
Preface

The state-space approach in the time domain and the transfer-function approach in the frequency domain are the two major methods for the design of linear control systems. The frequency-domain approach allows an equivalent representation of the time-domain results and it can be formulated on the basis of polynomial matrix fraction descriptions. These polynomial matrix representations generalize the concept of transfer functions for single-input, single-output (SISO) systems to multivariable systems.

The motivation for formulating the results of the state-space approach in the frequency domain can be understood when considering the observer-based control in the time domain. First, a state feedback is designed to assign the eigenvalues of the resulting state feedback loop. In order to implement this control the states of the system have to be estimated by an observer. This leads to an observer-based compensator that assigns the eigenvalues of the state feedback loop and of the observer to the resulting closed-loop system. This property is the well-known separation principle of state feedback control. Since the corresponding compensator is driven by the input and the output of the system only the input–output behaviour of the controller influences the properties of the closed-loop system, and this input–output behaviour is completely characterized by the transfer functions of the compensator. Therefore, it seems reasonable to determine directly the transfer behaviour of the compensator when designing control systems. This has the advantage that the additional degrees of freedom that exist in choosing a suitable realization of the compensator can be used, for example, to achieve further robustness properties. In a time-domain design it is not at all obvious how the choice of the free parameters in the design of the state feedback and the observer will influence the internal realization of the resulting compensator.

In this book it is shown that the transfer behaviour of the observer-based compensator can be determined on the basis of the transfer behaviour of the system. This is achieved by introducing a parameterization of the state feedback loop and of the observer in the frequency domain. An important property of these parameterizations is that the number of independent design parame-

ters coincides in the time-domain and the frequency-domain approaches and that connecting relations exist that allow an establishment of a one-to-one relationship between the time- and the frequency-domain parameterizations. Thus, every time-domain result can be transferred to an equivalent frequency-domain result and *vice versa*. As a consequence, each control problem can either be solved in the time domain or in the frequency domain, so that the best method for the solution can be chosen. For example, the decoupling of a reference transfer behaviour is easily stated and solved in the frequency domain since the design specification is to obtain a diagonal reference transfer matrix.

The general approach taken in this book is to formulate the time-domain solution first. Assuming that the reader is familiar with the basics of the state-space approach the presentation of the time-domain results is usually kept comparatively short. The development of the state-space methods given here is only more elaborate for those results which cannot be found in standard text books. Therefore, the book also provides a fast access to the controller design in the time domain.

Practical applications usually lead to system descriptions of high orders, so that the control design can only be carried out using computer assistance. In the last decades there have been important improvements in the algorithms for polynomial matrices, so that the polynomial approach can be used to design control systems for practical applications. All examples in the book were computed with the aid of the Polynomial Toolbox for MATLAB[®]. The m-files and the SIMULINK[®] files of the examples are available by writing an email to polybook@rt.eei.uni-erlangen.de.

Whereas the state-space descriptions of linear dynamic systems can be regarded as a standard tool, the frequency-domain representation of multiple-input, multiple-output systems on the basis of polynomial matrix fraction descriptions is not so widely known. Therefore, the *first chapter* of this book contains a short résumé of the basic facts on polynomial matrices used in the context of the design methods presented. A comprehensive introduction to polynomial methods, however, is not intended here.

The *second chapter* is devoted to the time-domain and the frequency-domain parameterizations of state feedback control. In the time domain the dynamics of the state feedback loop can be assigned by choosing the feedback gain K . It is shown that the resulting dynamics of the closed-loop system can be parameterized by the polynomial matrix $\tilde{D}(s)$ in the frequency domain. A relation is derived that establishes a one-to-one connection between the constant matrix K and the polynomial matrix $\tilde{D}(s)$, so that the time-domain parameterization can be obtained from the frequency-domain parameterization and *vice versa*.

In *Chapter 3* the design of reduced-order state observers is covered, which also includes the full-order observer. The time-domain formulation of the reduced-order observer used in this book was first proposed by Uttam and O'Halloran [62]. This particular form of the reduced-order observer has the

advantage of allowing a parameterization by two gain matrices that completely characterize the observer dynamics influencing the closed-loop system. Consequently, a connecting relation can be determined between the observer's gains and the polynomial matrix $\tilde{D}(s)$ that characterizes the observer dynamics in the frequency domain.

In the time domain the observed-based compensator is specified by the gain matrices of the state feedback and the observer, and the state-space model of the system is used in the design. In the frequency domain one starts with the parameterizing polynomial matrices of the state feedback and the observer and needs to compute the transfer behaviour of the corresponding observer-based compensator. In *Chapter 4* it is demonstrated that this transfer behaviour can be computed on the basis of the transfer behaviour of the system. As the eigenvalues of the closed-loop system are arbitrarily assignable, compensators with high gain can result. Such compensators may give rise to output signals that pass the input limitations existing in every technical system. The resulting restrictions of the plant input signals can cause undesired effects in the transients of the closed-loop system and they can even lead to limit cycles. These undesired effects of input saturation are called windup. It is shown in this chapter that the frequency-domain representation of the observer-based compensator allows the formulation of systematic measures to prevent windup effects.

In single-input systems, a set of desired eigenvalues completely specifies the feedback gain K . If more than one input exists, only part of K is specified by the desired eigenvalues. Since the remaining parameters in K also influence the properties of the closed-loop system they need to be determined when designing the control. The so-called parametric approach allows a choice of the desired eigenvalues while assigning the additional degrees of freedom in K to meet other design specifications. The same approach can also be used to parameterize the observer gains. This parametric design method was originally formulated in the time domain. *Chapter 5* presents an equivalent frequency-domain approach that uses the poles and the so-called pole directions to obtain a complete parameterization of the observer-based controller. This parametric compensator design also leads to a new time-domain parameterization of the reduced-order observer of Uttam and O'Halloran.

In a system with p inputs the modification of one input usually affects all outputs, *i.e.*, there is a cross coupling between the inputs and outputs. When the set point of one controlled output is changed this coupling can lead to an undesired reaction in the remaining outputs. Using a decoupling control, the closed-loop system behaves to reference inputs as if p single-input, single-output systems were operated in parallel. Not all systems are decouplable by static state feedback. Therefore, also a partial decoupling has to be considered. In *Chapter 6* the frequency-domain conditions for the complete decoupling and the partial decoupling are derived followed by the frequency-domain design of the corresponding controllers.

An observer-based compensator as designed in Chapter 4 does not asymptotically compensate persistently acting disturbances. It is well known that constant disturbances are rejected by an integral controller action. This concept can be generalized to more general signal forms that are modelled as outputs of a linear time-invariant dynamical system. By incorporating a model of this signal process in the compensator, also called the internal model principle, such persistently acting disturbances can be asymptotically rejected. In *Chapter 7* the robust design introduced by Davison [10] is modified in the time domain by formulating the driven signal model as an observer. This, on the one hand, avoids undesired effects on the reference transfer behaviour if the signal forms of the references do not coincide with the signal forms of the disturbances. On the other hand, it allows a systematic prevention of windup. In the second part of this chapter this new approach is formulated in the frequency domain.

The time- and the frequency-domain designs of optimal state feedback controllers and observers are presented in *Chapter 8*. The state feedback control can be designed in an optimal way by minimizing a quadratic performance index. The solution of the corresponding optimization problem leads to an algebraic Riccati equation (ARE). By using the connecting relations between the time- and the frequency-domain parameterizations of state feedback a polynomial matrix equation is obtained for $\tilde{D}(s)$ that parameterizes the optimal state feedback control in the frequency domain. By solving this equation, the design of the optimal controller can be carried out directly in the frequency domain. Also, an optimal design of observers is possible and it yields optimal filters. Assuming that the system is driven by white noise and that its measurements are also corrupted by white noise with known covariance matrices, an asymptotic reconstruction of the state is not possible. However, an optimal state observer can be designed to yield a state estimate with minimal error variance in the stationary case. The frequency-domain design of this stationary Kalman filter also leads to a design equation for $\tilde{D}(s)$ that directly follows from the corresponding Riccati equation when using the connecting relations between the time- and the frequency-domain parameterizations of observers. Given the polynomial matrices parameterizing the optimal state feedback and the Kalman filter, the resulting observer-based compensator solves the so-called linear quadratic Gaussian (LQG) control problem in the frequency domain.

The usual structure of observer-based state feedback control has the drawback that the disturbance behaviour cannot be designed independently from the reference transfer behaviour. In *Chapter 9* a two degrees of freedom structure for state feedback control is introduced. A feedforward control assures a reference behaviour of the system that coincides with the reference transfer behaviour of a controlled model of the system when no disturbances are present. Only in the presence of disturbances does an observer-based feedback controller become active to assure asymptotic tracking. As a consequence, the reference and the disturbance behaviour of the resulting closed-loop system

can be designed independently, *i.e.*, the closed-loop system has two degrees of freedom. The time-domain design of such model-matching control systems in Chapter 9 is followed by an equivalent frequency-domain formulation.

Most control applications are based on computer control. Therefore, the discrete-time results for the design of observer-based controllers with disturbance rejection are presented in *Chapter 10*. The differences between the continuous-time and the discrete-time descriptions of observer-based compensators are minor, so that the presentation in Chapter 10 can be kept short. In *Chapter 11* the time- and the frequency-domain design of optimal observer-based controllers for discrete-time systems is covered.

The presentation of the design methods is always supplemented by accompanying examples, which can easily be replicated, since all necessary parameters are provided. Thus, any result not directly shown can be obtained by the corresponding simulations.

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