Preface

During the past fifteen years, there have been many exciting developments at the interface of mathematical control theory and control engineering. Many of these developments were based on Lyapunov methods for analyzing and controlling nonlinear systems. Constructing strict Lyapunov functions is a challenging, central problem. By contrast, *non*-strict Lyapunov functions are often readily constructed from passivity, backstepping, or forwarding (especially in the time-varying context), or by using the Hamiltonian in Euler-Lagrange systems. Roughly speaking, strict Lyapunov functions are characterized by having negative *definite* time derivatives along all trajectories of the system, while non-strict Lyapunov functions have negative *semi-definite* derivatives along the trajectories. Even when a system is known to be globally asymptotically stable, one often still needs an explicit strict Lyapunov function, e.g., to build feedbacks that provide input-to-state stability to actuator errors.

One important research direction involves finding necessary and sufficient conditions for various kinds of stability, in terms of the existence of Lyapunov functions, such as Lyapunov characterizations for hybrid systems, or for systems with outputs and measurement uncertainty. Converse Lyapunov function theory guarantees the existence of strict Lyapunov functions for many globally asymptotically stable nonlinear systems. However, the Lyapunov functions provided by converse theory are often abstract and non-explicit, because they involve suprema or infima over infinite sets of trajectories, so they may not always lend themselves to feedback design. Explicit strict Lyapunov functions are also useful for quantifying the effects of uncertainty, since for example they can be used to construct the comparison functions in the input-to-state stability estimate, or to guarantee that a model reduction based on singular perturbation analysis can be done. In fact, once an appropriate global strict Lyapunov function has been constructed, several important robustness and stabilization problems can be solved almost immediately, through standard arguments.

In some cases, non-strict Lyapunov functions are sufficient, because they can be used in conjunction with Barbalat's Lemma or the LaSalle Invariance Principle to prove global stability. In other situations, it is enough to analyze the system near an equilibrium point, or around a reference trajectory, so linearizations and simple local quadratic Lyapunov functions suffice. However, it has become clear in the past two decades that non-strict Lyapunov functions and linearizations are insufficient to analyze general nonlinear time-varying systems. Non-strict Lyapunov functions are not well suited to robustness analysis, since their negative semi-definite derivatives along trajectories could become positive under arbitrarily small perturbations of the dynamics. Moreover, there are important nonlinear systems (e.g., chemostat models) that naturally evolve far from their equilibria. This has motivated a great deal of significant research on methods to explicitly construct global strict Lyapunov functions.

One approach to building explicit strict Lyapunov functions, which has received a considerable amount of attention in recent years, is the so-called *strictification method*. This involves transforming given non-strict Lyapunov functions into strict Lyapunov functions. Strictification reduces strict Lyapunov function construction problems to oftentimes much easier non-strict Lyapunov function construction problems. This book brings together a broad but unifying repertoire of strictification based methods. Much of this work appears here for the first time. We cover many important classes of nonlinear dynamics, including Jurdjevic-Quinn systems, time-varying systems satisfying LaSalle or Matrosov Conditions, adaptively controlled dynamics, slowly and rapidly time-varying systems, and hybrid time-varying systems. In fact, under a very mild extra assumption, we show how strict Lyapunov functions can be constructed for systems satisfying the conditions of the LaSalle Invariance Principle. The simplicity of our constructions makes them suitable for quantifying the effects of uncertainty, and for feedback design, including cases where only an output is available for measurement. We illustrate this in several applications that are of compelling engineering interest.

This work complements several books on nonlinear control theory, such as [149] by Sepulchre, Janković, and Kokotović. While many texts include Lyapunov function constructions, our work provides a systematic, design-oriented approach to building global strict Lyapunov functions, including simplified constructions that are more amenable to feedback design and robustness analysis. In fact, many of the systems covered by our approaches are beyond the scope of the well-known explicit strict Lyapunov constructions. Our book will be easily understood by readers who are familiar with the nonlinear control theory in the textbooks of Khalil [70] and Sontag [161]. We review much of the prerequisite material in the first two chapters. The remaining chapters can be used as supplemental reading in a first graduate control systems theory, or for a second course on Lyapunov based methods. Engineers and applied mathematicians interested in nonlinear control will also find our book useful.

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