Preface

The purpose of this volume is to provide a comprehensive view of the theory of periodic systems, by presenting the fundamental analytical tools for their analysis and the state of the art in the associated problems of filtering and control. This book is intended to be useful as a reference for all scientists, students, and professionals who have to investigate periodic phenomena or to study periodic control systems. For teaching purposes, the volume can be used as a textbook for a graduate course, especially in engineering, physics, economics and mathematical sciences. The treatment is self-contained; only a basic knowledge of input–output and state–space representations of dynamical systems is assumed.

Periodic Systems

Ordinary differential equations with periodic coefficients have a long history in physics and mathematics going back to the contributions of the 19th century by Faraday [134], Mathieu [230], Floquet [145], Rayleigh [250] and [251], Hill [184], and many others. As an intermediate class of systems bridging the time-invariant realm to the time-varying one, periodic systems are often included as a regular chapter in textbooks of differential equations or dynamical systems, such as [123, 175, 237, 256, 303]. In the second half of the 20th century, the development of systems and control theory has set the stage for a renewed interest in the study of periodic systems, both in continuous and in discrete-time, see e.g., the books [136, 155, 228, 252, 312] and the survey papers [29, 42]. This has been emphasized by specific application demands, in particular in industrial process control, see [1, 43, 76, 266, 267, 299], communication systems, [119, 144, 282], natural sciences, [225] and economics, [148, 161].

Periodic Control

The fact that a periodic operation may be advantageous is well-known to mankind since time immemorial. All farmers know that it is not advisable to always grow the same crop over the same field since the yield can be improved by adopting a crops rotation criterion. So, cycling is good.

In more recent times, similar concepts have been applied to industrial problems. Traditionally, almost every continuous industrial process was set and kept, in presence of disturbances, at a suitable steady state. It was the task of the designer to choose the optimal stationary regime. If the stationary requirement can be relaxed, the question arises whether a periodic action can lead to a better performance than the optimal stationary one. This observation germinated in the field of chemical engineering where it was seen that the performance of a number of catalytic reactors improved by cycling, see the pioneer contributions [17, 153, 189]. Suitable frequency domain tests have been developed to this purpose in the early 1970s, [25, 62, 162, 163, 172, 173, 275]. Unfortunately, as pointed out in [15], periodic control was still considered "too advanced" in the scenario of industrial control, in that "the steady-state operation is the norm and unsteady process behavior is taboo". Its interest was therefore confined to advanced applications, such as those treated in [274] and [257]. However, in our days, the new possibilities offered by the control technology, together with the theoretical developments of the field, opened the way for a wide use of periodic operations. For example, periodic control is useful in a variety of problems concerning under-actuated systems, namely systems with a limited number of control inputs with respect to the degrees of freedom. In this field, control is often performed by imposing a stable limit cycle, namely an isolated and attractive periodic orbit, [97, 150, 179]. Another example comes from non-holonomic mechanical systems, where in some cases stabilization cannot be achieved by means of a time-invariant differentiable feedback control law, but it is achievable with a periodic control law, [14, 86].

In contemporary literature, the term periodic control takes a wider significance, and includes problems where either the controller or the system under control is a proper periodic system.

Periodic Models in Time-Series and Signal-Processing

Periodic models are useful in signal-processing and time-series analysis as well. In digital signal processing, periodicity arises whenever filter banks are used to impose different sampling rates for data compression, see [282] for a reference book. In time-series analysis, there are important phenomena that exhibit a periodic-like behavior. A typical example is constituted by the Wolf series of the sun spot numbers, a series which presents a sort of periodicity of 11.4 years. A distinctive feature of some seasonal series is that its lack of stationarity is not limited to the non-stationarity of the mean value: higher order statistics do not match those of a stationary process. In such cases, a model with periodic coefficients is more accurate than time-invariant modeling, especially for prediction purposes. The theory of such models is the subject of many publications, both theoretical and applied. The corresponding stochastic processes are named *cyclostationary processes* or *periodic correlated signals*, [79, 146, 155, 160, 190–192, 199, 222–225].

Motivating Applications

Among all possible fields where periodic control has a major impact, we focus here on a couple of motivating applications.

In aerospace, a considerable effort has been devoted to the development of active control systems for the attenuation of vibrations in helicopters. Besides improving the comfort of the crew and passengers, such attenuations would be profitable to reduce fatigue in the rotor structure of the aircraft and to protect on-board equipment from damage. Various approaches to this problem have been proposed in the literature, [280]. Among them, the focus is on the Individual Blade Control (IBC) strategies, [176, 196, 280] and references quoted therein. In an IBC framework, the control input is the pitch angle of each blade of the main rotor, while the output is the acceleration of the vibratory load transmitted from the blades to the rotor hub. This IBC vibration control problem arises in various flight conditions. While in hovering (motionless flight over a reference point), the dynamics of the rotor blade can be described by a time-invariant model; in forward flight the dynamics turns out to be time-periodic, with period $2\pi/\Omega$, where Ω is the rotor revolution frequency, see [198]. More precisely, the matrices of the model are periodic and take different values depending on the forward velocity of the aircraft. As for the vibration effect, if only the vertical component of the vibratory load is considered, it can be conveniently modeled by a periodic additive disturbance with frequency $N\Omega$, where N is the number of blades. The reader interested in more details is referred to [11, 71, 73, 75].

Another aerospace application can be found in satellite attitude control. The recent interest in small earth artificial satellites has spurred research activity in the attitude stabilization and control. The typical actuator is the magneto-torque, a device based on the interaction between the geomagnetic field and the magnetic field generated on-board by means of a set of three magnetic coils, typically aligned with the spacecraft principal axis. The current circulating in the coils is produced by means of solar paddles. A time history of the geomagnetic field encountered by the satellite along a (quasi) polar orbit shows a very strong periodic behavior, [302]. Consequently, the effect of the interaction between the geomagnetic field and the on-board magnetic field is periodically modulated with a period equal to the period of rotation of the satellite around the earth. It is therefore not surprising that the attitude model obtained by linearization of the satellite dynamics around the orbit is essentially periodic, [12, 135].

Turning to the field of economics and finance, a longstanding tradition in data analysis is to adopt additive models where trend, seasonality, and irregularity are treated as independent components none of which can be separately observed, [84, 177, 188, 232, 283]. In economic data, the seasonal term is determined by the natural rhythm of the series, typically 12 months or four quarters. The decomposition can be performed with various techniques. The US Bureau of Census seasonal adjustment toolkit known as $X - 11$ is one of the most used, [217]. The basic rationale behind "seasonal adjustment", is to pre-filter data so as to obtain a new series that can be described by an ARMA (AutoRegressive Moving Average) model. Solid identification and forecasting methods are then available. Reality, however, is often complex, and periodicity may be hidden in a more subtle form. This is the case of the STANDARD & POOR's 500 (S&P 500) stock index, or the UK non-durable consumer's expenditures, [147, 148, 161, 240]. The higher-order periodicity is better captured by Periodic ARMA or Periodic GARCH (Generalized AutoRegressive Conditional Heteroscedastic) models, so obtaining a significant improvement in the accuracy of forecasts, [241].

Organization of the Book

As already said, this book aims to provide a theoretical corpus of results relative to the analysis, modeling, filtering, and control of periodic systems. The primary objective is to bring together in a coordinated view the innumerable variety of contributions appeared over many decades, by focusing on the theoretical aspects.

We concentrate on discrete-time signals and systems. However, in Chap. 1 we provide an overview of the entire field, including the continuous-time case. In particular, we will present the celebrated Π-test aiming at establishing whether a periodic operation of a time-invariant plant may lead to better performances than the optimal steady-state operation, see [25, 62, 162, 163, 172, 274–276], in continuous-time and [64, 65] in discrete-time.

The book proceeds with Chap. 2, devoted to the basic models and tools for the analysis. Among the key ingredients of this chapter, we mention the periodic transfer operator, the PARMA representation and the periodic state–space representation.

In Chap. 3, the discrete-time version of the Floquet theory is presented and the characterization of stability is provided also in terms of Lyapunov difference inequalities. Robust stability criteria are provided as periodic linear matrix inequalities. Finally, the companion forms for periodic systems are introduced.

Chapter 4 is devoted to structural properties. Here we provide a thorough analysis of reachability, controllability, observability, reconstructability, stabilizability, and detectability. From this study, the canonical decomposition of a periodic system follows. The structural properties allow the derivation of the so-called inertia results for the Lyapunov difference equations.

The study of periodic systems as input–output periodic operators is the subject of Chap. 5. Here, a single-input single-output (SISO) periodic system is seen as the (left or right) ratio of periodic polynomials. Instrumental to this study is the algebra of such polynomials. The chapter ends with the definition and characterization of input–output stability. A main tool for the analysis of periodic systems in discrete-time is provided by the reformulation technique. This amounts to the rewriting of a periodic system as a time-invariant one via a suitable transformation of variables. This is indeed a powerful technique, often encountered in the literature. However, in any design steps one has to explicitly take into account the constraints which a time-invariant system must meet in order to be transformed back into a periodic one.

These considerations are at the basis of Chap. 6, where three time-invariant reformulations are treated, namely, the time-lifted, the cyclic and the frequency-lifted reformulations.

The state–space realization problem, in its various facets, is studied in Chap. 7. After the definition of minimality for periodic systems, we first consider the determination of a state–space model starting from the time-lifted transfer function. Then, the realization of SISO systems from polynomial fractions is analyzed. The chapter ends with the study of balanced realization.

The notions of poles and zeros are dealt with in Chap. 8. These notions are seen from different angles, depending on the adopted type of model. Related to this study is the analysis of the delay structure, leading to the notion of spectral interactor matrix.

Chapter 9 is devoted to the norms of periodic systems. In particular, the notions of L_2 and L_∞ norms are widely discussed, and their characterizations are given both in the time and frequency domain. The definition of entropy is also introduced. Finally the role of the periodic Lyapunov difference equation in the computation of the l_∞-l_2 gain is pointed out.

Chapter 10 deals with the problem of factorization of a periodic system. The spectral factorization and the J-spectral factorization are characterized in terms of the periodic solutions of suitable periodic Riccati equations. Finally, the parametrization of all stabilizing controllers is treated both in the field of rational periodic operators and in the field of polynomial periodic operators.

Cyclostationary processes are concisely studied in Chap. 11, with special references to their spectral properties.

Chapter 12 includes the typical estimation problems. First, filtering, prediction and fixed-lag smoothing in L_2 are considered via state–space and factorization tools. In particular, the celebrated periodic Kalman filtering theory is recovered. Next, the same problems are tackled in the L_∞ framework.

In the last chapter, namely Chap. 13, we treat a large number of stabilization and control problems. We begin with the characterization of the class of periodic state-feedback controllers in terms of periodic linear matrix inequalities. The extension to uncertain systems is also dealt with. Next, the classical problem of pole assignment is considered. Both sampled and memoryless state-feedback control laws are considered. The chapter proceeds with the description of the exact model matching problem and the H_2 and H_∞ state-feedback control problems. The static output-feedback stabilization problem is dealt with next. The last section regards the dynamic output-feedback problem, including robust stabilization via a polynomial description, the LQG and the H_∞ problems.

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