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Introduction

The purpose of this book is to give insight into some exciting recent developments in the area of fractals, which are deeply influenced by the close interplay between geometry, analysis and probability, as well as by algebraic approaches as a new direction of research. In many of the topics the influence of randomness plays a major role. This reflects, in particular, natural relationships to models in statistical physics, mathematical genetics, population biology, finance and economics. For the convenience of the reader we have divided the book into five parts, corresponding to different directions of current research.

In Part 1 certain classes of fractals are treated in the more general frameworks of analysis on metric measure spaces and of self-similar algebraic groups acting on homogeneous rooted trees. The article of A. Grigor'yan, J. Hu and K.-S. Lau gives a survey on recent developments in the study of heat kernels on metric measure spaces. It explains several results obtained by the authors and their collaborators in a line of papers. In particular, it illustrates how heat kernel estimates of rather general form imply certain doubling properties of the underlying measure. Also relations to embeddings of the associated Dirichlet space into Besov type spaces and vice versa are shown. In addition a parabolic maximum principle is proved, along with some applications. Fractal sets with certain self-similarity properties serve as special examples. The latter may also be obtained as limit sets in the recently developed theory of self-similar groups. This leads, in particular, to an algebraic approach to Laplacians on such fractals. In the contribution of V. Kaimanovich a survey on new methods and results for self-similar groups is presented focusing on relationships between random walks on these structures and their amenability. Self-similar groups generated by bounded automata are discussed as a special case. The paper is completed by many references to the current literature on general self-similar groups.

Part 2 deals with a modern field in conformal dynamics. The Schramm-Loewner evolution (SLE) is a conformally invariant stochastic process consisting of a family of random planar curves. They are generated by solving Charles Loewner's differential equation with Brownian motion as input. SLE was discovered by Oded Schramm (2000) and developed by him together with Gregory Lawler and Wendelin Werner in a series of joint papers, for which they were awarded several prizes. SLE is conjectured or proved to be the scaling limit of various critical percolation models, and other stochastic processes in the plane with important applications in statistical physics. In the present book G.F. Lawler derives some new results: Using the Girsanov transformation and stochastic calculus he obtains large deviation (multifractal) estimates for a reverse flow associated with SLE. Moreover, novel fractal tools are developed to give a more accessible proof of Beffara's theorem (2008) on the Hausdorff dimension of the SLE curves.

In Part 3 some old and new relationships between fractal geometry and stochastic processes are reviewed by D. Khoshnevisan, where Lévy processes are of special interest. Connections to stochastic partial differential equations associated with the generator of such a process are also discussed. For proving geometric properties of occupation measure, range and local time Fourier transformation arguments are used as a main tool.

In Part 4 we show how recent developments in various areas of probability have led to the discovery and study of new fractal objects. The article of J. Steif surveys the model of dynamical percolation. Generically, this can be interpreted as a family of strongly coupled percolation processes indexed by a continuous time parameter. This setup allows to ask whether there exist exceptional times when the process has a property that has probability zero at any fixed time. In the cases described in this survey, the exceptional times form an interesting random fractal. The contribution of J. Blath describes a class of processes arising in mathematical genetics and population biology from the point of view of their fractal properties and, in particular, a new fractal phenomenon, the flickering of random measures, is described. G. Miermont deals with the classical probabilistic question of convergence of rescaled probabilistic objects to universal objects in the new context of random planar maps, such as random quadrangulations of a 2-sphere. The resulting limiting object, which is homeomorphic to the 2-sphere but has Hausdorff dimension 4, is again an exciting new fractal object.

Part 5 concerns (random) fractals generated by iterated function systems in an extended sense. M.F. Barnsley studies homeomorphisms between the corresponding attractors for equivalent address structures of the coding maps. In particular, the notion of fractal tops is discussed. Furthermore, generalized Minkowski metrics are considered which make affine iterated function systems hyperbolic. The paper of M. Furukado, S. Ito and H. Rao contains some new constructions and original results on the theory of Rauzy fractals based on the interplay between symbolic dynamics and domain-exchange transformations. The special class of (non-linear) Cantor sets is of traditional interest in fractal geometry and physical applications. F.M. Dekking answers in his article the question, whether the algebraic difference of two independent copies contains an interval or not, for two families of random Cantor sets. The survey is based on earlier papers by him, K. Simon and other coauthors.

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