

Preface

This book is an introduction to the remarkable range and variety of finite group theory for undergraduate and beginning graduate mathematicians, and all others with an interest in the subject. My original plan was to develop the theory to the point where I could present the proofs and supporting material for some of the main results in the subject. These were to include the theorems of Lagrange, Sylow, Burnside (Normal Complement), Jordan–Hölder, Hall and Schur–Zassenhaus amongst others, and to provide an introduction to character theory developed to the point where Burnside’s $p^r q^s$ -theorem could be derived and Frobenius kernels and complements could be introduced. I have come to realise that this would have resulted in a rather long book and so some material would have to go. It was at this point that modern technology came to my aid. Solutions to the problems were also to be included, but these would have taken at least 90 rather dense pages and an appendix to this book was perhaps not the best place for this material. A number of textbooks now put solutions on a web site attached to the book which is maintained jointly by the author and the publisher. Extending this idea has allowed me to fulfil my original intentions and keep the printed text to manageable proportions. So the web site now attached to this book, which can be found by going to

www.springer.com

and following the product links, includes not only the Solution Appendix but also extra sections to many of the chapters and two extra web chapters. These items are listed on the contents pages, and present work that is not basic to a chapter’s topic being either slightly more specialised or slightly more challenging. Also, perhaps unfortunately, all work on character theory and applications (Chapters 13 and 14) is now on the web. As this book goes to press, about half of this web material is written and ‘latexed’, it is hoped that the remaining half will be available when the book is published or soon after. Of course, more web items could be added later. I attended Muchio Suzuki’s graduate group theory lectures given at the University of Illinois in 1974 and 1975, and so in tribute to him and the insight he gave into modern finite group theory I have ended the extended text with a discussion of his simple groups $Sz(2^n)$ as an application of the Frobenius theory.

Prerequisites

This book begins with the definition of a group, and Appendices A and B give a brief résumé of the background material from Set Theory and Number Theory that is required. So in one sense, the book needs no prerequisites, only the ability to ‘think-straight’ and a desire to learn the subject. On the other hand, it would help if the reader had undertaken the following.

- (a) We are assuming that the reader is familiar with the material of a basic abstract algebra course, and so he or she has seen at least a few examples of groups and fields, associative and commutative operations, *et cetera*, and also has had some experience working in an abstract setting.
- (b) We are also assuming that the reader is familiar with the basics of linear algebra including facts about vector spaces, matrices and determinants, and the definitions of inner and Hermitian forms. We also use the elementary operations, similarity and rational canonical forms, and related topics. Most standard one-semester linear algebra textbooks provide more than is required.
- (c) It would also help if the reader had undertaken a first course on analysis which included the basic set operations, elementary properties of the standard number systems: integers \mathbb{Z} , rational numbers \mathbb{Q} , real numbers \mathbb{R} , and the complex numbers \mathbb{C} , and the standard set-theoretic methods summarised in Appendix A.
- (d) Lastly, some familiarity with elementary number theory would be an asset, Appendix B summarises most that is required. The Euclidean Algorithm is used widely in this book, as are the basic congruence properties.

Plan of the Book

The author of an introductory group theory text has a problem: the theory is self-contained and coherent, many topics are interconnected, and several are needed more or less from the start. On the other hand, the material in a book has perforce to be presented linearly starting at Page 1. During the planning and writing of this book, I have assumed that most readers will not read it sequentially from cover to cover, but will occasionally ‘dot-about’. Hence I have allowed some ‘forward reference’, mostly for examples.

The essential topics that the reader should ‘get to grips with’ first include the basic facts about groups and subgroups, homomorphisms and isomorphisms, direct products and solubility. Also some aspects of the theory of *actions*—conjugacy, the centraliser and the normaliser—are not far behind. Of course, as noted above, although the material has to be presented linearly, it need not be read linearly, and there are considerable advantages in presenting the basic facts of a topic—homomorphisms, for example—in one place. One consequence of this fact is that the order of the chapters has some flexibility. So Chapter 7 could be read before Chapters 5 and 6 with only a small amount of back-reference in the examples. Some group-theorists may consider it essential for students to have a good grounding in the Abelian theory before the non-Abelian theory is tackled. Similarly, Chapters 10

and 11 can be read in either order with little back-reference required. So a possible non-linear reading of the text is

Sections 2.1, 2.3, 2.4 and 4.1—the basic core of the subject, then the rest of Chapter 2, Sections 4.2, 4.3, 7.1 and 11.1 in this order,

then the following sections where the reading order might be varied

Part or all of Chapters 3 and 5, Sections 7.2, 7.3, and 9.1, and Chapters 6 and 10.

Following this the remaining printed sections or possibly some of the web sections could be tackled. In the text, I have sometimes introduced topics ‘early’ and out of their logical order, for example, isomorphisms in Chapter 2, to deal with this point. Also, as a general rule, the ‘easier’ and/or more elementary parts of a topic come near the beginning of the chapter, and so the final sections often contain more specialised and/or challenging material.

Further Reading

The reader would do not harm studying any of the books listed in the bibliography, we suggest a few concentrating on the more recent titles. For a general further development of the finite theory try:

Robinson (1982), Suzuki (1982, 1986), Aschbacher (1986), Kurzweil and Stellmacher (2004), and Isaacs (2008).

Also the three volume Huppert and Blackburn (1967, 1982a, 1982b) is very comprehensive and deals with many topics not found elsewhere. For more specialised topics, the following should be read:

Doerk and Hawkes (1992) for soluble groups,
Carter (1972), the ATLAS (1985), and Conway and Sloane (1993) for finite simple groups,
James and Liebeck (1993), Huppert (1998) and Isaacs (2006) for character theory, and
Kaplansky (1969), Fuchs (1970, 1973), and Rotman (1994) for infinite Abelian groups.

Of course, some of the older books still have much to offer, these include

Burnside (1911, reprinted 2004), Kurosh (1955), Scott (1964) and Rose (1978)—no relation!

Although 45 years old, in my opinion, Scott’s book remains one of the best introductions to the subject.

All errors and omissions that are still present in the text and/or web pages are entirely my fault, please contact me with details at

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General comments, including comments on the correctness and/or clarity of the text, or shorter, clearer or better solutions to the problems (which could be added to the web site), are also welcome.

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