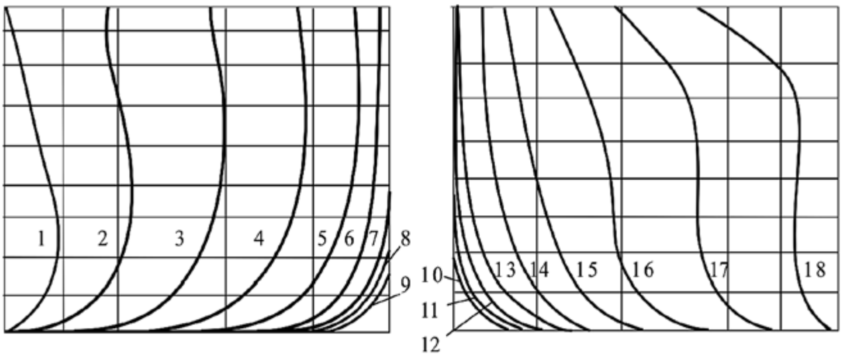


Fig. 2.48 Shipframe shapes for ship B

where

$$a = \frac{3}{2}\left(1+\frac{2T}{B}\right) - \frac{1}{2}\sqrt{1+\frac{20T}{B}+\frac{4T^2}{B^2}-\frac{64\beta T}{\pi B}}. \tag{2.28}$$

The expression under the square root is negative when $\beta > (\pi/32)(2T/B + B/2T + 10)$. This inequality distinguishes the shipframes of bulb-type shape. Therefore, for the shipframes of bulb-type shape the formula (2.27) is not applicable. For three main positions of the bulb-type shipframe with respect to the water surface (Fig. 2.50) Ivanuta and Boyanovsky derived the following approximate formulas:

- for position I: $\lambda_{33} = 0.5\rho\pi b_S^2$,
- for position II: $\lambda_{33} = \rho\pi b_S(1 - b_S^2/2H_S^2)$,
- for position III: $\lambda_{33} = \pi\rho b_{S1}^2(1 - b_{S1}^2/2H_{S1}^2)$,

where the characteristic lengths b_S, b_{S1}, H_S, H_{S1} are shown in Fig. 2.50. The values in the brackets take into account the influence of water surface in the first approx-

Table 2.5 Coefficients of added masses of shipframes

S.F. No.	Ship A				Ship B			
	$B/2T$	β	$k_{220} = c_h$	$k_{33} = c_v$	$B/2T$	β	$k_{220} = c_h$	$k_{33} = c_v$
1	–	–	0.515	–	–	–	0.585	–
2	0.240	1.035	0.950	2.20	0.320	1.13	0.960	2.81
3	0.520	0.788	0.965	1.04	0.675	0.99	1.18	1.56
4	0.773	0.736	0.978	0.872	0.970	0.94	1.22	1.225
5	0.985	0.775	1.07	0.880	1.15	0.91	1.255	1.15
6	1.170	0.762	1.13	0.850	1.23	0.926	1.370	1.19
7	1.19	0.886	1.21	1.065	1.26	0.985	1.49	1.32
8	1.19	0.930	1.27	1.160	1.27	0.99	1.51	1.37
9	1.19	0.945	1.285	1.205	1.27	0.99	1.51	1.44
10	1.19	0.960	1.304	1.235	1.27	0.99	1.51	1.42
11	1.19	0.993	1.350	1.280	1.27	0.98	1.49	1.38
12	1.19	0.990	1.348	1.275	1.27	0.975	1.44	1.26
13	1.19	0.960	1.304	1.235	1.195	0.945	1.39	1.18
14	1.19	0.930	1.348	1.160	1.195	0.930	1.34	1.15
15	1.17	0.865	1.160	1.030	1.10	0.875	1.155	1.04
16	1.15	0.790	1.10	0.920	0.960	0.785	1.05	0.917
17	1.07	0.733	1.145	0.823	0.745	0.635	1.05	0.740
18	0.933	0.666	1.110	0.729	0.497	0.308	0.70	0.397
19	0.773	0.505	0.946	0.650	–	–	–	–
20	0.586	0.303	0.794	0.790	–	–	–	–
21	0.320	0.927	1.570	0.742	–	–	–	–

imation. More precise formulas for taking into account the influence of the water surface on the added masses of a circular cylinder can be found in Chap. 5.

If $0.5 \leq \beta \leq 0.9$, then for approximate calculations one can use the formula

$$\lambda_{33} = \frac{1}{2}\rho\beta B^2.$$

2.5 Added Masses of an Inclined Shipframe

The above results can be generalized [121, 225] to computation of added masses λ_{22} and λ_{24} of inclined shipframes, whose contour can not be considered symmetric with respect to any vertical axis. The problem was solved under the assumption that on the water surface (which was assumed to be flat) the water-tightness condition is imposed.

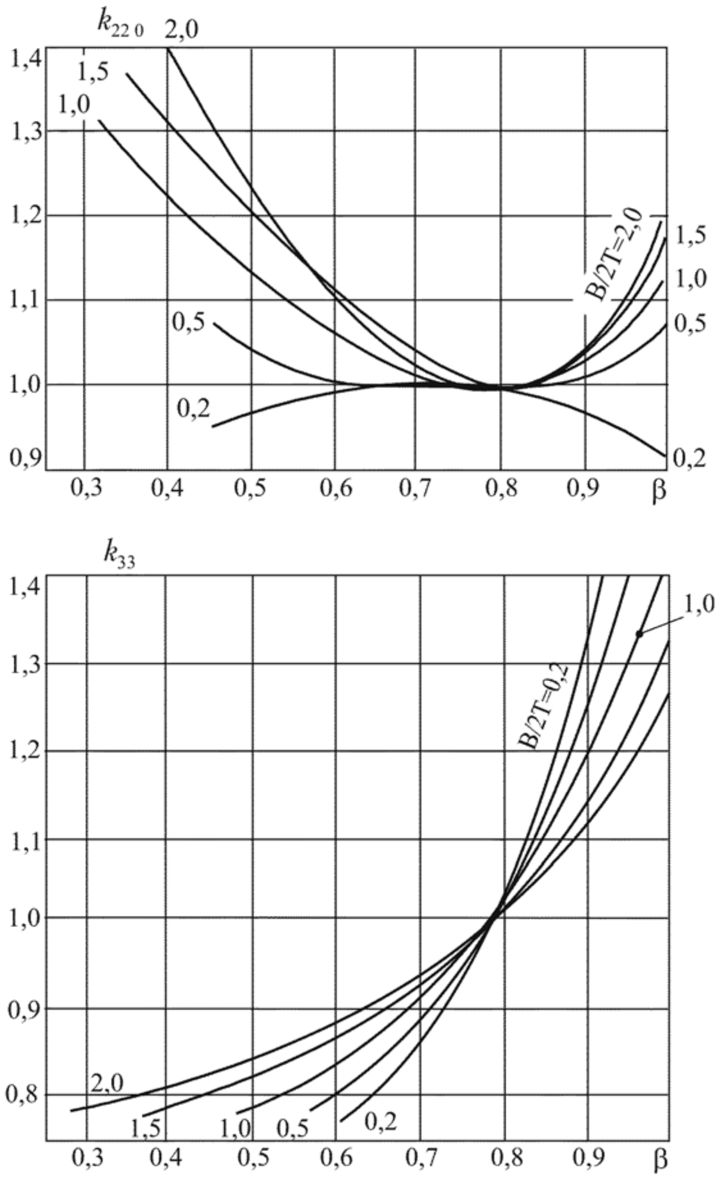


Fig. 2.49 Graphs of analytic representation of coefficients k_{220} and k_{33} in terms of characteristic dimensions of shipframes

Mapping the exterior of the contour of a duplicated shipframe in the τ-plane (Fig. 2.51) to the exterior of the unit circle in the ζ-plane by the function

$$\tau = y + iz = f(\zeta) = \frac{B}{2(1+p+q)}\left(\zeta + p\zeta^{-1} + a\zeta^{-2} + q\zeta^{-3} - a\right),$$

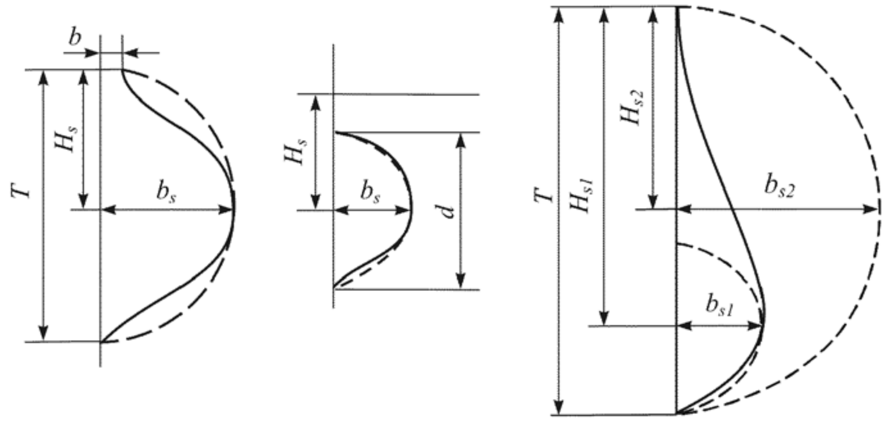


Fig. 2.50 Typical positions of bulb-type shipframes: from left to right: position I, position II and position III

where B is the shipframe width computed at the waterline, we can get the parametrical expression for the frame contour:

$$y = \frac{B}{2(1+p+q)}\big[(1+p)\cos\theta + a\cos 2\theta + q\cos 3\theta - a\big];$$
$$z = \frac{B}{2(1+p+q)}\big[(1-p)\sin\theta - a\sin 2\theta - q\sin 3\theta\big]. \tag{2.29}$$

The difference between formulas (2.29) and formulas (2.20) is in the presence of a non-zero parameter a and, also, in coefficients in front of the brackets in the formulas for $f(\zeta)$.

The graphs of coefficients $k_{22} = (\beta, \bar{y}_m, B/T) = 2\lambda_{22}/(\pi\rho T^2)$ and $k_{24} = \lambda_{24}/(\rho T^3)$ calculated by Usachev [225] are shown in Figs. 2.51–2.54. The value $\bar{y}_m := 2y_m/B$ (Fig. 2.51) determines the asymmetry of the contour.

2.6 Added Masses of Catamarans and Twin Rudders

The added masses of catamarans can be determined via the added masses of a single body [8][3]:

$$\lambda_{11c} = \kappa_1\lambda_{11}, \qquad \lambda_{22c} = \kappa_2\lambda_{22},$$
$$\lambda_{66c} = \kappa_2\left(\lambda_{66} + \frac{B_1^2}{4}\lambda_{22}\right) + \kappa_1\frac{B_1^2}{4}\lambda_{11}$$

[3]This section was written by A.I. Nemzer.

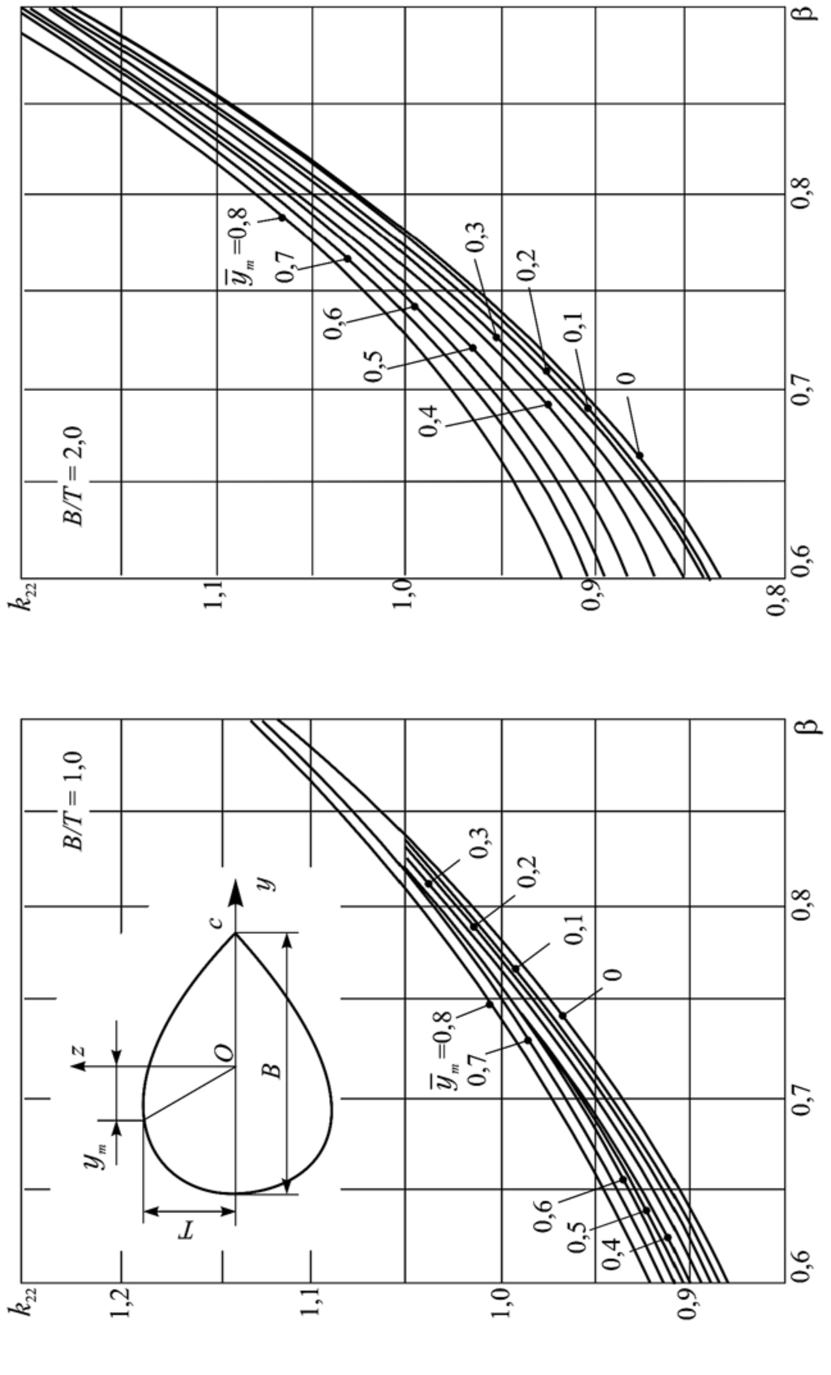


Fig. 2.51 Coefficients of added masses of an inclined shipframe

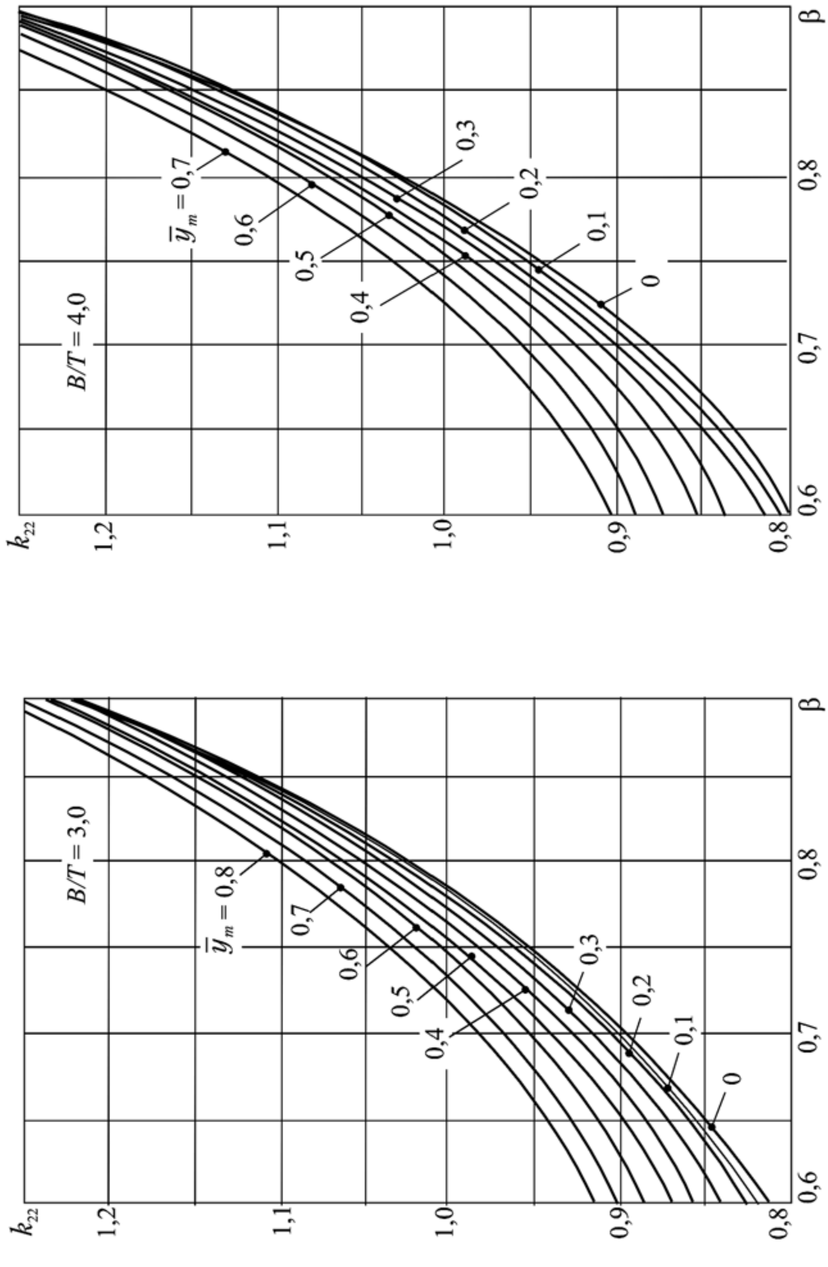


Fig. 2.52 Coefficients of added masses of an inclined shipframe

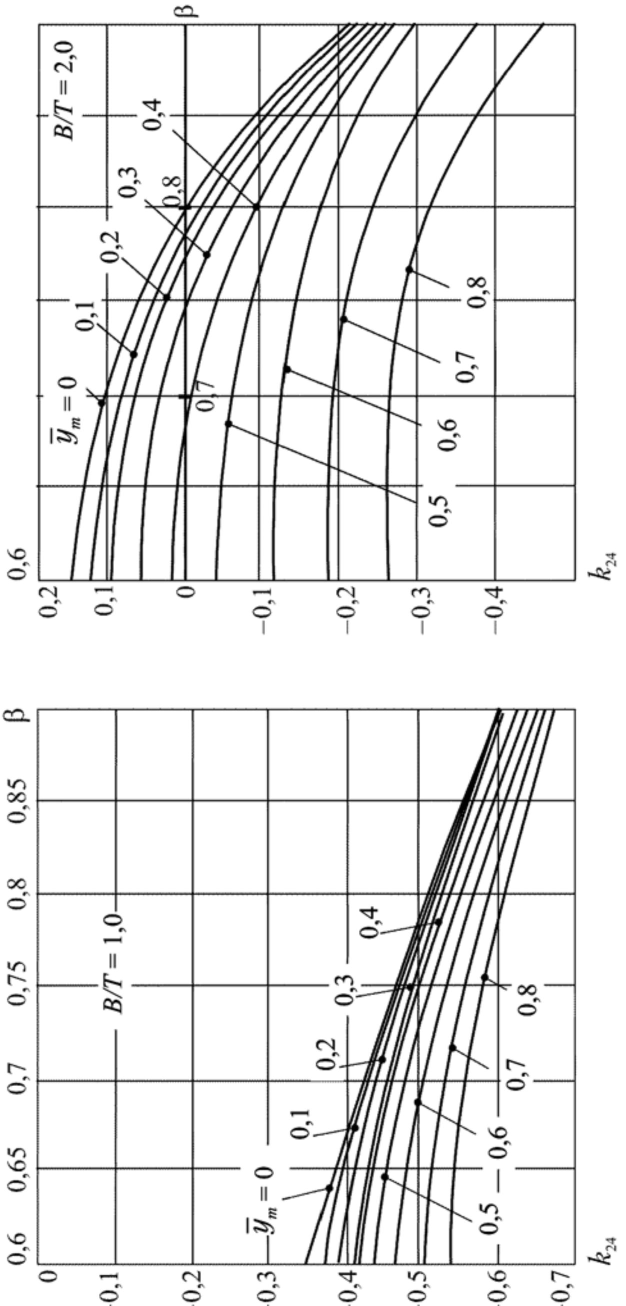


Fig. 2.53 Coefficients of added masses of an inclined shipframe

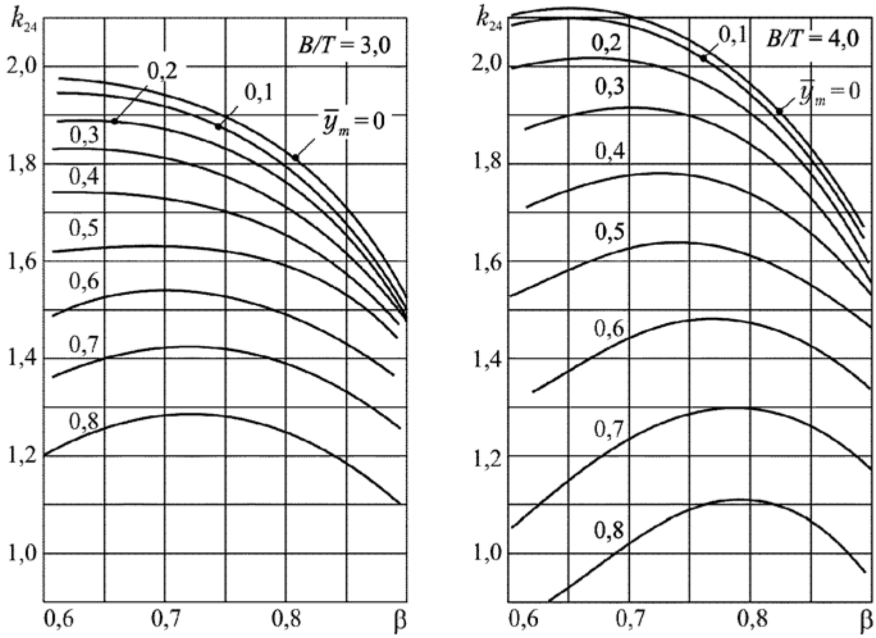


Fig. 2.54 Coefficients of added masses of an inclined shipframe

where $\lambda_{11}, \lambda_{22}$ and λ_{66} are added masses and added moment of inertia of the single body; B_1 is the distance between the symmetry planes (diameter planes) of the bodies; κ_1 and κ_2 are coefficients taking into account the mutual position of the bodies. These coefficients are determined by the formulas:

$$\kappa_1 = 2 + e^{-\bar{c}}, \qquad \kappa_2 = 2 - 0.8e^{-2\bar{c}}, \qquad \bar{c} = \frac{C}{B},$$

where C is the distance between the internal surfaces of the bodies, measured along the waterline, B is the width of one body of the catamaran.

Other interesting results for added masses of a shipframe considered as a part of a catamaran obtained in [48]:

$$\lambda_{22csf} = k_{22}\lambda_{22sf}, \qquad \lambda_{33csf} = k_{33}\lambda_{33sf},$$

where $\lambda_{22sf}, \lambda_{33sf}$ are added masses of the shipframe of each of the bodies forming the catamaran; k_{22} and k_{33} are coefficients taking into account the mutual position of the bodies.

The coefficients k_{22} and k_{33} for the different shapes of the shipframes are shown in Fig. 2.55.

On catamarans, as well as on single-body vessels, one could use twin rudders (Fig. 2.56). The added mass of the rudders in the direction of the y axis is determined as follows [2, 230]:

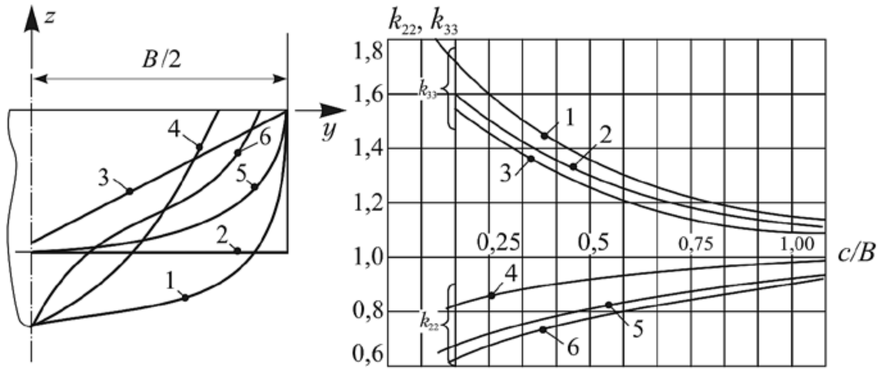


Fig. 2.55 Coefficients of added masses of a single hull considered as part of a catamaran

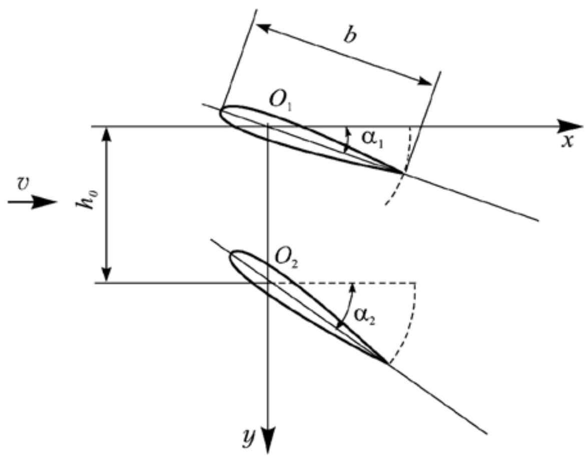


Fig. 2.56 Positions of twin rudders

$$\lambda_y = K_y \rho b^2 l,$$

where b is the chord of the rudder, l is the span of the rudder, K_y is the coefficient determined from the graphs shown in Figs. 2.57–2.60 as functions of parameters

$$\alpha_1, \alpha_2, \bar{\alpha}, \bar{h}_0/b, \lambda,$$

where α_1 and α_2 are the attack angles of the twin rudders, $\bar{\alpha} := \alpha_2/\alpha_1$, h_0 is the distance between the axes of the stocks, and λ is the elongation (ratio of the chord to the span) of the rudder.

Graphs shown in Figs. 2.57–2.60 are obtained under assumption that $\alpha_2 \geq \alpha_1$ and therefore $\bar{\alpha} \geq 1$. Therefore if $\alpha_1 \neq \alpha_2$ we choose the larger angle α_2 in determining the coefficient K_y. If $\alpha_1 > \alpha_2$, then the indices of the angle should be interchanged, i.e., α_1 should be chosen as an argument on the graphs in Figs. 2.57–2.60, and parameter $\bar{\alpha}$ is determined as $\bar{\alpha} = \alpha_1/\alpha_2$.

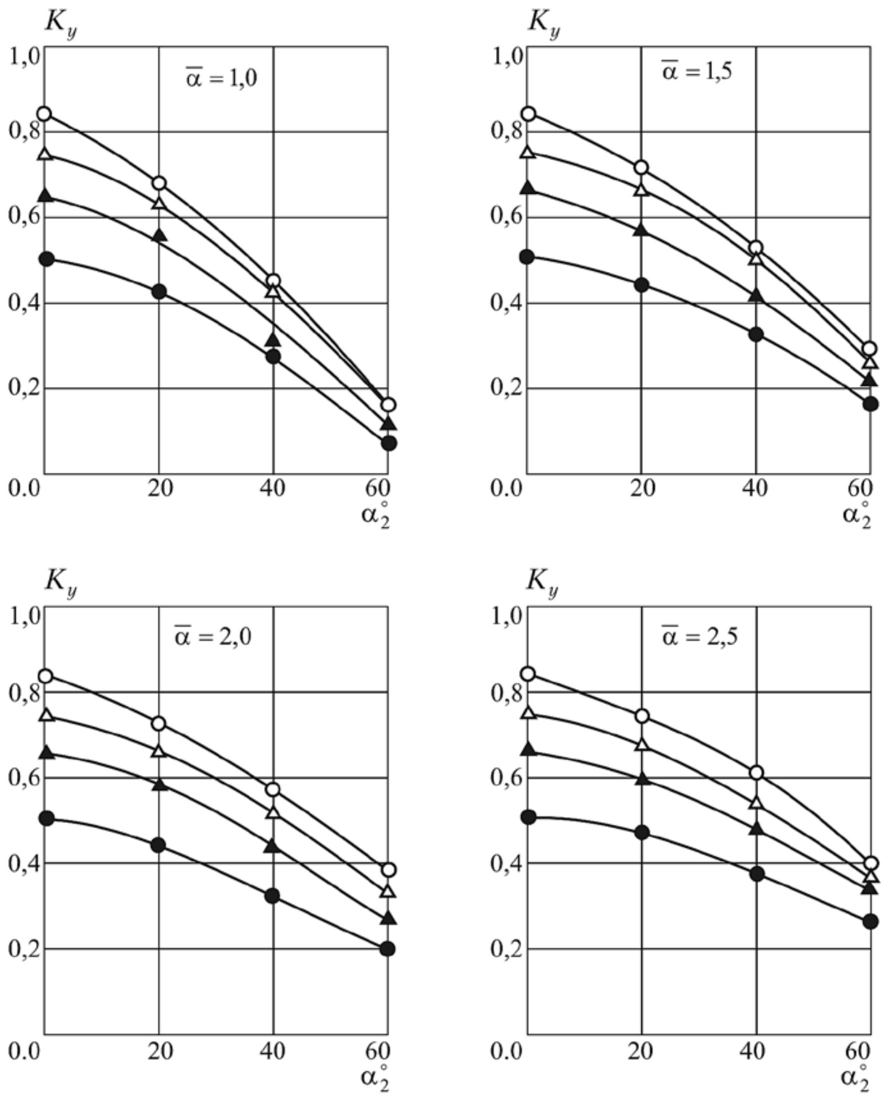


Fig. 2.57 Values of coefficient K_y for $h_0/b = 0.8$. *Black circles* correspond to $\lambda = 0.5$, *black triangles* to $\lambda = 0.75$, *white triangles* to $\lambda = 1.0$, *white circles* to $\lambda = 1.25$

The total added masses of the system vessel-rudder are determined by the formulas

$$\lambda_{22}^{\text{total}} = \lambda_{22} + \lambda_y, \qquad \lambda_{26}^{\text{total}} = \lambda_{26} - l_b\lambda_y, \qquad \lambda_{66}^{\text{total}} = \lambda_{66} - l_b^2\lambda_y,$$

where l_b is the distance from the rudder stock to the center of mass of the vessel.

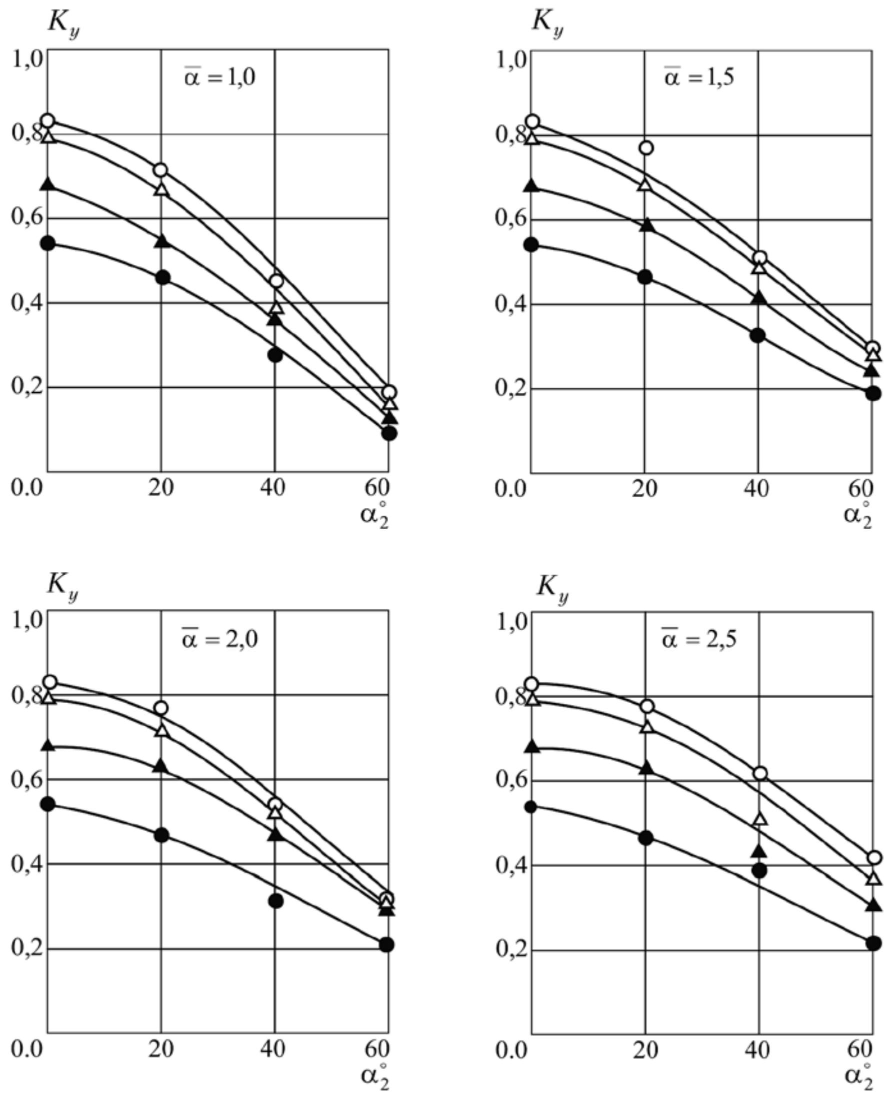


Fig. 2.58 Values of coefficient K_y for $h_0/b = 1.0$. *Black circles* correspond to $\lambda = 0.5$, *black triangles* to $\lambda = 0.75$, *white triangles* to $\lambda = 1.0$, *white circles* to $\lambda = 1.25$

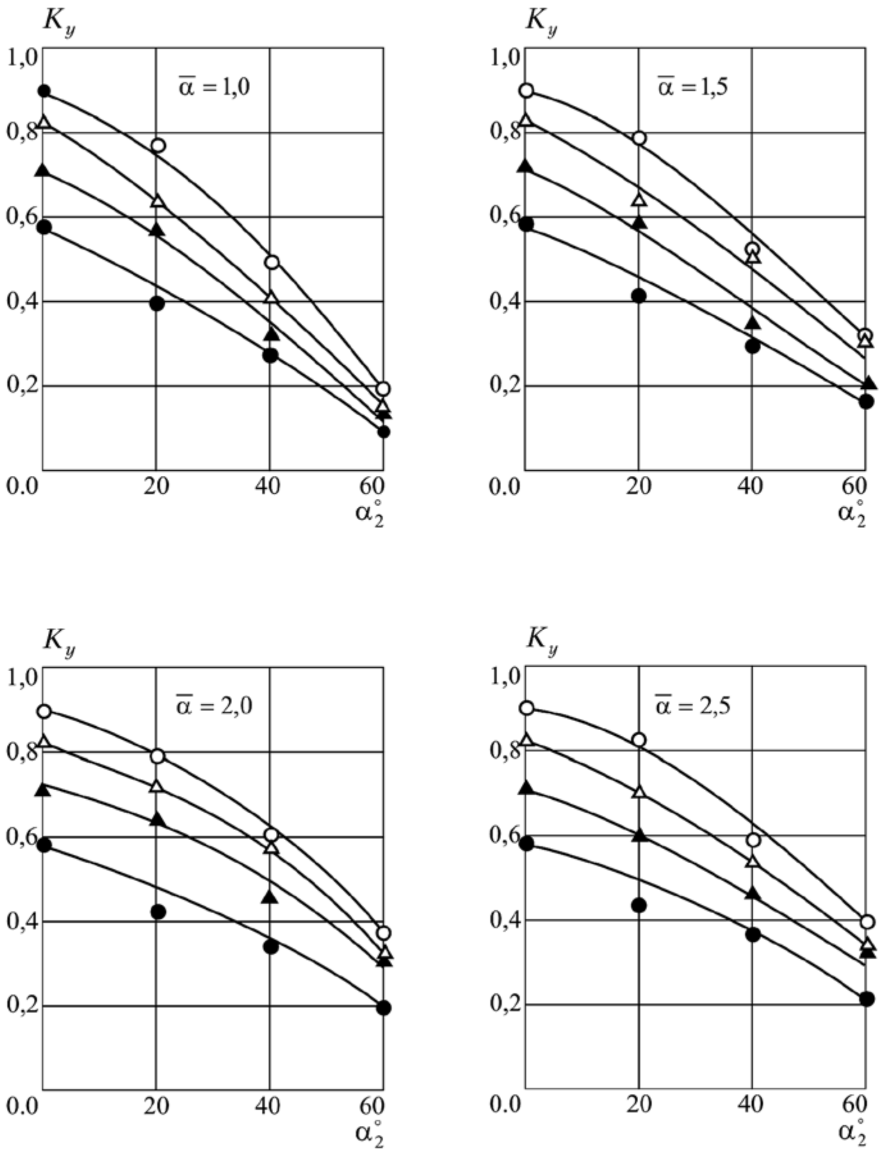


Fig. 2.59 Values of coefficient K_y for $h_0/b = 1.2$. *Black circles* correspond to $\lambda = 0.5$, *black triangles* to $\lambda = 0.75$, *white triangles* to $\lambda = 1.0$, *white circles* to $\lambda = 1.25$

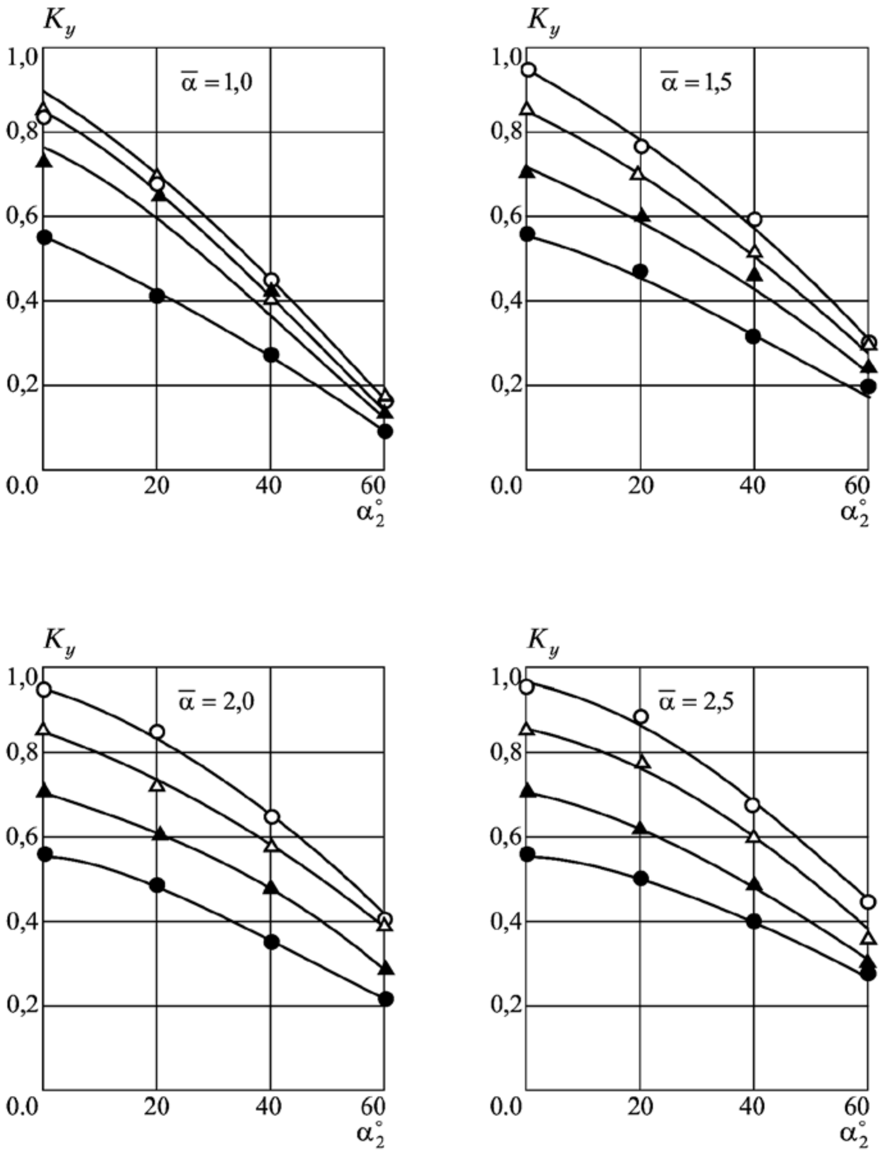


Fig. 2.60 Values of coefficient K_y for $h_0/b = 1.4$. *Black circles* correspond to $\lambda = 0.5$, *black triangles* to $\lambda = 0.75$, *white triangles* to $\lambda = 1.0$, *white circles* to $\lambda = 1.25$

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