

# Contents

<b>Preface</b>	<b>vii</b>
<b>Preface to the second edition</b>	<b>ix</b>
<b>1 Introduction to tropical geometry</b>	<b>1</b>
1.1 Images under the logarithm . . . . .	1
1.2 Families of amoebas . . . . .	4
1.3 Non-Archimedean amoebas . . . . .	5
1.4 Non-standard complex numbers . . . . .	7
1.5 The tropical semifield $\mathbb{T}$ . . . . .	9
1.6 Tropical curves and integer affine structure . . . . .	11
1.7 Exercises . . . . .	14
<b>2 Patchworking of algebraic varieties</b>	<b>17</b>
2.1 Introduction: A general idea of the patchworking construction . . .	17
2.2 Elements of toric geometry . . . . .	19
2.2.1 Construction of toric varieties . . . . .	19
2.2.2 A toric variety associated with a fan . . . . .	20
2.2.3 A toric variety associated with a convex lattice polyhedron	21
2.2.4 Embedding of $\text{Tor}(\Delta)$ into a projective space . . . . .	22
2.2.5 The real part of a toric variety and the moment map . . . .	22
2.2.6 Hypersurfaces in toric varieties . . . . .	24
2.3 Viro's patchworking method . . . . .	25
2.3.1 Chart of a real polynomial . . . . .	25
2.3.2 Patchworking of real nonsingular hypersurfaces . . . . .	27
2.3.3 Combinatorial patchworking . . . . .	30
2.3.4 A tropical point of view on the combinatorial Viro patch- working . . . . .	34
2.3.5 Patchworking of pseudo-holomorphic curves on ruled surfaces	37
2.4 Patchworking of singular algebraic hypersurfaces . . . . .	45
2.4.1 Initial data . . . . .	46
2.4.2 Transversality conditions . . . . .	46

2.4.3	The patchworking theorem . . . . .	47
2.4.4	Some $\mathcal{S}$ -transversality criteria . . . . .	50
2.5	Tropicalization and patchworking in the enumeration of nodal curves	51
2.5.1	Plane tropical curves . . . . .	52
2.5.2	Algebraic enumerative problem and its tropical analogue . .	54
2.5.3	Tropical formulas for the Gromov–Witten and Welschinger invariants . . . . .	56
2.5.4	Tropical limit . . . . .	56
2.5.5	Tropicalization of nodal curves . . . . .	57
2.5.6	Reconstruction of a simple tropical curve . . . . .	61
2.5.7	Reconstruction of the limit curve $C^{(0)}$ . . . . .	63
2.5.8	Refinement of a tropical limit . . . . .	64
2.5.9	Refinement of the condition to pass through a fixed point .	67
2.5.10	Refined patchworking theorem . . . . .	69
2.5.11	The real case: Welschinger invariants . . . . .	73
2.6	Exercises . . . . .	74
<b>3</b>	<b>Applications of tropical geometry to enumerative geometry</b>	<b>77</b>
3.1	Introduction . . . . .	77
3.2	Tropical hypersurfaces in $\mathbb{R}^n$ . . . . .	78
3.3	Geometric description of plane tropical curves . . . . .	81
3.4	Count of complex nodal curves . . . . .	83
3.5	Correspondence theorem . . . . .	84
3.6	Mikhalkin’s algorithm . . . . .	85
3.7	Welschinger invariants . . . . .	86
3.8	Welschinger invariants $W_m$ for small $m$ . . . . .	88
3.9	Tropical calculation of Welschinger invariants . . . . .	89
3.10	Asymptotic enumeration of real rational curves . . . . .	90
3.11	Recurrence formula for Welschinger invariants . . . . .	92
3.12	Welschinger invariants $W_{m,i}$ . . . . .	93
3.13	Exercises . . . . .	95
	<b>List of Figures</b>	<b>97</b>
	<b>Bibliography</b>	<b>99</b>



<http://www.springer.com/978-3-0346-0047-7>

Tropical Algebraic Geometry

Itenberg, I.; Mikhalkin, G.; Shustin, E.I.

2009, IX, 104 p., Softcover

ISBN: 978-3-0346-0047-7

A product of Birkhäuser Basel