Preface

A major problem in control engineering is a robust feedback design that stabilizes a nominal plant, while also attenuating the influence of parameter variations and external disturbances. In the last decade, this problem was heavily studied and considerable research efforts have resulted in the development of systematic design methodology for nonlinear feedback systems. This methodology was, however, basically confined to smooth feedback systems, whereas motivated by modern applications, significant interest has emerged in extending this methodology to complex, particularly, electromechanical systems with hard-to-model nonsmooth phenomena such as friction and backlash. Since ignoring these phenomena may severely limit the achievable performance, the practical utility of the existing smooth control algorithms becomes questionable for many electromechanical applications. Thus, nonsmooth feedback design is continuing to be developed.

The primary concern of this book is stability analysis and robust control synthesis of uncertain discontinuous systems within the framework of methods of nonsmooth Lyapunov functions. Motivating examples for studying discontinuous systems are mechanical systems with complex nonlinear phenomena (collision, backlash, dry friction). Another motivation comes from the rapidly developed area of variable structure control.

A discontinuous system is typically viewed as a simple model of hybrid systems, consisting of a finite family of continuous-time subsystems, equipped with a rule of switching between them. Whenever the system trajectory hits a switching surface, the continuous state makes a jump, specified by a restitution rule. A special case is when state jumps are absent. Just in case, the state trajectory is always continuous, but in general it is not differentiable when it hits a switching surface. After hitting a switching surface, the trajectory can either cross it or evolve along the surface on a finite time interval. In the latter case, sliding motions occur in the system.

Although the Lyapunov methods have been widely used in practice and the need of nonsmooth Lyapunov functions has particularly been recognized for nonsmooth dynamic systems [94, 214], these methods do not admit a straightforward extension to discontinuous dynamic systems. In this regard, recall (see [146] for details) that Krasovskii–LaSalle's invariance principle [118, 121], generally speaking, fails to hold for dynamic systems, governed by differential inclusions and, in particular, by differential equations with discontinuous right-hand sides (see [6, 91, 97, 145, 165, 202, 223] for various extensions of the invariance principle).

There has been work on the Lyapunov stability theory in discontinuous systems. Smooth Lyapunov functions have successfully been used by V. A. Yakubovich, G. A. Leonov, and A. Kh. Gelig [245] to analyze dynamic systems with discontinuous nonlinearities. For a class of discontinuous systems, admitting a finite frequency of switches only, the theory of multiple Lyapunov functions was initiated by M. Branicky [30] and then developed by D. Liberzon [132]. This theory does not capture discontinuous systems with sliding motions and it can be viewed as a complementary to analysis tools of sliding mode control theory pioneered by V. I. Utkin [227].

In order to analyze discontinuous systems with sliding modes, differentiable Lyapunov functions suffice in many cases. D. Shevitz and B. Paden [207] invoked the nonsmooth Lyapunov analysis for studying discontinuous systems where "kinks" form an essential part of dynamics. While being confined to a class of discontinuous systems, trajectories of which are unambiguously defined to the right, their analysis becomes restrictive in many practical situations.

In order to avoid relating to this restrictive uniqueness condition, a novel technique, recently proposed in [165], is developed in the present book. In addition to a nonsmooth Lyapunov function with a non-positive time derivative along the system trajectories, the technique involves an auxiliary indefinite (rather than a definite) function that allows one to derive a certain integral inequality, which by Barbalat's lemma ensures the asymptotical stability of the closed-loop system. In contrast to Krasovskii–LaSalle's invariance principle [118, 121], the technique, which is referred to as the extended invariance principle, is applicable to general time-varying systems with discontinuous nonlinearities as well.

The extended invariance principle, while being applied to a homogeneous discontinuous system, proves to be capable of revealing not simply asymptotic stability but also the finite time stability of such a system. In turn, the finite time stability of a homogenous discontinuous system is known [168, 169] to persist regardless of inhomogeneous perturbations. This fundamental property constitutes the quasihomogeneity principle, and along with the extended invariance principle, it forms the core of the stability analysis developed in the book.

Based on these fundamental principles, a synthesis of globally stabilizing controllers of uncertain dynamic systems is subsequently developed. The present synthesis does not necessarily rely on the generation of sliding modes, while retaining robustness features, similar to those possessed by their sliding mode counterparts. The strategy of the discontinuous controllers constructed is to drive the system to the zero dynamics manifold in finite time and to maintain it there in spite of the parameter uncertainties and external disturbances, both with a priori known norm bounds. Desired robustness properties and an asymptotic stability of the the closed-loop system are thus provided.

Attractive features of the discontinuous controllers proposed are illustrated by applications to electromechanical systems. Allowing relatively strong Coulomb friction in these applications precludes the use of continuous regulators, because the closed-loop system, in that case, would have a nontrivial set of equilibrium points and it would therefore be driven to a wrong endpoint. As opposed to continuous controllers, the discontinuous controllers are demonstrated to be capable of providing the desired system performance in spite of significant uncertainties in the system description, as is typically the case in the control of electromechanical systems with complex backlash/friction phenomena.

Besides the finite-dimensional treatment, robust discontinuous control algorithms, recently developed in [162, 171, 174] for infinite-dimensional systems such as time delay systems and distributed parameter systems, are also presented.

The book is intended for graduate students and control specialists interested in the discontinuous systems theory and control applications. No background in discontinuous systems is required, as such systems are conceptually introduced at the appropriate level.

Some related topics, however, are either not covered or only partially covered in this book. No specific study is proposed for impulsive systems, for switched systems with isolated switching events, and for sliding mode systems. These kinds of systems are treated within the general paradigm of discontinuous systems, whereas their rather comprehensive studies can be found in [13, 91, 123, 201, 204, 246], regarding impulsive systems, in [97, 132], regarding switched systems, and in [65, 99, 227, 228], regarding sliding modes (see also surveys [76, 77] and the special issue [208] for advanced results on higher-order sliding modes). General hybrid systems, to be involved into hybrid synthesis of underactuated systems in Sect. 10, are only briefly commented in Chap. 1. For a deeper insight on hybrid systems, see [34, 35, 82, 143, 202, 230].

Analysis, synthesis, and applications to electromechanical systems are developed under uncertainty conditions, including model inadequacies. While being less useful for modelers, this adopted framework is attractive for control engineers interested in coming up with schemes that allow one to successfully address stability/stabilization issues in all circumstances. Adequate models of complex phenomena in electromechanical systems can be found, e.g., in [10, 41, 56, 158] (friction modeling), [142, 156, 221] (backlash modeling), [33, 81, 125, 150] (post-impact restitution).

The book consists of an introduction and four parts, and it is organized as follows. Chapter 1 previews issues that will arise in the sequel.

Mathematical tools of discontinuous systems are reviewed in Part I of the book. Differential equations with piece-wise continuous right-hand sides are accepted in Chap. 2 as a basic mathematical model of such a system. While allowing Dirac functions in the coefficients, the equations admit instantaneous jumps of the state of the system. The instantaneous impulse response of the system is adequately defined according to Schwartz' distribution theory in a nonlinear setting. Various solution concepts (Filippov solution [71], Utkin solution [227], and the vibroimpact solution [164]) are introduced for these equations. The existence and uniqueness of the solutions as well as their physical sense, and applications to modeling of nonlinear phenomena in electromechanical systems are discussed.

In Chap. 3, the nonsmooth Lyapunov analysis of discontinuous systems and those of discrete-continuous dynamics are developed side by side. Several stability concepts such as stability and exponential/asymptotic stability are addressed locally and in large. Semiglobal Lyapunov functions are particularly introduced to subsequently address a relatively new kind of robust stability when the system is required to be asymptotically stable, equiuniformly in admissible non-vanishing external disturbances. In addition, $\mathscr{L}_2$-gain analysis in a finite-dimensional setting and LMI-based analysis in an infinite-dimensional setting are presented.

Chapter 4 deals with homogeneous systems. These systems are of a particular interest, because under appropriate conditions on the homogeneity degree, their equiuniform asymptotic stability ensures that the state of the system escapes to zero in finite time. The finite-time stability property persists, even if the system is affected by inhomogeneous external disturbances. This result constitutes the quasihomogeneity principle whose capabilities are illustrated by several examples.

Part II of the book is devoted to robust discontinuous control synthesis. The quasihomogeneous design is constituted in Chap. 5.

Sliding mode-based unit feedback controller design is developed in Chap. 6. Following the Lyapunov minmax approach, a discontinuous feedback controller, that counteracts non-vanishing disturbances and parameter variations, is synthesized to guarantee that the time derivative of a Lyapunov function, selected for a nominal system, is negative definite on the trajectories of the perturbed system. The approach gives rise to the so-called unit control feedback, the norm of which is equal to one everywhere but on the manifold where the feedback undergoes discontinuities. The resulting closed-loop system is shown to never pass through the switching manifold. The system stability is thus analyzed beyond the manifold. Once the trajectory is on the switching manifold, a smooth dynamic is restored and standard Lyapunov theory is in force. In addition, how undesired high frequency state oscillations, caused by fast switching in the unit controller, can be removed by smoothing the unit signal is discussed.

In Chap. 7, $\mathscr{H}_\infty$-control synthesis is presented for nonsmooth time-varying systems via measurement feedback. Being in a certain sense an extension of the unit feedback synthesis, the $\mathscr{H}_\infty$-control design aims to guarantee both the internal asymptotic stability of the (disturbance-free) closed-loop system and the dissipative inequality (in that the size of an error signal is uniformly bounded with respect to the worst case size of an external disturbance signal). Similar to [107, 229], the approach here is to construct an energy or storage function, resulting in the dissipative inequality. Once the storage function is found, it can be used as a Lyapunov function, guaranteeing the internal stability requirement. Sufficient conditions for the storage function to exist are given in terms of the solvability of two nonsmooth Hamilton–Jacobi–Isaacs inequalities which arise in the state-feedback and, respectively, the output-injection design. The development follows the line of reasoning proposed in [4] where the corresponding Hamilton–Jacobi–Isaacs expressions are required to be negative definite rather than semidefinite. This feature allows one to develop an $\mathscr{H}_\infty$-design procedure with no a priori-imposed stabilizability-detectability conditions on the control system. Although the design procedure results in an infinite-dimensional problem, this difficulty is circumvented by solving the problem locally. The distribution-based formalism is then developed to straightforwardly derive a local solution of the sampled-data measurement feedback $\mathscr{H}_\infty$-control problem from that of the time continuous measurement feedback $\mathscr{H}_\infty$-control problem.

Part III develops the unit feedback synthesis for infinite-dimensional systems driven in a Hilbert space. The presence of an unbounded operator in the state equation precludes from a simple extension of the finite-dimensional control algorithms. Theoretical results obtained in an abstract infinite-dimensional setting are then supported by applications to distributed parameter and time delay systems.

In Chap. 8, the unit feedback synthesis is developed for a class of linear infinite-dimensional systems with a finite-dimensional unstable part using finite-dimensional sensing and actuation. An output feedback controller is synthesized by coupling an infinite-dimensional Luenberger state observer and unit state feedback controller. In order to obtain the fully practical finite-dimensional framework for controller synthesis, a finite-dimensional approximation of the Luenberger observer as well as a continuous approximation of the unit feedback controller are carried out at the implementation stage. Implementation, performance, and robustness issues of the unit output feedback control design are illustrated in a simulation study of the linearization of the Kuramoto–Sivashinsky equation (KSE) around the spatially-uniform steady-state solution with periodic boundary conditions. While being unforced, the KSE describes incipient instabilities in a variety of physical and chemical systems and a control problem that occurs here is to avoid the appearance of the instabilities in the closed-loop system.

In Chap. 9, the unit control approach is extended to Hilbert space-valued minimum phase semilinear systems. Control algorithms presented ensure global or local asymptotic stability, according as state feedback or output feedback is available. The desired robustness properties of the closed-loop system against external disturbances with a priori known norm bounds make the algorithms extremely suited for stabilization of the underlying system operating under uncertainty conditions. It is, in particular, shown that discontinuous feedback stabilization is constructively available in the case where complex nonlinear dynamics of the uncertain system do not admit factoring out a destabilizing nonlinear gain, and thus the destabilizing gain cannot be handled through nonlinear damping. The theory is applied to the stabilization of chemical processes around pre-specified steady-state temperature and concentration profiles corresponding to a desired coolant temperature. Performance issues of the unit feedback design are illustrated in a simulation study of the plug flow reactor.

In Chap. 10, the unit feedback synthesis is generalized for a class of uncertain time-delay systems with nonlinear disturbances and unknown delay values whose unperturbed dynamics are linear. The controller constructed proves to be robust against sufficiently small delay variations and weak external (possibly, unmatched) disturbances. It is worth noticing that allowing unmatched disturbances is a step beyond a standard sliding mode control treatment. Specifically, the critical delay value when the closed-loop system, corresponding to this value, becomes asymptotically unstable, is explicitly calculated as a function of linear growth constants of the unmatched disturbances. Performance issues of the controller are illustrated by means of a numerical example.

In Part IV of the book, performance issues of the developed controllers are experimentally tested in engineering applications to electromechanical systems with complex nonlinear phenomena.

Chapter 11 develops the local nonsmooth $\mathscr{H}_\infty$-synthesis of Lagrangian systems which is capable of accounting for hard-to-model friction forces and backlash effects. The resulting control algorithms are illustrated in an experimental study of the position feedback regulation of a laboratory three-link robot manipulator with frictional joints and in that of the output feedback regulation of a servomechanism with backlash.

Chapter 12 deals with the quasihomogeneous synthesis which is shown to be extremely suited for fully actuated systems with dry friction. Since external disturbances, affecting these systems, represent discontinuous functions in the state variables and meet the matching condition, their influence is not simply attenuated as it would be the case under $\mathscr{H}_\infty$-synthesis, but it is fully rejected under the quasihomogeneous synthesis. Moreover, global position regulation becomes possible provided that an upper bound on the magnitude of the external disturbances is known a priori. Attractive features of the quasihomogeneous synthesis are illustrated by means of orbital stabilization of a simple inverted pendulum, and by means of the global position regulation of a multi-link robot manipulator.

In Chap. 13, the quasihomogeneous design is developed for 2-DOFs underactuated systems by including it into a unified hybrid synthesis framework. Being verified experimentally, the proposed hybrid synthesis appears to provide desired robustness properties against friction forces. Its capabilities are illustrated in experimental studies made for laboratory test beds such as a horizontal double pendulum, an inverted double pendulum (Pendubot), and a pendulum, located on a cart.

The presentation in Chaps. 11-13 emphasizes the control algorithms, rather than their technical implementation. Such a technical consideration would somehow fall out of the general theoretical line of the book. A consideration of the specific applications, however, makes the book appropriately complete.

Ensenada, Mexico, July 2008 *Yury Orlov* http://www.springer.com/978-1-84800-983-7

Discontinuous Systems
Lyapunov Analysis and Robust Synthesis under Uncertainty Conditions
Orlov, Y.V.
2009, XIX, 320 p., Hardcover
ISBN: 978-1-84800-983-7