

Superconductivity and Superfluidity (What was Done and What was Not Done)^{1*}

2.1 Introduction. Early Works

I, the author of the present paper, am 80 years old (this paper was written in 1997) and I cannot hope to obtain new, important scientific results. At the same time, I feel I need to summarize my work of over 50 years. I do not mean now my work in general (I have been engaged in solving quite a variety of physical and astrophysical problems, see [1], p. 312 and also Chap. 6 below) but my activity in the field of superconductivity and superfluidity. In general, it is not traditional to write such papers. In my opinion, however, this comes from a certain prejudice. In any case, I decided to try and write such a paper, something like a scientific autobiography, but devoted only to two related problems – superconductivity and superfluidity. I may say that it is not associated with some priority or any other claims: it is only a desire to continue my work, though in an unusual form. I leave it to the reader to judge whether this attempt has been pertinent and successful.

I began working, i.e., obtaining some results in physics in 1938–1939, when I graduated from the physics faculty of Moscow State University. Before the Second World War, i.e., until mid-1941, I was engaged in classical and quantum electrodynamics, as well as the theory of higher-spin particles. We somehow felt that war would break out and were scared of it, but were unprepared, and lived with the hope that the danger would pass. I am not going to generalize, but this atmosphere reigned in the Department of Theoretical Physics of FIAN (the P.N. Lebedev Physical Institute of the USSR Academy of Sciences). When the danger did not pass by, we began looking, while waiting for the call-up or some other changes in our lives, for an application of our abilities which might be of use for defense. I, for one, was engaged in problems of radio-wave propagation in the ionosphere (see [1, 2]). But these and similar subjects remained, at least in my case, far from finding an application in defense. Therefore, I went on working in various fields under these or other influences.

The most important such influence, not to mention the continuation of research in the field of the relativistic theory of spin particles, was exerted by L.D. Landau. In 1939, after a year's confinement in prison, Landau started working on the theory of the superfluidity of Helium II.¹ I was present, if I am not mistaken, in 1940, at Landau's talk devoted to this theory (the corresponding paper, [4], was submitted for publication in 15 May, 1941). At the end of the paper [4], he also considered superconductivity interpreted as the superfluidity of an electron liquid in a metal. I do not know whether an assertion of the kind had ever been expressed before but it is hardly probable. (Some hint, in this respect, was made in [5].) The point is that superfluidity in the proper sense of the word was discovered only in 1938 independently and simultaneously by P.L. Kapitza [5] and G.F. Allen and A.D. Misener [6].

We mean here a frictionless flow through capillaries and gaps. As to the anomalous behavior of liquid Helium (^4He) below the λ -point, i.e., at a temperature $T < T_\lambda = 2.17\text{ K}$, the study of this issue began, in effect, in 1911. Precisely in the year when superconductivity was discovered [7] (for more details, see [8, 9]; paper [7] is also included in [9] as an appendix), Kamerlingh Onnes reported a Helium density maximum at T_λ [10, 11, 127]. It was only in 1928 that the existence of two phases – Helium I and Helium II – became obvious and, in 1932, a clear λ -shaped curve for the temperature dependence of the specific heat near the λ -point was obtained. The superhigh thermal conductivity of Helium II was discovered by W. Keezom and A. Keezom (see the references in [11, 12]) in 1936 and, finally, superfluidity was revealed [5, 6] in 1938. One can thus say that it took 27 years (from 1911 to 1938) to discover superfluidity [127]. Such a long process is in obvious contrast with the discovery of superconductivity, which was practically a one-stroke occurrence [7] (for details, see [8, 9] and Chap. 6 in [2]). One can hardly doubt that the reason lies in the different methods. Superconductivity was discovered when the electrical resistance of a wire (or, more precisely, a capillary filled with mercury) was being measured. It is a much more difficult task to investigate the character of liquid flow (concretely, Helium II) through gaps and capillaries and, besides, one must hit upon the idea of carrying out such experiments.

At the same time, the origin of superfluidity remained obscure. Landau believed [4] that the responsibility rested with the spectrum of 'elementary excitations' in a liquid, while Bose statistics and Bose–Einstein condensation had nothing to do with it. F. London and L. Tisza [12], on the contrary, associated superfluidity with Bose–Einstein condensation. The validity of the latter opinion became obvious in 1949 after liquid ^3He with atoms obeying Fermi statistics, the properties differing radically from those of liquid ^4He , had been obtained. Theoretically, the same conclusion was drawn by Feynman (see [13]). But nothing could be derived from it in respect of superconductivity because

¹ As is well known, P.L. Kapitza's plea for Landau's discharge from prison was motivated by the very wish to have his assistance in the field of superfluidity theory (see [3]).

electrons obey Fermi statistics. As we know today, the solution of the problem (or rather the puzzle) lies in the fact that electrons in a superconductor form ‘pairs’ with zero spin. Such pairs can undergo Bose–Einstein condensation with which the transition to a superconducting state is associated. My fairly modest contribution to this subject consists in pointing out that, in a Bose gas of charged particles, the Meissner effect must be observed [14]. The idea of ‘pairing’ itself did not occur to me. To the best of my knowledge, R.A. Ogg was the first to suggest it in 1946 [15]. This viewpoint was supported and further developed by M.R. Schafroth [16]. However, the cause and mechanism of pairing remained absolutely vague, and it was only in 1956 that L.N. Cooper [17] pointed out a concrete mechanism of pairing in a Fermi gas with attracting particles. This was the basis on which J. Bardeen, L.N. Cooper, and J.R. Schrieffer (BCS) finally formulated the first consistent, though model-type microtheory of superconductivity [18] in 1957. It is curious that [18] contains no indications of Bose–Einstein condensation, while it is, in fact, the crucial point.

However, I am running many years ahead as far as my own work is concerned. Concretely, in 1943, I tried [19], on the basis of the Landau theory [4] of superfluidity, to construct a quasi-microscopic theory of superconductivity [19]. The paper postulated a spectrum of electrons (charged ‘excitations’) in a metal with a gap Δ . For such a spectrum, superconductivity (the superfluidity of a charged liquid) must be observed. The introduction of a gap provided the critical field with a dependence on temperature and penetration depth into a superconductor, which approximately corresponded to the actual one. A comparison between the theory and the experiment gave the value $\Delta/k_B T_c = 3.1$. As is well known, in BCS theory $2\Delta_0/k_B T_c = 3.52$ but the most important point is that $\Delta_0 \equiv \Delta(0)$ is the value of the gap at $T = 0$ and, with increasing temperature, the gap decreases to yield $\Delta(T_c) = 0$. In my paper, the gap Δ was assumed to be constant and a satisfactory agreement with the experiment is possibly explained by the inaccuracy of the experimental data employed. I do not think that a more detailed analysis of this question is pertinent because model [19] is of no more than historical value now. Nonetheless, [19] did have some ideas that could have been of interest; for example, the occurrence of resonance phenomena for incident radiation at a frequency $\nu = \Delta/h$ was mentioned. In any case, the fact is that in his well-known review [20], published in 1956, Bardeen covered the results of paper [19] rather extensively. Notice that paper [19] also presented a survey of the macrotheory of superconductivity. It was followed by [21] considering gyromagnetic and electron inertia experiments with superconductors. Finally, in the same year, 1944, paper [22], devoted to thermoelectric phenomena in superconductors, was published.² This latter paper remains topical even now and we shall return to it in Sect. 2.5. The previously mentioned papers [19, 21, 22] were in-

² It should be noted that all three papers [19, 21, 22] were submitted for publication on the same date (23 November, 1943). I do not remember why this happened.

cluded in the monograph *Superconductivity* [24] written in 1944. Before taking up superconductivity, I analyzed [23], on the basis of the Landau theory, the problem of light scattering in Helium II. In what follows, I shall consider this and some other papers devoted to superfluidity (see Sect. 2.6).

2.2 The Ψ -Theory of Superconductivity (The Ginzburg–Landau Theory)

Within the first two decades after the discovery of superconductivity, its study went rather slowly compared to today’s standards. This does not seem strange if we remember that liquid helium, which was first obtained in Leiden in 1908, became available elsewhere only after 15 years, i.e., in 1923. Without plunging into the history (see [8, 9, 11]; see also Chap. 6 in [2]), I shall restrict myself to the remark that the Meissner effect was only discovered [25] in 1933, i.e., 22 years after the discovery of superconductivity. Only after that did it become clear that a metal in normal and superconducting states can be treated as two phases of a substance in the thermodynamic sense of this notion. As a result, in 1934, there appeared [20, 26] the so-called two-fluid approach to superconductors and also the relation:

$$F_{n0}(T) - F_{s0}(T) = \frac{H_{cm}^2(T)}{8\pi}, \quad (2.1)$$

where F_{n0} and F_{s0} are free-energy densities (in the absence of a field) in the normal and superconducting phase, respectively, and H_{cm} is the critical magnetic field destroying superconductivity. A differentiation of expression (2.1) with respect to T leads to expressions for the differences of entropy and specific heat.

According to the two-fluid picture, the total electric current density in a superconductor is

$$\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n, \quad (2.2)$$

where \mathbf{j}_s and \mathbf{j}_n are the densities of the superconducting and normal current.

The normal current in a superconductor does not, in fact, differ from the current in a normal metal and, in the local approximation, we have

$$\mathbf{j}_n = \sigma_n(T) \mathbf{E}, \quad (2.3)$$

where \mathbf{E} is the electric field strength and σ_n is conductivity of the ‘normal part’ of the electron liquid; for simplicity, we henceforth take $\mathbf{j}_n = 0$, unless otherwise specified.

In 1935, F. London and H. London proposed [27] for \mathbf{j}_s the equations (now referred to as Londons’ equations):

Most probably it was connected with some special conditions pertaining to the war.

$$\text{rot}(\Lambda \mathbf{j}_s) = -\frac{1}{c} \mathbf{H} \quad (2.4)$$

$$\frac{\partial(\Lambda \mathbf{j}_s)}{\partial t} = \mathbf{E}, \quad (2.5)$$

where Λ is a constant and the magnetic field strength \mathbf{H} here and later does not differ from the magnetic induction \mathbf{B} .

We arrive at such equations, for example, proceeding from the hydrodynamic equations for a conducting ‘liquid’ which consists of particles with charge e , mass m , and velocity $\mathbf{v}_s(\mathbf{r}, t)$:

$$\begin{aligned} \frac{\partial \mathbf{v}_s}{\partial t} &= -(\mathbf{v}_s \nabla) \mathbf{v}_s + \frac{e}{m} \mathbf{E} + \frac{e}{mc} [\mathbf{v}_s \mathbf{H}] \\ &= \frac{e}{m} \mathbf{E} - \nabla \frac{\mathbf{v}_s^2}{2} + \left[\mathbf{v}_s \left(\text{rot} \mathbf{v}_s + \frac{e}{mc} \mathbf{H} \right) \right]. \end{aligned} \quad (2.6)$$

Such an equation corresponds to infinite (ideal) conductivity [28] and is not an obstruction to the presence of a constant magnetic field in a superconductor, which contradicts the existence of the Meissner effect. Therefore, the Londons imposed, so to say, an additional condition $\text{rot} \mathbf{v}_s + e\mathbf{H}/mc = 0$, interpreted as the condition of a vortex-free motion for a charged liquid. If \mathbf{j}_s is written in the form $\mathbf{j}_s = en_s \mathbf{v}_s$, where n_s is the charge concentration, the additional condition for $n_s = \text{const}$ assumes precisely form (2.4) and

$$\Lambda = \frac{m}{e^2 n_s}. \quad (2.7)$$

Equation (2.6) transforms to (2.5) up to a small term proportional to $\nabla \mathbf{v}_s^2$ (see Sect. 2.5). Within such an approach, the principal Londons’ equation (2.4) is, of course, merely postulated. This condition is an effect of quantum nature and follows from the Ψ -theory of superconductivity [29] considered later and from the microtheory of superconductivity [18, 30] which, in turn, transforms near T_c to Ψ -theory [31].

Londons’ equation (2.4), along with the Maxwell equation

$$\text{rot} \mathbf{H} = \frac{4\pi}{c} \mathbf{j}_s \quad (2.8)$$

at $\Lambda = \text{const}$ (we are obviously dealing with the quasi-stationary case), leads to the equations:

$$\Delta \mathbf{H} - \frac{1}{\delta^2} \mathbf{H} = 0, \quad \Delta \mathbf{j}_s - \frac{1}{\delta^2} \mathbf{j}_s = 0 \quad (2.9)$$

$$\delta^2 = \frac{\Lambda c^2}{4\pi} = \frac{mc^2}{4\pi e^2 n_s}. \quad (2.10)$$

Equation (2.9) implies that the magnetic field \mathbf{H} and the current density \mathbf{j}_s exponentially decay through the superconductor depth (for example, in the

field parallel to and near a flat boundary, we have $H = H_0 \exp(-z/\delta)$, where z is the distance from the boundary), i.e., the Meissner effect arises. The Londons' equations still hold true but only in the case of a weak field:

$$H \ll H_c, \quad (2.11)$$

where H_c is the critical magnetic field destroying superconductivity (in the case of non-local coupling between the current and the field, Londons' equations do not hold either [20, 30] but we do not consider such cases here). We mean here type I superconductors. For type II superconductors, the Londons' theory has a wider limit of applicability, including the vortex phase for $H \ll H_{c2}$ at any temperature. But if the field is strong, i.e., comparable with H_c , Londons' theory may be invalid or otherwise insufficient. So, from Londons' theory, it follows that the critical magnetic field H_c , in which the superconductivity of a flat film of thickness $2d$ is destroyed (in the field parallel to it), is

$$H_c = \left(1 - \frac{\delta}{d} \operatorname{th} \frac{d}{\delta}\right)^{-1/2} H_{\text{cm}},$$

where H_{cm} is the critical field for a massive specimen (see [24, 32, 33] and references therein). This expression for H_c , however, contradicts experimental data. The situation can be improved by introducing different surface tensions σ_n and σ_s on the boundaries of the normal and the superconducting phases with a vacuum [32]. It turns out, however, that

$$\frac{\sigma_n - \sigma_s}{H_{\text{cm}}^2/8\pi} \sim \delta \sim 10^{-5} \text{ (cm)}.$$

σ is a conductivity in (2.3) but I hope this will not lead to any misunderstanding.

At the same time, it might be expected that $(\sigma_n - \sigma_s) \sim (10^{-7}-10^{-8}) H_{\text{cm}}^2/8\pi$, i.e., is of the order of the volume energy $H_{\text{cm}}^2/8\pi$ multiplied by an atomic scale length. Moreover, in the theory based on Londons' equations, on the boundary between the normal and superconducting phases, the surface tension (surface energy) connected with the field and the current is $\sigma_{\text{ns}}^{(0)} = -\delta H_{\text{cm}}^2/8\pi$. Consequently, to obtain a positive surface tension $\sigma_{\text{ns}} = \sigma_{\text{ns}}^{(0)} + \sigma_{\text{ns}}^{(j)}$ observed for a stable boundary, it is necessary to introduce a certain surface energy $\sigma_{\text{ns}}^{(j)} > \delta H_{\text{cm}}^2/8\pi$ of non-magnetic origin. The introduction of such a comparatively high energy is totally ungrounded. On the contrary, one can think that a rational theory of superconductivity must automatically lead to the possibility of expressing the energy σ_{ns} in terms of parameters characterizing the superconductor.

Such a theory that generalized the Londons' theory, eliminated the indicated difficulties, and suggested some new conclusions, was the Ψ -theory [29]

formulated in 1950.³ In the same year, I wrote a review [33] devoted to the macro-theory of superconductivity, including the Ψ -theory.

In the absence of a magnetic field, the superconducting transition is a second-order transition. The general theory of such transitions always includes [34] a certain parameter (the order parameter) η which, when in equilibrium, differs from zero in the ordered phase and equals zero in the disordered phase. For example, in the case of ferroelectrics, the role of η is played by the spontaneous electric polarization \mathbf{P}_s and, in the case of magnetics, by the spontaneous magnetization \mathbf{M}_s (not long before the appearance of our paper [29]; both these cases were discussed in the review [35]). In superconductors, where the ordered phase is superconducting, for the order parameter we chose a complex function Ψ which plays the role of an ‘effective wavefunction of superconducting electrons’. This function can be so normalized that $|\Psi|^2$ is the concentration n_s of ‘superconducting electrons’.

The free energy density of a superconductor and the field was written in the form:

$$F_{sH} = F_{s0} + \frac{H^2}{8\pi} + \frac{1}{2m} \left| -i\hbar\nabla\Psi - \frac{e}{c}\mathbf{A}\Psi \right|^2, \\ F_{s0} = F_{n0} + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4, \quad (2.12)$$

where \mathbf{A} is the vector potential of the field $\mathbf{H} = \text{rot } \mathbf{A}$. Without the field, in the state of thermodynamic equilibrium $\partial F_{s0}/\partial|\Psi|^2 = 0$, $\partial^2 F_{s0}/\partial^2|\Psi|^2 > 0$ and we must have $|\Psi|^2 = 0$ for $T > T_c$ and $|\Psi|^2 > 0$ for $T < T_c$. This implies that $\alpha_c \equiv \alpha(T_c) = 0$ and $\beta_c \equiv \beta(T_c) > 0$, and $\alpha < 0$ for $T < T_c$. Within the validity limits of expansion (2.12) in $|\Psi|^2$, one can put $\alpha = \alpha'_c(T - T_c)$ and $\beta(T) = \beta_{T_c} \equiv \beta_c$. From this, at $T < T_c$ [see also (2.1)], we have:

$$|\Psi|^2 \equiv |\Psi_\infty|^2 = -\frac{\alpha}{\beta} = \frac{\alpha'_c(T_c - T)}{\beta_c}, \\ F_{s0} = F_{n0} - \frac{\alpha^2}{2\beta} = F_{n0} - \frac{(\alpha'_c)^2(T_c - T)^2}{2\beta_c} = F_{n0} - \frac{H_{cm}^2}{8\pi}. \quad (2.13)$$

In the presence of the field, the equation for Ψ is derived upon varying the free energy $\int F_{sH} dV$ with respect to Ψ^* and, obviously, has the form:

$$\frac{1}{2m} \left(-i\hbar\nabla - \frac{e}{c}\mathbf{A} \right)^2 \Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0. \quad (2.14)$$

³ This theory is usually called the Ginzburg–Landau theory. I try, however, to avoid this term, not out of false modesty but rather because in such cases the use of one’s own name is not conventional in Russian. Furthermore, in its application to superfluidity (not superconductivity) the Ψ -theory was developed not with L.D. Landau but with L.P. Pitaevskii and A.A. Sobyenin (see Sect. 2.4). The article [29] is included in this book.

If, on the superconductor boundary, the variation $\delta\Psi^*$ is arbitrary, i.e., no additional condition is imposed on Ψ and no additional term corresponding to the surface energy is introduced in (2.12), then the condition of minimal free energy is the so-called natural boundary condition on the superconductor boundary:

$$\mathbf{n} \left(-i\hbar\nabla\Psi - \frac{e}{c} \mathbf{A}\Psi \right) = 0, \quad (2.15)$$

where \mathbf{n} is the normal to the boundary (for more details, see [29] and Sect. 2.3). Condition (2.15) refers to the case of a boundary between a superconductor and a vacuum or a dielectric. As regards the equation for \mathbf{A} , under the condition $\text{div } \mathbf{A} = 0$, and after variation of the integral $\int F_{sH} dV$ over \mathbf{A} , it becomes

$$\Delta\mathbf{A} = -\frac{4\pi}{c} \mathbf{j}_s, \quad \mathbf{j}_s = -\frac{ie\hbar}{2m} (\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) - \frac{e^2}{mc} |\Psi|^2 \mathbf{A}. \quad (2.16)$$

Here, of course, we assume that $\mathbf{j}_n = 0$, i.e., the total current is superconducting. An expression similar to (2.14) is, of course, also obtained for Ψ^* and, as expected, we have $\mathbf{j}_s \mathbf{n} = 0$ on the boundary [see (2.15)]. The solution of the problem of the distribution of the field, current, and function Ψ in a superconductor is reduced to the integration of the system of (2.14) and (2.16). An expression similar to (2.14) is, of course, also obtained for Ψ^* and, as expected, we have $\mathbf{j}_s \mathbf{n} = 0$ on the boundary [see (2.15)]. The solution of the problem of the distribution of the field, current, and function Ψ in a superconductor is reduced to the integration of the system of (2.14) and (2.16). Assuming $\Psi = \Psi_\infty = \text{const}$, the superconducting current density is $\mathbf{j}_s = -e^2 |\Psi_\infty|^2 \mathbf{A} / mc = -e^2 n_s \mathbf{A} / mc$ (with normalization $|\Psi_\infty|^2 = n_s$). Applying the operation rot to this expression, we obtain London's equation (2.4) [see also (2.7)]. Thus, the Ψ -theory generalizes the London's theory and passes over into it in the limiting case $\Psi = \Psi_\infty = \text{const}$.

Paper [29] is rather long (19 pages) and solves several problems to which we shall return in what follows. After that, I myself, sometimes with co-authors, devoted a number of papers to the development of the Ψ -theory of superconductivity. These papers are mentioned later. Moreover, this theory was further promoted and accounted for in a lot of papers and books (see, for example, [20, 30, 33, 36–41, 229, 236–239]). I do not follow the corresponding literature now, the more so as (2.14) and its extensions are widely used outside superconductivity or only in applications to superconductors (see, for example, [42–44]). This equation is also being investigated by mathematicians whose works are incomprehensible to me (see, for example, [45]). The relativistic generalization of the equations of the Ψ -theory and some of the concepts associated with this theory also enjoy wide applications in quantum field theory (for example, spontaneous symmetry breaking, etc; see [46]).⁴ In

⁴ To confirm this, I would cite paper [46] (see p. 184; p. 480 in the English translation): ‘It is easy to see that the Higgs model is fully analogical to the Ginzburg–

such a situation, it seems absolutely impossible to elucidate here the present-day state of the Ψ -theory or even focus in detail on the original paper [29] and my subsequent papers.

However, what I think is necessary is to tell the story of the appearance of paper [29] and to speak about the role of Landau and myself. Nobody else can do this because regrettably Lev Davidovich Landau passed away long ago (he stopped working in 1962 and died in 1968). At the same time, this is, of course, a very delicate question. That is why, when 20–25 years ago I was approached by the bibliographical magazine *Current Contents* with a request to elucidate the history of the appearance of [29], I refused. My refusal was motivated by the fact that my story might be interpreted as an attempt to exaggerate my role. And, in general, I had no desire to prove that I was indeed a full co-author and not a student or a postgraduate to whom Landau ‘had set a task’, whilst actually doing everything himself. Without such a premise it is difficult to explain why our paper has been frequently cited as Landau and Ginzburg, although it is known to have Ginzburg and Landau in the title. Of course, I have never made protestations concerning this point and, in general, consider it to be a trifle, but still I believe that such a citation with a wrong order of authors is incorrect. It would certainly still be incorrect even if my role had indeed been a secondary one. But I did not think so, and neither did Landau, and this fact was well known to his circle, and generally in the USSR. As to foreigners, they really did not know much about scientific research in the USSR at that time, for in 1950 the Cold War was at its height. As far back as 1947, the *USSR Journal of Physics*, which was a good journal, stopped being published and [29] appeared only in Russian. We could not go abroad at that time. Perhaps we sent a reprint to D. Shoenberg or he himself came across this article in *Zh. Eksp. Teor. Fiz* (JETP). In any event, Shoenberg translated the paper into English on his own initiative, and distributed it among some people and it then became available at least to some colleagues. Landau’s name played, of course, a positive role and stimulated a lively interest in the paper.

One way or another, I decided to dwell on the history of the appearance of the work [29] because the present paper would be incomplete if I did not.

I regard the already mentioned paper [32] as being accomplished as far back as 1944 (it was submitted for publication on 21 December, 1944), as initial. From [32], it is quite clear that the London theory is invalid for the description of the behavior of superconductors in strong enough fields and, in particular, for the calculation of the critical field in the case of films. The introduction of the surface energies σ_n and σ_s was an artificial technique, and these quantities were absurdly large new constants whose values were not predicted by the the-

Landau theory and is its relativistic generalization. It turned out that this conclusion bears an important heuristic value by allowing to establish direct analogs between superconductivity theory and theories of elementary particles, including the Higgs model.’ (See also [265].)

ory. The same applies to the surface energy σ_{ns} on the boundary between the normal and superconducting phases. It was also absolutely unclear how the critical current should be calculated in the case of small-sized superconductors. Therefore, it was necessary somehow to generalize the Londons' theory to overcome its limits. Unfortunately, advancement in this direction was slow. One of the possible explanations is that, like many theoretical physicists of my generation and the previous, I was simultaneously engaged in the solution of various problems and did not concentrate on anything definite (it can be seen, for instance, from the bibliographical index [47]). But there was gradual progress. So, on the basis of the conception of the Landau theory [4], I came to the conclusion [48] that electromagnetic processes in superconductors must be nonlinear and, incidentally, suggested a possible experiment for revealing such nonlinearity. The main point is that, in note [48], I made the following remark: 'The indication of a possible inadequacy of the classical description of superconducting currents consists in the fact that the zero energy of excitation in a superconductor is equal in order of magnitude to $\hbar^2 n / m \delta \sim 1 \text{ erg cm}^{-2}$ (for $\delta \sim 10^{-5} \text{ cm}$ and $n \sim 10^{22} \text{ cm}^{-3}$) and is thus higher than the magnetic energy $\delta H^2 / 8\pi \sim 0.1 \text{ erg cm}^{-2}$ (for $H \sim 500 \text{ Oe}$.)' The feeling that the theory of superconductivity should take into account quantum effects was also reflected in [49], devoted for the most part to critical velocity in Helium II. At the same time, in that paper, I also tried to apply the theory of second-order phase transitions to the λ -transition in liquid helium.

It seems surprising, and unfortunately, it did not occur to me at that time to ask why Landau, the author of the theory of phase transitions [34] and the theory of superfluidity [4], had never posed the question of the order parameter η for liquid helium. In [49], I chose as such a parameter ρ_s , i.e., the density of the superfluid phase of Helium II. However, this choice raises doubts because the expansion of the free energy (thermodynamic potential) begins with the term $\alpha \rho_s$, whereas, in the general theory, the first term of the expansion has the form $\alpha \eta^2$. Hence, $\sqrt{\rho_s}$ is a more pertinent choice as the order parameter. But $\sqrt{\rho_s}$ is proportional to a certain wavefunction Ψ , so far as it is precisely the quantity $|\Psi|^2$ that is proportional to the particle concentration. Unfortunately, I do not remember exactly whether or not it was these arguments alone that prompted me to introduce the order parameter $\eta = \Psi$ and nothing is said about it in [49]. More important for me was the desire to explain the surface tension σ_{ns} by the gradient term $|\nabla \Psi|^2$. In quantum mechanics, this term has the form of kinetic energy $\hbar^2 |\nabla \Psi|^2 / 2m$. It was precisely this idea that I suggested to Landau, probably in late 1949 (paper [29] was submitted on 20 April, 1950 but it had taken a great deal of time to prepare it). I was on good terms with Landau; I attended his seminars and often asked his advice on various problems. Landau supported my idea of introducing the 'effective wavefunction Ψ of superconducting electrons' as the order parameter, and so we were immediately led to the free energy (2.12). The thing I do not remember exactly (and certainly do not want to contrive) is whether I came to him with the ready expression:

$$\frac{1}{2m} \left| -i\hbar\nabla\Psi - \frac{e}{c} \mathbf{A}\Psi \right|^2$$

or with an expression without the vector potential. The introduction of the latter is obvious by analogy with quantum mechanics, but perhaps this was made only during a conversation with Landau. I feel I should present my apologies to the reader for such reservations and uncertainty but since that time nearly 50 years have passed (!), no notes have remained, and I never thought that I would have to recall those remote days.

After the basic equations (2.12), (2.14), and (2.16) of the Ψ -theory were derived, one had to solve various problems on their basis and compare the theory with experiments. Naturally, it was myself who was mostly concerned with this but I regularly met with Landau to discuss the results. What has been said may produce the impression that my role in the creation of the Ψ -theory was even greater than that of Landau. But this is not so. One should not forget that the fundamental basis was the theory [34,50] of second-order phase transitions developed by Landau in 1937 which I had employed in a number of cases [35,49] and applied to the theory of superconductivity in paper [29]. Moreover, I find it necessary to note that the important remark made in [29] concerning the meaning of the Ψ -function used as an order parameter was due to Landau himself. I shall cite the relevant passage from [29]:

“Our function $\Psi(\mathbf{r})$ may be thought of as immediately related to the density matrix $\rho(\mathbf{r}, \mathbf{r}') = \int \Psi^*(\mathbf{r}, \mathbf{r}'_i) \Psi(\mathbf{r}', \mathbf{r}'_i) d\mathbf{r}'_i$, where $\Psi(\mathbf{r}, \mathbf{r}'_i)$ is the true Ψ -function of electrons in a metal which depends on the coordinates of all the electrons \mathbf{r}_i ($i = 1, 2, \dots, N$) and \mathbf{r}'_i are the coordinates of all the electrons except a distinguished one (its coordinates are \mathbf{r} and at another point \mathbf{r}'). One may think that for a non-superconducting body, where the long range order is absent, as $|\mathbf{r} - \mathbf{r}'| \rightarrow \infty$ we have $\rho \rightarrow 0$, while in the superconducting state $\rho(|\mathbf{r} - \mathbf{r}'| \rightarrow \infty) \rightarrow \rho_0 \neq 0$. In this case it is natural to assume the density matrix to be related to the introduced Ψ -function as $\rho(\mathbf{r}, \mathbf{r}') = \Psi^*(\mathbf{r})\Psi(\mathbf{r}')$.”

Accordingly, the superconducting (or superfluid) phase is characterized by a certain long-range order which is absent in ordinary liquids (see also [30], Sect. 26; [51,52], [53], Sect. 9.7.). This result is usually ascribed to C.N. Yang [51] and is referred to as off diagonal long-range order (ODLRO) [53]. However, as we can see, Landau realized the possibility of the existence of this long-range order 12 years before Yang. I mentioned this fact in [54].

In (2.12) and subsequent expressions, the coefficients e and m appear. These designations were, of course, chosen by analogy with the quantum-mechanical expression for the Hamiltonian of a particle with charge e and mass m . Our Ψ -function is, however, not the wavefunction of electrons. The coefficient m can be taken arbitrarily [29] because the Ψ -function is not an observed quantity: an observed quantity is the penetration depth δ_0 of a weak magnetic field (see (2.12), (2.13), and (2.16)):

$$\delta_0^2 = \frac{mc^2\beta_c}{4\pi e^2|\alpha|} = \frac{mc^2}{4\pi e^2|\Psi_\infty|^2}. \quad (2.17)$$

Since the Ψ -theory in a weak field (2.11) transforms to the London theory (though a number of problems cannot be stated in the London theory even in this case), the penetration depth δ_0 is frequently called the London penetration depth and is denoted by δ_L or λ_L . If we assume [29] e and m to correspond to a free electron ($e_0 = 4.8 \times 10^{-10}$ CGS, $m_0 = 9.1 \times 10^{-28}$ g), then $|\Psi_\infty|^2 = n_s$, where n_s is the ‘superconducting electron’ concentration thus defined. In fact, one can choose any arbitrary value of m [29, 37] which will only affect the normalization of the observed quantity $|\Psi_\infty|^2$. In the literature, $m = 2m_0$ occasionally occurs, which corresponds to the mass of a ‘pair’ of two electrons. As to the charge e in (2.12) and subsequent expressions, it is an observed quantity (see later). It seemed to me from the very beginning that one should regard the charge e in (2.12) as a certain ‘effective charge’ e_{eff} and take it as a free parameter. But Landau objected and, in paper [29], it is stated as a compromise that ‘there is no reason to assume the charge e to be other than the electron charge’. Running ahead, I shall note that I still went on thinking of the question of the role of the charge $e \equiv e_{\text{eff}}$ as open and pointed out the possibility of clarifying the situation by comparing the theory with the experiment (see [14], p. 107). The point is that the essential parameter involved in the Ψ -theory is the quantity:

$$\varkappa = \frac{mc}{e\hbar} \sqrt{\frac{\beta_c}{2\pi}} = \frac{\sqrt{2}e}{\hbar c} H_{\text{cm}} \delta_0^2. \quad (2.18)$$

In [29], we set $e = e_0$ and could, therefore, determine \varkappa from experimental data on H_{cm} and δ_0 . At the same time, the parameter \varkappa enters the expressions for the surface energy σ_{ns} , for the penetration depth in a strong field ($H \gtrsim H_{\text{cm}}$) and the expressions for superheating and supercooling limits. Using the approximate data of measurements available at the time, I came to the conclusion [55] (this paper was submitted for publication on 12 August, 1954) that the charge $e \equiv e_{\text{eff}}$ in (2.18) is two to three times greater than e_0 . When I discussed this result with Landau, he put forward a serious objection to the possibility of introducing an effective charge (he had apparently had this argument in mind before, when we discussed paper [29] but did not then advance it). Specifically, the effective charge might depend on the composition of a substance, its temperature and pressure, and, therefore, might appear to be a function of coordinates. But, in that case, the gradient invariance of the theory would be broken, which is inadmissible. I could not find arguments against this remark and, with the consent of Landau, I included it in paper [55]. The explanation seems now to be quite simple. No, an effective charge e_{eff} , which might appear to be coordinate-dependent, should not have been introduced. But it might well be supposed that, say, $e_{\text{eff}} = 2e_0$. And this was exactly the case, but it became obvious only after the creation of BCS theory [18] in 1957, and after the appearance of the paper by L.P. Gorkov [31] who showed that the Ψ -theory near T_c follows from the BCS theory. More precisely, the Ψ -theory near T_c is certainly wider than the BCS theory in the sense that it is independent of some particular assumptions used in the BCS

theory. But this is a different subject. The formation of pairs with charge $2e_0$ is a very general phenomenon, too. I have already emphasized that the idea of pairing and, what is important, the realistic character of such pairing, was far from trivial.

So, in the Ψ -theory, we have $e = 2e_0$ and, consequently [see (2.18)]:

$$\varkappa = \frac{2\sqrt{2}e_0}{\hbar c} H_{\text{cm}} \delta_0^2. \quad (2.19)$$

As can be seen from the calculations, the surface tension σ_{ns} is positive only for $\varkappa < 1/\sqrt{2}$. An analytical calculation of σ_{ns} encounters difficulties. In paper [29], this was only done for a sufficiently small \varkappa :

$$\sigma_{\text{ns}} = \frac{\delta_0 H_{\text{cm}}^2}{\sqrt{2} \cdot 3\pi \varkappa}, \quad \Delta = \frac{\sigma_{\text{ns}}}{H_{\text{cm}}^2/8\pi} = \frac{1.89\delta_0}{\varkappa}, \quad \sqrt{\varkappa} \ll 1. \quad (2.20)$$

From this, it is already seen that the Ψ -theory leads to σ_{ns} values of the required order of magnitude. It is only in paper [56] that the energy σ_{ns} is calculated analytically up to the terms of the order of $\varkappa\sqrt{\varkappa}$. The result is as follows [the value $\Gamma = 2\sqrt{2}/3$ corresponds to expression (2.20)]:

$$\sigma_{\text{ns}} = \frac{\delta_0 H_{\text{cm}}^2}{4\pi \varkappa} \Gamma, \quad \Gamma = \frac{2\sqrt{2}}{3} - 1.02817\sqrt{\varkappa} - 0.13307\varkappa\sqrt{\varkappa} + \dots \quad (2.21)$$

As \varkappa increases, the energy σ_{ns} decreases and, in [29], it was pointed out that, according to numerical integration:

$$\sigma_{\text{ns}} = 0, \quad \varkappa = \frac{1}{\sqrt{2}}. \quad (2.22)$$

But it was also shown that for $\varkappa > 1/\sqrt{2}$, there occurs some specific instability of the normal phase, namely, nuclei of the superconducting phase are formed in it. Concretely, this instability arises in the field:

$$H_{c2} = \sqrt{2} \varkappa H_{\text{cm}}. \quad (2.23)$$

(It should be noted that (2.23) is present in [29] in an implicit form, and it was written explicitly in [57].) In the case $\varkappa < 1/\sqrt{2}$, the field H_{c2} corresponds to the limit of a possible supercooling of the normal phase (for $H < H_{c2}$, this phase becomes metastable; see also [57], where, as in some of my other papers, the field H_{c2} is denoted by H_{k1}). When $\varkappa > 1/\sqrt{2}$, it is clear from (2.23) that superconductivity is preserved in some form in the field $H > H_{\text{cm}}$ too and vanishes only in the field H_{c2} . Generally, it is just for $\varkappa = 1/\sqrt{2}$ that the change in the behavior of a superconductor becomes pronounced. Hence, there were no doubts in the validity of the result (2.22). Analytically this is proved, for example, in [30,37,38]. It turns out that for pure, superconducting metals we typically have $\varkappa < 1/\sqrt{2}$ or even $\varkappa \ll 1/\sqrt{2}$ (for instance, according

to [30], κ is equal to 0.01 for Al, 0.13 for Sn, 0.16 for Hg, and 0.23 for Pb). Such superconductors are called type I superconductors. If $\kappa > 1/\sqrt{2}$, the surface tension σ_{ns} is negative and we then deal with type II superconductors (for the most part alloys) whose behavior was first investigated thoroughly in experimental studies by L.V. Shubnikov⁵ and co-authors as far back as 1935–36 (for references and explanations see [24, 58]). In [29], we considered only type I superconductors, and we read such a phrase there: ‘For sufficiently large κ , on the contrary, $\sigma_{\text{ns}} < 0$, which is indicative of the fact that such large κ do not correspond to the typically observed picture’. So we, in fact, overlooked the possibility of the existence of type II superconductors. Neither was I engaged in the study of type II superconductors later on. In this respect, I only made a remark in [57]. The theory of the behavior of type II superconductors based on the Ψ -theory was constructed in 1957 by A.A. Abrikosov [59] (see also [30, 41]). As indicated in [59] and [30], p. 191, Landau was the first to suggest that in alloys $\kappa > 1/\sqrt{2}$.

Allowing for (2.13) and (2.17), one can write:

$$H_{\text{cm}} = \left(\frac{4\pi(\alpha'_c)^2}{\beta_c} \right)^{1/2} (T_c - T), \quad \delta_0 = \left(\frac{m_0 c^2 \beta_c}{16\pi e_0^2 \alpha'_c} \right)^{1/2} (T_c - T)^{-1/2}. \quad (2.24)$$

These expressions, the same as the whole Ψ -theory are, strictly speaking, valid only in the vicinity of T_c , i.e., the condition $(T_c - T) \ll T_c$ is needed. However, the condition of applicability of the theory for small κ is, in fact, more rigorous because to satisfy the local approximation, the penetration depth δ_0 must significantly exceed the size ξ_0 of the Cooper pair (the corresponding condition written in [30], Sect. 45 has the form $(T_c - T) \ll \kappa^2 T_c$ but in [29] this, of course, could not yet be discussed). Along with the penetration depth δ_0 , the Ψ -theory involves one more parameter which has the dimension of length – the so-called coherence length or the correlation radius (length):

$$\xi = \frac{\hbar}{\sqrt{2m_0|\alpha|}} = \frac{\hbar}{\sqrt{2m_0\alpha'_c(T_c - T)}} = \frac{\hbar\tau^{-1/2}}{\sqrt{2m_0\alpha'_c T_c}} = \xi(0)\tau^{-1/2}, \quad (2.25)$$

where $\tau = (T_c - T)/T_c$ and $\xi(0) = \hbar/\sqrt{2m_0\alpha'_c T_c}$ is a conditional correlation radius for $T = 0$ (we call it conditional because the Ψ -theory is, strictly speaking, applicable only in the vicinity of T_c). To compare the formulae written here with those of [30], one should bear in mind that in our expression (2.12) in [30] $m = 2m_0$ and, of course, $e = 2e_0$.

As is readily seen [see (2.18), (2.19), and (2.24)]:

$$\kappa = \frac{m_0 c}{2e_0 \hbar} \sqrt{\frac{\beta_c}{2\pi}} = \frac{\delta_0(T)}{\xi(T)}. \quad (2.26)$$

⁵ L.V. Shubnikov, a prominent experimental physicist, was guiltlessly executed in 1937.

In addition to these mentioned problems, some more points were considered in [29], namely the field in a superconducting half-space and critical fields for plates (films) in the case where superconductivity is destroyed by the field and current. The penetration depth of the field in a superconducting half-space adjoining a vacuum has the form:

$$\delta = \delta_0 \left[1 + f(\kappa) \left(\frac{H_0}{H_{\text{cm}}} \right)^2 \right], \quad f(\kappa) = \frac{\kappa(\kappa + 2\sqrt{2})}{8(\kappa + \sqrt{2})^2} \quad (2.27)$$

where H_0 is the external field (the field for $z = 0$) and, by definition, $\delta = \int_0^\infty H(z) dz / H_0$. The nonlinearity of the electrodynamics of superconductors, which was assumed already in [48] and is reflected in the dependence of δ on H_0 , is fairly small. So, even for $\kappa = 1/\sqrt{2}$ and $H_0 = H_{\text{cm}}$, the depth is $\delta = 1.07\delta_0$. In 1950, there were no accurate enough experimental measurements of $\delta(H)$. I am not sure that they have yet been carried out, though it is probable.

Now I should make or, rather, repeat one general remark. I was never long engaged in studying only superconductivity but researched various fields (see [1], p. 309 and [47] and Chap. 6 in [2]). As to the macroscopic theory of superconductivity (the Ψ -theory and its development), it was generally beyond the scope of my interest from a certain time (see Sect. 2.3). As a result, I am ignorant of the current state of the problem as a whole. Unfortunately, neither am I aware of the existence of a monograph compiling all the material (I am afraid there is no such book). Moreover, I forgot much of what I had done myself and now recollect the old facts, sometimes with surprise, when reading my own papers. That is why I cannot be convinced that my old calculations were unerring; I do not know the subsequent calculations and the results of their comparison with experiment. However, the present paper does not even claim to make a current review; it is only an attempt to elucidate some problems of the history of studies of superconductivity and superfluidity in an autobiographical context. Those uninterested will just not read it and, in this, I find some consolation.

The concluding part of paper [29] is devoted to a consideration of superconducting plates (films) of thickness $2d$ in an external magnetic field H_0 parallel to the film and also in the presence of a current $J = \int_{-d}^{+d} j(z) dz$ (where $j(z)$ is the current density) flowing through the film. Instead of J , it is convenient to work in terms of the field $H_J = 2\pi J/c$ created by the current outside the film.

In the absence of current, the critical field H_c destroying superconductivity for thick films with $d \gg \delta_0$ is [see (2.27)]:

$$\frac{H_c}{H_{\text{cm}}} = 1 + \frac{\delta_0}{2d} \left(1 + \frac{f(\kappa)}{2} \right), \quad d \gg \delta_0. \quad (2.28)$$

For sufficiently thin films, a transition to the normal state is a second-order transition (i.e., for $H_0 = H_c$, the function Ψ is equal to zero) and, for small

\varkappa , we have:

$$\left(\frac{H_c}{H_{cm}}\right)^2 = 6\left(\frac{\delta_0}{d}\right)^2 - \frac{7}{10}\varkappa^2 + \frac{11}{1400}\varkappa^4\left(\frac{d}{\delta_0}\right)^2 + \dots, \quad d \ll \delta_0. \quad (2.29)$$

For films with a half-thickness $d > d_c$, where:

$$d_c^2 = \frac{5}{4}\left(1 - \frac{7}{24}\varkappa^2 + \dots\right)\delta_0^2 \quad (2.30)$$

we are already dealing with first-order transitions with a release of latent transition heat (in other words, d_c is a tricritical point or, as it was termed before, a critical Curie point).

In the presence of a current and field (for $\varkappa = 0$):

$$\frac{H_{J_c}}{H_{cm}} = \frac{2\sqrt{2}}{3\sqrt{3}}\frac{d}{\delta_0}\left[1 - \left(\frac{H_0}{H_c}\right)^2\right]^{3/2}, \quad d \ll \delta_0, \quad (2.31)$$

where H_c is the critical field for a given film in the absence of a current [see (2.29)], H_0 is the external field, and J_c is the critical current destroying superconductivity ($H_{J_c} = 2\pi J_c/c$).

The field H_c for such films is much larger than the critical field⁶ H_{cm} for bulk samples and $H_{J_c} \ll H_{cm}$. It is interesting, however, that according to (2.29) and (2.31) (for $\varkappa = 0$ and $H_0 = 0$), we are led to

$$H_c H_{J_c} = \frac{4}{3} H_{cm}^2. \quad (2.32)$$

In [29] we certainly tried to compare the theory with the then available experimental data. But, the latter was not numerous and, particularly importantly, their accuracy was low. To the best of my knowledge, all the results of the theory were later confirmed by experiment.

2.3 The Development of the Ψ -Theory of Superconductivity

In [29], neither did we solve all the problems, nor even those which were easy to formulate. Therefore, I naturally continued, although with some intervals, to develop the Ψ -theory for several years. For example, in paper [60] (see also [14]), I considered in more detail than in [29] the destruction of the superconductivity of thin films having half-thickness $d > d_c$ [see (2.30)]: the condition $(\varkappa d/\delta_0)^2 \ll 1$ was used). Critical fields were found for supercooling

⁶ The critical field for superconducting films was calculated with allowance for corrections of the order of \varkappa^2 in [280], where the theory was compared with the experiment.

and superheating. I note that not for films, but for cylinders and balls, critical fields were calculated (on the basis of the Ψ -theory) by V.P. Silin in [61] and myself in [62] (see also [229]). The critical current for superconducting films deposited onto a cylindrical surface was found in [63]. The question of normal phase supercooling [see (2.23)] was discussed in [57], which has already been mentioned, and the critical field for superheating of the superconducting phase in bulk superconductors was calculated in [62]. So, for a small \varkappa , the critical field for superheating (denoted as the field H_{k2} in [62]) is

$$\frac{H_{c1}}{H_{cm}} = \frac{0.89}{\sqrt{\varkappa}}, \quad \sqrt{\varkappa} \ll 1, \quad (2.33)$$

where the coefficient is obtained from numerical integration.^{2*}

In several papers (see [14,32,55,64]), I discussed, in particular, the behavior of superconductors in a high-frequency field, but later on showed no interest in this issue and am now unaware whether these papers were of interest and importance for experiments (in respect of the behavior in a high-frequency field).

As I have already emphasized, the Ψ -theory can be immediately applied only in the vicinity of T_c . Naturally, I wished to extend the theory to the case of any temperature. In the framework of the phenomenological approach this goal can be achieved in different ways. So, Bardeen [65] suggested replacing the expression for the free energy F_{s0} from (2.12) with another expression involving a more complicated dependence of $F_{s0}(|\Psi|^2)$ on $|\Psi|^2$. The same object can, however, be attained [66] without the changing expression (2.12) but by assuming a certain dependence of the coefficients α and β on temperature, or, more precisely, on the ratio T/T_c . A somewhat different approach to the problem consists [67] not in assuming the dependence $F_{s0}(|\Psi|^2)$ in advance, but rather in finding it from comparison with the experiment.

After the creation of the BCS theory in 1957 and the papers [31] by Gorkov, I almost lost interest in the theory of superconductivity. Superconductivity was no longer an enigma (it had been an enigma for a long 46 years after its discovery in 1911). Quite a lot of other attractive problems existed, and I thought that I would drop superconductivity for ever. It was merely by inertia that, in 1959, when it became finally clear that the effective charge in the Ψ -theory was $e_{\text{eff}} \equiv e = 2e_0$, I compared [68] the Ψ -theory with the available experimental data and made sure that everything was all right. I will also mention the note [69] devoted to the allowance for pressure in the theory of second-order phase transitions as applied to a superconducting transition.

It was F. London [70] who had already pointed out that a magnetic flux through a hollow massive superconducting cylinder or a ring must be quantized, and that the flux quantum must be $\Phi_0 = hc/e$ and the flux $\Phi = k\Phi_0$, where k is an integer and e is the charge of the particles carrying the current. Naturally, London assumed $e = e_0$ to be a free electron charge. It was only in 1961 that the corresponding experiments were carried out (for references and a description of the experiments see, for example, [71]) demonstrating that,

in fact, $e = 2e_0$. The latter is quite clear from the point of view of the BCS theory according to which it is pairs of electrons that are carried over. Thus

$$\Phi = \frac{\hbar ck}{2e_0} = \frac{\pi \hbar ck}{e_0} = \Phi_0 k, \quad \Phi_0 = 2 \times 10^{-7} \text{ G cm}^2 \quad (k = 0, 1, 2, \dots). \quad (2.34)$$

This result (2.34) refers, however, only to the case of doubly connected bulk samples, for instance, hollow cylinders with wall thicknesses substantially exceeding the magnetic field penetration depth δ in a superconductor. And yet, samples of any size, as well as those located in an external magnetic field, etc., are also of interest. Within the framework of the Ψ -theory, I solved this problem in paper [72]. A similar but less thorough and comprehensive analysis appeared nearly simultaneously in [73, 74] (all the papers [72–74] were submitted for publication in mid-1961).

I have not yet mentioned my papers [75] and [76], which were written before the creation of the BCS theory which, however, fell out of the scope of direct application of the Ψ -theory [29]. So, in [75], the Ψ -theory was extended to the case of anisotropic superconductors. In the ‘low-temperature’ (conventional) superconductors known at that time, anisotropy is either absent altogether (isotropic and cubic materials) or is fairly small. It was apparently for this reason that in [29] we assumed, even without reservations, that metals are isotropic. But as early as in paper [22], when I considered thermoelectric phenomena, I had to examine an anisotropic (i.e., non-cubic) crystal and, in view of this, I generalized the London theory (2.4), (2.5) by introducing a symmetric tensor of rank two, A_{ik} , instead of the scalar A so that $\text{rot } \mathbf{A}(\mathbf{j}) = -\mathbf{H}/c$, $A_i(\mathbf{j}) = A_{ik} j_k$, (here $\mathbf{j} = \mathbf{j}_s$ is the superconducting current density). Such a generalization is, of course, obvious enough but I mention it here because in the extensive review [20] Bardeen refers in this connection only to papers [78, 79] by M. Laue which appeared later.

In [75], the complex scalar function $\Psi(\mathbf{r})$ for anisotropic material is introduced as before but the free energy is written not in the form (2.12) but as

$$F_{sH} = F_{s0} + \frac{H^2}{8\pi} + \frac{1}{2m_k} \left| -i\hbar \frac{\partial \Psi}{\partial x_k} - \frac{2e_0}{c} A_k \Psi \right|^2, \quad (2.35)$$

where doubly occurring indices are summed and, in [75], the charge e is taken instead of $2e_0$, and, for an isotropic or cubic material, $m_1 = m_2 = m_3 = m$, and we obtain (2.12).

As mentioned previously, the anisotropy in ‘conventional’ superconductors is not large, i.e., the ‘effective masses’ m_k little differ from one another. But, in the majority of high-temperature superconductors, in contrast, the anisotropy is very large and it is (2.35) and the corollaries to it, partially mentioned already in [75], that are widely used. An interesting effect related to the anisotropy of a superconductor is noted in [238].

Among the superconductors known in the 1950s, there was not a single ferromagnetic. This is, of course, not accidental. The point is that even digressing from microscopic reasons, the presence of ferromagnetism hampers

the occurrence of superconductivity [76]. Indeed, one can see that in the depth of a ferromagnetic superconductor the magnetic induction \mathbf{B} must also be zero. However, spontaneous magnetization \mathbf{M}_s causes induction $\mathbf{B} = 4\pi\mathbf{M}_s$. Consequently, in a ferromagnetic superconductor, even in the absence of an external magnetic field, there must flow a surface superconducting current compensating for the ‘molecular’ current responsible for magnetization. From this, it follows that a thermodynamic critical magnetic field for a ferromagnetic superconductor is

$$H_{\text{cm}}(T) = \frac{H_{\text{cm}}^{(0)}(T)}{\sqrt{\mu}} - \frac{4\pi M_s}{\mu}, \quad H_{\text{cm}}^{(0)} = \sqrt{8\pi(F_{\text{n}0} - F_{\text{s}0})}, \quad (2.36)$$

where the ferromagnetic is assumed to be ‘ideal’, i.e., for it $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} = \mu\mathbf{H} + 4\pi\mathbf{M}_s$ (μ is magnetic permittivity) and $F_{\text{n}0}$ and $F_{\text{s}0}$ are free energies for the normal and superconducting phases of a given metal in the absence of magnetization and a magnetic field. Obviously, superconductivity is only possible under the condition $H_{\text{cm}}^{(0)}(0) > 4\pi M_s/\sqrt{\mu}$ which can hold, in fact, only for ferromagnetics with a not very large spontaneous magnetization M_s . With the appearance of the BCS theory, it became clear that superconductivity and ferromagnetism obstruct each other, even irrespective of the previously mentioned so-called electromagnetic factor. Indeed, conventional superconductivity is associated with the pairing of electrons with oppositely directed spins, while ferromagnetism corresponds to parallel spin orientation. Thus, the exchange forces that lead to ferromagnetism obstruct the appearance of superconductivity. Nevertheless, ferromagnetic superconductors were discovered, but naturally with fairly low values of T_c and the Curie temperature T_M (see [77, 217, 266] and also Chap. 6 in [2]).

Unfortunately, I am not aware of corresponding experiments and wish to emphasize here that the ‘electromagnetic factor’ was allowed for in only the simplest, trivial case of an equilibrium uniform magnetization of bulk metal. However, there exist alternative possibilities [76] (see also [230]).

For example, let us assume that a ferromagnetic metal possesses a large coercive force and that in the external field $H_c < H_{\text{coer}}$ magnetization can remain directed opposite to the field (for simplicity, we consider cylindrical samples in a parallel field). Then, for $M_s < 0$ (the magnetization is directed oppositely to the field), superconductivity may exist under the condition $H_{\text{cm}}^{(0)}(0) > 4\pi|M_s|/\sqrt{\mu} - \sqrt{\mu}H_{\text{coer}}$, i.e., in principle, the ‘electromagnetic factor’ may be absolutely insignificant. Of even greater interest are possibilities arising in the case of thin films and generally small-size samples. For them, the critical field $H_c^{(0)}$, as is well known and has already been mentioned, may substantially exceed the field $H_{\text{cm}}^{(0)}$ for bulk metal. At the same time, a critical field for a ferromagnetic superconducting film, even for $M_s > 0$ (when the magnetization is directed along the field), has, as before, the form (2.36) but with $H_{\text{cm}}^{(0)}$ replaced by $H_c^{(0)}$. Now, the presence of magnetization M_s may already be of no importance. Thus, additional possibilities open up for inves-

tigating ferromagnetic superconductors. I do not know if these possibilities have ever been considered.⁷

We have up to now discussed only equilibrium or metastable (superheated or supercooled) states of superconductors, fluctuations being totally ignored. Meanwhile, fluctuations near phase transition points, especially for second-order transitions, generally speaking, play an important role (see, for example, [34], Sect. 146). In the case of superconductors, one should expect fluctuations of the order parameter Ψ both below and above T_c . I can tell the reader about my activity in this field. In 1952, at the end of [80], it was noted that fluctuations of the ‘concentration of superconducting electrons’ n_s must also be present above T_c and that this must affect, first of all, the complex dielectric constant of a metal. At the end of review [14], this remark was made again, with an emphasis on the fact that as $T \rightarrow T_c$ the fluctuations must be large. However, I never elaborated upon this observation later. Fourteen years had passed before V.V. Schmidt [81] (whose untimely death occurred in 1985) went farther and considered (with a reference to [80]) the question of the fluctuational specific heat of small balls above T_c , and also mentioned the possibility of observing the fluctuational diamagnetic moment of such balls. It is curious that another two physicists with this name investigated [82, 83] the same issue and, moreover, considered fluctuational conductivity above T_c (for the fluctuation effects see also [30, 84, 85]).

Let us now turn to a very important question of the applicability limits of Landau’s phase transition theory, both in the general context and in its application to superconductors [86].

Landau’s phase transition theory [34, 50] is well known to be the mean field theory (or, as it is sometimes referred to, the molecular or self-consistent field theory). This means that the free energy (or a corresponding thermodynamic potential) of the type:

$$F = F_0 + \alpha\eta^2 + \frac{\beta}{2}\eta^4 + \frac{\gamma}{6}\eta^6 + g(\nabla\eta)^2 \quad (2.37)$$

does not include the contribution from the fluctuations of η .

As we have seen in the example of a superconductor, when $\eta = \Psi$ [see (2.12), (2.13)], below the second-order transition point (we set $\gamma = 0$), the equilibrium value is

$$\eta_0^2 = -\frac{\alpha}{\beta} = \frac{\alpha'_c(T_c - T)}{\beta_c}. \quad (2.38)$$

Taking the Landau theory as the first approximation and using it as a basis, one can find the fluctuations of various quantities, in particular, the parameter η itself. Naturally, the Landau theory holds true and the fluctuations calculated on its basis hold true only as long as they are small compared to

⁷ The superconductivity in ferromagnetics has attracted much attention [?, 253–255, 258, 273, 274, 276].

the mean values obtained within the Landau theory. In application to η , this means that the condition

$$\overline{(\Delta\eta)^2} \ll \eta_0^2 \quad (2.39)$$

must hold, where obviously $\overline{(\Delta\eta)^2}$ is the statistical mean of the fluctuation of the quantity η (the fluctuation $(\Delta\eta)$ is zero because we calculate the deviations from the value η_0 corresponding to the minimum free energy).

The use of criterion (2.39) leads to the following condition of applicability of the Landau theory (see [86–88]):

$$\tau \equiv \frac{T_c - T}{T_c} \gg \frac{k_B^2 T_c \beta_c^2}{32\pi^2 \alpha'_c g^3} \quad (2.40)$$

where k_B is the Boltzmann constant. This means that the Landau theory can be exploited within the temperature range in the vicinity of the transition point T_c satisfying inequality (2.40). A condition of type (2.40) or similar was derived in different but close ways in [34, 86–88]. For example, in [88] the condition of applicability of the Landau theory is written in the form (in our notation; moreover, in [34, 88] k_B was set unity):

$$\text{Gi} = \frac{T_c \beta_c^2}{\alpha'_c g^3} \ll \tau \ll 1, \quad \tau = \frac{T_c - T}{T_c}. \quad (2.41)$$

The number Gi in [88] was called the Ginzburg number but I never employ this terminology for the reason mentioned earlier in respect to the Ψ -theory. In my opinion it is more appropriate to employ a criterion of the form (2.40) because the coefficient $1/32\pi^2$ is fairly small and this extends, in fact, the limits of applicability of the Landau theory (note that in [86] the coefficient $1/32\pi^2$ in the final expression (2.5b) is omitted but it is clear from (2.4) for $(\Delta\eta)^2$).

Obviously, the smaller the number Gi is, the closer to the transition point the Landau theory can be used, in which, in particular, the specific heat simply undergoes a jump (without λ -singularity) and $\eta_0^2 \sim (T_c - T)$. This immediately implies, for example, that in liquid helium (^4He) the parameter Gi is large and this results in the existence of the λ -singularity. In [86], various transitions are discussed, the most detailed consideration being given to ferroelectrics to which the Landau theory is generally well applicable, as to other structure phase transitions. This subject was discussed many years later in paper [89] but we shall not touch upon it here; see Chap. 5 in [2]. In the present paper, we are concerned with superconducting transitions and the λ -transition in liquid helium. The latter is dealt with in Sect. 2.4. As far as superconductors are concerned, from comparison of the expressions in (2.12) with $e = 2e_0$ and $m = m_0$, (2.25), (2.26), (2.37), and (2.40), it follows that condition (2.40) takes on the form

$$\tau \equiv \frac{T_c - T}{T_c} \gg \tau_G \equiv \frac{(k_B \beta_c)^2}{32\pi^2 (\alpha'_c)^4 T_c^2 [\xi(0)]^6}. \quad (2.42)$$

This expression, however, bears no specific features for superconductors and refers to any second-order transition described by the Landau theory. In the framework of this theory, as is clear from [34] and, for example, from (2.13) or (2.37), the jump ΔC of specific heat $C = T dS/dT$, where $S = -\partial F/\partial T$ is entropy, at transition is

$$\Delta C = \frac{(\alpha'_c)^2 T_c}{\beta_c}. \quad (2.43)$$

From (2.43), it is clear that condition (2.42) involves, in particular, the directly measurable quantity ΔC . Next, for superconductors [see (2.13), (2.23), (2.25), (2.26), and (2.34)],

$$\begin{aligned} H_{\text{cm}}^2 &= \frac{4\pi(\alpha'_c)^2}{\beta_c} (T_c - T)^2 = \frac{4\pi(\alpha'_c)^2 T_c^2}{\beta_c} \tau^2 \equiv H_{\text{cm}}^2(0) \tau^2, \\ H_{c2}^2 &= 2\kappa^2 H_{\text{cm}}^2, \quad \xi^2 = \frac{\hbar^2}{2m_0\alpha'_c T_c} \tau^{-1} \equiv \xi^2(0) \tau^{-1}, \\ \kappa^2 &= \frac{m_0^2 c^2 \beta_c}{8\pi e_0^2 \hbar^2}, \quad \xi^{-2}(0) = \frac{2e_0}{\hbar c} H_{c2}(0) = \frac{2\pi H_{c2}(0)}{\Phi_0}, \quad H_{c2}^2(0) = 2\kappa^2 H_{\text{cm}}^2(0). \end{aligned} \quad (2.44)$$

To avoid misunderstanding, we shall stress that all our consideration, as well as the Ψ -theory itself, refers directly only to the region near T_c . Consequently, the quantities $H_{\text{cm}}(0)$ and $H_{c2}(0)$ are somewhat formal and are not at all the true values of the fields $H_{\text{cm}}(T)$ and $H_{c2}(T)$ at $T = 0$. In view of this, it would be more correct to employ the derivatives $(dH_{\text{cm}}/dT)_{T=T_c} = -H_{\text{cm}}(0)/T_c$ and $(dH_{c2}/dT)_{T=T_c} = -H_{c2}(0)/T_c$ which can be measured in the experiment.

Allowing for (2.43) and (2.45), one can rewrite condition (2.42) in the form

$$\tau \gg \tau_G = \left(\frac{2\pi}{\Phi_0} \right)^3 \frac{H_{c2}^3(0)}{32\pi^2(\Delta C)^2}, \quad \Phi_0 = \frac{\pi \hbar c}{e_0}. \quad (2.45)$$

For type I superconductors, the substitution in (2.42) and (2.45) of the values of $\xi(0)$ (or $H_{c2}(0)$) and ΔC known from the experiment, even without account of the factor $1/32\pi^2 \sim 3 \times 10^{-3}$, yields the estimate $\tau_G \sim 10^{-15}$ (see [86] for $T_c \sim 1$ K) or, on the basis of the BCS model, the estimate $\tau_G \sim (k_B T_c / E_F)^4 \sim 10^{-12} - 10^{-16}$ (here E_F is the Fermi energy; see [30], Sect. 45; 88). Physically, it is obvious that the smallness of the value τ_G for superconductors is due to the high value of the correlation radius $\xi(0)$ in type I superconductors. In this case, the characteristic value $\xi(0) \sim \xi_0 \sim 10^{-4} - 10^{-5}$ cm is of the order of the size of a Cooper pair. For structure phase transitions, $\xi(0) \sim d \sim 3 \times 10^{-8}$ cm and is of the order of interatomic length and the fluctuation region must be seemingly large. But, in this case (in particular, in ferroelectrics), the relative smallness of τ_G is caused by other factors (see [86, 89]).

Thus, the Ψ -theory is, generally speaking, well applied to superconductors. The words ‘generally speaking’ refer to several circumstances. Firstly, we have considered here the three-dimensional case. For quasi-two-dimensional (thin

films), quasi-one-dimensional (thin wires, etc.), and quasi-zero-dimensional (small seeds, say, balls) superconductors, the conditions of applicability of the theory are different: the fluctuation region is wider than for a three-dimensional system. Unfortunately, I do not know all aspects of the problem (see, however, [90]). Secondly, as has already been emphasized, good applicability of the mean field approximation (the Landau theory and, in particular, the Ψ -theory) is in no way an obstruction to the calculation of various fluctuation effects, as long as they are sufficiently small (see, for example, [81–85, 90, 91]). It is of importance, especially in application to high-temperature superconductors (HTSCs), that paper [90] analyses, on the basis of (2.35), the anisotropic case. Third, in a number of superconductors (dirty alloys, HTSCs), the parameter κ is large or even very large (reaching hundreds) while the correlation length is small. Then the fluctuation region, i.e., the temperature range in which inequality (2.42), (2.45) is violated, is not so small. So, in [90], we present the values $\tau_G = (0.2\text{--}2) \times 10^{-4}$ for HTSCs. Somewhat lower values are reported in [92]. For $\tau_G \sim 10^{-4}$ and $T_c \sim 100$ K, the width of the fluctuation region is $\Delta T \sim 10^{-2}$ K (in this region the fluctuations are already high and are, therefore, not a small correction). This region does not seem to be so very large, but in experiments the variation of the specific heat of some HTSCs near T_c has a clearly pronounced λ -shaped form similar to the one we observe in Helium II (see [93], p. 2; 132), where the original literature is cited).

In view of the latter circumstance, it seems interesting to extend the Ψ -theory to the fluctuation region. We shall touch upon this issue in Sect. 2.4 because this extension was proposed in the application to liquid helium. But after the discovery of HTSCs in 1986–1987, such a ‘generalized Ψ -theory’ was suggested in the application to superconductors as well [54, 90, 94].

Underlying the ‘generalized’ Ψ -theory of superconductivity is, for instance, the following expression:

$$\begin{aligned} \tilde{F} = \tilde{F}_{n0} + \frac{C_0 T_c}{2} \tau^2 \ln \tau + \int \left[-a_0 \tau^{4/3} |\Psi|^2 + \frac{b_0}{2} \tau^{2/3} |\Psi|^4 \right. \\ \left. + \frac{g_0}{3} |\Psi|^6 + \frac{\hbar^2}{4m_k} \left| \left(\nabla_k - i \frac{2e_0}{\hbar c} A_k \right) \Psi \right|^2 \right] dV \end{aligned} \quad (2.46)$$

for the free energy which leads to the following equation for Ψ :

$$-\frac{\hbar^2}{4m_k} \left(\nabla_k - i \frac{2e_0}{\hbar c} A_k \right)^2 \Psi + (-a_0 \tau^{4/3} + b_0 \tau^{2/3} |\Psi|^2 + g_0 |\Psi|^4) \Psi = 0. \quad (2.47)$$

If one neglects anisotropy and sets $m_k = m_0/2$, then (2.47) will differ from (2.14) by a transformed temperature dependence of the coefficients and by the presence of the term proportional to $|\Psi|^4 \Psi$. Taking the example of helium II, we shall see in Sect. 2.4 that the ‘generalized’ Ψ -theory entails a number of consequences near T_c which correspond in reality in the case of liquid helium.

One might think that this could also be extended to superconductors with a very small correlation length. Such a case corresponds in a certain measure to the Schafroth model [16] which involves small-sized pairs. One of the directions of HTSC theory is based precisely on this model [93].

Another example of generalization of Ψ -theory near the transition point can be seen in [277].

An important point in the ‘generalized’ Ψ -theory is the problem of boundary conditions. Condition (2.15) is, generally speaking, already insufficient here and should be replaced [37, 90, 95] by a more general condition:

$$n_k \Lambda_k \left[\frac{\partial \Psi}{\partial x_k} - i \frac{2e_0}{\hbar c} \Lambda_k \Psi \right] = -\Psi \quad (2.48)$$

on the boundary with a vacuum or a dielectric, where all the quantities are, of course, taken on the boundary, n_k are the components of the unit vector \mathbf{n} perpendicular to the boundary, and Λ_k are some coefficients having dimensions of length, sometimes referred to as extrapolation lengths. For the isotropic case, when $\Lambda_k = \Lambda$, (2.48) takes on the form:

$$\mathbf{n} \left(\nabla \Psi - i \frac{2e_0}{\hbar c} \mathbf{A} \Psi \right) = -\frac{1}{\Lambda} \Psi \quad (2.49)$$

[this Λ should not be confused with the coefficient (2.17) involved in the London theory (2.4), (2.5)].

For $\Lambda_k \gg \xi_k(T)$, condition (2.49) becomes (2.15) because, generally speaking, $\partial \Psi / \partial x_k \sim \Psi / \xi_k$. In the case $\Lambda_k \ll \xi_k(T)$, however, we arrive at the boundary condition

$$\Psi = 0. \quad (2.50)$$

This condition on a rigid wall was chosen in the initial Ψ -theory of superfluidity [94, 96]. As far as I know, the ‘generalized’ Ψ -theory of superconductivity was never used after paper [90]. Two reasons for this are possible. On the one hand, the ‘generalized’ Ψ -theory has no reliable microscopic grounds (as distinct from the conventional Ψ -theory of superconductivity considered earlier). On the other hand, the investigations of HTSC are obviously at such a stage now that it has probably not yet become necessary to solve problems requiring the application of the ‘generalized’ Ψ -theory. As far as the conventional Ψ -theory is concerned, its application to HTSC is also now only rather small scale.

It should be remarked that the Ψ -theory of superconductivity [29] might be, and sometimes has to be, generalized in view of introducing a more complex Ψ -functions. In paper [275], for instance, the author considered a generalization of the Ψ -theory in the context of MgB_2 superconductivity by introducing two functions Ψ_1 and Ψ_2 (so that the order parameter has now the Ψ_1 and Ψ_2 components [275]; see also [277]).

I have dwelt on the development of the initial Ψ -theory [29] in three directions: allowing for anisotropy [75], for ferromagnetic superconductors [76],

and in a fluctuation region [90]. Also of importance are extensions in another two directions, namely to the non-stationary case, when the function Ψ is time dependent, and to superconductors in which the order parameter is not reduced to the scalar complex function $\Psi(\mathbf{r})$. I obtained no results in either of these two directions. True, in what concerns the non-stationary generalization of the Ψ -theory, I already understood [64] in 1950 that this task did exist, but restricted myself to the remark that (2.14) might be supplemented with the term $i\hbar\partial\Psi/\partial t$. Meanwhile, an allowance for relaxation is more significant. The corresponding equations for $\Psi(\mathbf{r}, t)$ are discussed in reviews [85, 97, 237]. As to the so-called ‘unconventional’ superconductors in which Cooper (or analogous) pairs are not in the s-state, I not only failed to contribute to this field, I also have a poor knowledge of it. By the way, the possibility of ‘unconventional’ pairing was first pointed out [98] for superfluid ^3He , and this fact was later confirmed. In the case of superconductivity, the ‘unconventional’ pairing takes place for at least several superconductors with heavy fermions (UB_{13} , CeCu_2Si_2 , UPt_3) and, apparently, several HTSCs – the cuprates. I shall restrict myself only to pointing to one of the pioneering papers in this field [99] and reviews [100–103]. It is a pleasure to me to note also that ‘unconventional’ superconductors are now the subject of successful research by Y.S. Barash [104], my immediate colleague (our joint research was, however, conducted in quite a different field – the theory of Van der Waals forces [105]). It is noteworthy that an appropriately extended Ψ -theory is extensively used for ‘unconventional’ superconductors as well [99–102].^{9*,10*}

2.4 The Ψ -Theory of Superfluidity

As I have already mentioned, the behavior of liquid helium near the λ -point was beyond the scope of Landau’s interests. He also remained indifferent to the behavior of superfluid helium near a rigid wall. As for me, I was for some reason interested in both these questions from the very beginning of my work in the field of superfluidity, i.e., from 1943 on [19]. I have already mentioned the attempt [49] to introduce the order parameter ρ_s near the λ -point. As regards the behavior of helium near the wall, it looks like this. Helium atoms stick to the wall (they wet it, so to say). How can it be combined with a flow along the wall of the superfluid part of the liquid with density ρ_s and a velocity \mathbf{v}_s ? We know that in the Landau theory of superfluidity [4] the velocity \mathbf{v}_s along the wall (as distinct from the velocity \mathbf{v}_n of a normal liquid) does not become zero on the wall. This means that, on the wall, the velocity \mathbf{v}_s must become discontinuous (the velocity \mathbf{v}_s cannot tend gradually to zero because of the condition $\text{rot } \mathbf{v}_s = 0$). This velocity discontinuity must be associated with a certain surface energy σ_s [106]. Estimates show that this energy σ_s is rather high ($\sigma_s \sim 3 \times 10^{-2} \text{erg cm}^{-2}$) and its existence must have led to a pronounced effect. Specifically, something like dry friction must have been observed – to move a rigid body placed in Helium II, the energy

$\sigma_s S$ must have been expended, where S is the body (say, plate) surface area. However, specially conducted experiments showed [107] that no energy $\sigma_s S$ is actually needed and a possible value of σ_s is at least, by many orders of magnitude, smaller, than the previously mentioned estimates [106]. How can this contradiction be eliminated? The solution of the problem I saw in the assumption that the density ρ_s decreases on approaching the wall and, on the wall itself, $\rho_s(0) = 0$. Thus, the discontinuity of the velocity \mathbf{v}_s on the wall is of no importance because the flow $\mathbf{j}_s = \rho_s \mathbf{v}_s$ tends gradually to zero on the wall itself even without a change of velocity \mathbf{v}_s . By that time (1957), the Ψ -theory of superconductivity [29] had long since been constructed and there was no problem in extending it to the case of superfluidity and with the boundary condition $\Psi(0) = 0$ on the wall [see (2.50)], which provided the condition $\rho_s(0) = 0$, as well.

Unfortunately, I do not at all remember how far I had advanced in constructing the Ψ -theory of superfluidity before I learnt that L.P. Pitaevskii was engaged in solving the same problem. We naturally joined our efforts and the outcome was our paper [96] which we submitted for publication on 10 December, 1957.

The Ψ -theory of superfluidity constructed in [96] will, henceforth, be referred to as the initial Ψ -theory of superfluidity. The point is that this theory was later found to be inapplicable to Helium II in the quantitative respect and we had to generalize it. Such a generalized Ψ -theory of superfluidity, developed by A.A. Sobyenin and myself [108–112], is far from being so well grounded as the Ψ -theory of superconductivity. In this connection, and, I think, in view of an insufficient awareness of the distinction between the generalized theory and the initial one [96], the Ψ -theory of superfluidity has not attracted attention and, at the present time, remains undeveloped⁸ and not systematically verified. Meanwhile, the microtheory of superfluidity is not nearly so well developed as the microtheory of superconductivity, and the role of the macrotheory of superfluidity is particularly high. This has led Sobyenin and myself to the conviction that the development of the Ψ -theory of superfluidity and its comparison with an experiment would be highly appropriate.

The most comprehensive of the cited reviews devoted to the generalized Ψ -theory of superfluidity [110] amounts to 78 pages. This alone makes it clear that, in this article, I have no way of giving an in-depth consideration to the Ψ -theory of superfluidity. Here I shall restrict myself to brief remarks.

We shall begin with the initial theory [96]. It is constructed in much the same manner as the Ψ -theory of superconductivity [29]. As the order parameter, we took the function $\Psi = |\Psi| \exp i\varphi$ acting as an ‘effective wavefunction of the superfluid part of a liquid’ and so the density ρ_s and the velocity \mathbf{v}_s are expressed as

⁸ One of the reasons, and perhaps even the main one, is the fact that A.A. Sobyenin has become a politician and for several years now has not been working practically as a physicist.^{3*}

$$\begin{aligned}\rho_s &= m|\Psi|^2, \quad \mathbf{v}_s = \frac{\hbar}{m} \nabla \varphi, \\ \mathbf{j}_s &= \rho_s \mathbf{v}_s = -\frac{i\hbar}{2} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) = \hbar |\Psi|^2 \nabla \varphi,\end{aligned}\quad (2.51)$$

where $m = m_{\text{He}}$ is the mass of a helium atom and a convenient normalization of Ψ is chosen; in [96] it is shown (see also later) that in the expression for \mathbf{v}_s we have $m = m_{\text{He}}$, irrespective of the manner in which Ψ is normalized. Then there come the expressions

$$\begin{aligned}F &= F_0 + \frac{\hbar^2}{2m} |\nabla \Psi|^2, \\ F_0 &= F_I + \alpha |\Psi|^2 + \frac{\beta}{2} |\Psi|^4, \quad \alpha = \alpha'_\lambda (T - T_\lambda), \quad \beta = \beta_\lambda\end{aligned}\quad (2.52)$$

which are usual for the mean field theory (the Landau phase transitions theory), where $F_I(\rho, T)$ is the free energy of Helium I and T_λ is the temperature of the λ -point. In equilibrium, homogeneous Helium II

$$|\Psi_0|^2 = \frac{\rho_s}{m} = \frac{|\alpha|}{\beta_\lambda} = \frac{\alpha'_\lambda (T_\lambda - T)}{\beta_\lambda}, \quad \Delta C_p = C_{p,\text{II}} - C_{p,\text{I}} = T_\lambda \frac{(\alpha'_\lambda)^2}{\beta_\lambda}. \quad (2.53)$$

In inhomogeneous Helium II, the function Ψ obeys the equation:

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi + \alpha \Psi + \beta_\lambda |\Psi|^2 \Psi = 0 \quad (2.54)$$

which should be solved with the boundary condition (2.50) on a rigid wall.

As in (2.25), we introduce the correlation length (in [96] it is denoted by l):

$$\xi(T) = \frac{\hbar}{\sqrt{2m|\alpha|}} = \frac{\hbar \tau^{-1/2}}{\sqrt{2m\alpha'_\lambda T_\lambda}} = \xi(0) \tau^{-1/2}, \quad \tau = \frac{T_\lambda - T}{T_\lambda} = \frac{t}{T_\lambda}. \quad (2.55)$$

The estimate presented in [96] and based on the data of ΔC_p and ρ_s measurements [see (2.54)] gives approximately $\xi(0) \sim 3 \times 10^{-8} \text{ cm}$. At the same time, the Ψ -theory is applicable, provided only that the macroscopic Ψ -function changes little on atomic scales. This implies the condition $\xi(T) \gg a \sim 3 \times 10^{-8} \text{ cm}$ (here a is the mean interatomic distance in liquid helium). Consequently, the Ψ -theory can only hold near the λ -point for $\tau \ll 1$, say, for $(T_\lambda - T) < (0.1\text{--}0.2) \text{ K}$. Of course, proximity to T_λ is also the condition of applicability of expansion (2.52) in $|\Psi|^2$. The small magnitude of the length $\xi(0)$ in helium leads at the same time to considerable dimensions of the fluctuation region [86]. Indeed, applying criterion (2.42), we arrive at the value $\tau_G \sim 10^{-3}$ for helium (see [108], formula (2.46)). Thus, it turns out that the initial Ψ -theory of superfluidity can only hold under the condition $10^{-3} \text{ K} \ll (T_\lambda - T) \lesssim 0.1 \text{ K}$, i.e., it is practically inapplicable because in the studies of liquid helium, of particular interest is exactly the range of values

$(T_\lambda - T) \ll 10^{-3}\text{K}$. The fact that the mean field theory leading to the jump in specific heat (2.53) does not hold for liquid helium (we certainly mean ^4He) is attested by the existence of a λ -singularity in the specific heat, as well as the circumstance that the density ρ_s near T_λ does not behave at all proportional to $(T_\lambda - T)$, i.e., according to (2.53) but rather changes by the law

$$\rho_s(\tau) = \rho_{s0}\tau^\zeta, \quad \zeta = 0.6705 \pm 0.0006 \quad (2.56)$$

where the value of ζ is borrowed from the most recently reported data [113]. Note that, in [108], we gave the value $\zeta = 0.67 \pm 0.01$ and, in [110], the values $\zeta = 0.672 \pm 0.001$ and $\rho_s = 0.35\tau^\zeta \text{g cm}^{-3}$. Hence, to a high accuracy, we have

$$\zeta = \frac{2}{3}. \quad (2.57)$$

I cannot judge whether ζ actually differs from $2/3$ but if it does, the difference does not exceed 1%. It is noteworthy that, in 1957, when paper [96] was accomplished, the variation of ρ_s by the law (2.56) was not yet known. We, therefore, did not raise an alarm immediately (the λ -type behavior of specific heat is less crucial in this respect because it may not be associated with variations of Ψ , whereas the density ρ_s is proportional to $|\Psi|^2$).

Thus, the initial Ψ -theory of superfluidity [96] is inapplicable to liquid helium (^4He). However, owing to its simplicity, it has a qualitative and, occasionally, even quantitative significance for ^4He as well. The main thing is that liquid ^4He is not the only existing superfluid liquid, suffice it to mention liquid ^3He at very low temperatures, ^3He - ^4He solutions, non-dense ^4He films, and neutron liquid in neutron stars, as well as possible superfluidity in an exciton liquid in crystals, in supercooled liquid hydrogen [114], and in the Bose-Einstein condensate of the gas of various atoms (it is this very question that is presently commanding the attention of physicists; see, for example, [115] and references therein).^{4*} In some of these cases, the fluctuation region may appear to be small enough, so that the initial Ψ -theory of superfluidity may prove sufficient. This is apparently the situation in the particularly important case of superfluidity in ^3He (see note 8). We shall, therefore, dwell briefly on the results obtained in [96].

We found the distribution $\rho_s(z)$ near a rigid wall and in a liquid helium film of thickness d . The function $\Psi(z)$ and, of course, $\rho_s = m|\Psi|^2$, where z is the coordinate perpendicular to the film, has a dome-like shape because on the boundaries of the film we have $\Psi(0) = \Psi(d) = 0$ [see (2.50)]. Naturally, for a sufficiently small thickness d , the equilibrium value is $\Psi = 0$, i.e., the superfluidity vanishes. The corresponding critical value d_c (for $d < d_c$ a film is not superfluid) is equal to

$$d_c = \pi\xi(T) = \frac{\pi\hbar\tau^{-1/2}}{\sqrt{2m\alpha'_\lambda T_\lambda}}, \quad \tau = \frac{T_\lambda - T}{T_\lambda}. \quad (2.58)$$

This result implies that, for a film, the λ -transition temperature is lower than that for ‘bulk’ helium. Concretely, from (2.58), it follows that, for a film, the

λ -transition takes place at a temperature ($T_\lambda \equiv T_\lambda(\infty)$):

$$T_\lambda(d) = T_\lambda - \frac{\pi^2 \hbar^2}{2m\alpha'_\lambda d^2} = T_\lambda - \frac{\pi^2 T_\lambda \xi^2(0)}{d^2}. \quad (2.59)$$

The specific heat of the film changes with varying d , too. Such effects in small samples are observed experimentally. In [96] we also solved the problem of the vortex line, the value of Ψ on its axis being equal to zero and the velocity circulation around the line being

$$\oint \mathbf{v}_s \cdot d\mathbf{s} = \frac{2\pi \hbar k}{m}, \quad k = 0, 1, 2, \dots \quad (2.60)$$

In this formula, the ^4He atom mass $m = m_{\text{He}}$ should be used considering that the circulation cannot change with temperature and, as was shown by Feynman [116], at $T = 0$ it is the mass m_{He} that enters into (2.60). Finally, in [96], we found the surface energy on the boundary between Helium II and a rigid body and the vortex line energy.

The fact that for liquid helium and a number of other transitions, the mean field (Landau theory) does not hold led to the appearance of the generalized theory in which the free energy is written in the form (2.37) but with a different temperature dependence of the coefficients. Specifically, for the order parameter Ψ , we write

$$F_{\text{II}} = F_{\text{I}} - a_0 \tau |\tau|^{1/3} |\Psi|^2 + \frac{b_0}{2} |\tau|^{2/3} |\Psi|^4 + \frac{g_0}{3} |\Psi|^6. \quad (2.61)$$

Since for small $|\Psi|^2$ in equilibrium [see (2.53)] $|\Psi_0|^2 = \alpha/\beta = a_0 \tau^{2/3}/b_0$, this result is in agreement with (2.56), (2.57). Expression (2.61) is naturally so chosen as to correspond to the experiment. Parenthetically, the same method in application to the Ψ -theory of superconductivity was employed in paper [67], only not near but far from T_c . As far as I know, (2.61) was first applied by Y.G. Mamaladze [117]. Some other authors also discussed a generalization of the phase transition theory in the spirit of involving an equation of the type (2.61) (see references in [108]). Sobyenin and I developed the generalized Ψ -theory of superfluidity [108–112] on the basis of (2.61) which in turn underlay the ‘generalized’ Ψ -theory of superconductivity (see [90] and Sect. 2.3). But while the latter is of limited significance, the generalized Ψ -theory of superfluidity is a unique scheme capable of describing the behavior of liquid helium near the λ -point, not counting the incomparably more sophisticated approach based on the renormalization group theory (see [118] and references therein). In addition, this approach [118] is either of no or limited validity for the inhomogeneous and non-stationary cases.

Without going into details, we shall immediately present the expression for the involved free-energy density in some reduced units (instead of free energy, other thermodynamic potentials were used in [108–112] but this is of no importance):

$$F_{\text{II}} = F_1 + \frac{3\Delta C_p}{(3+M)T_\lambda} \quad (2.62)$$

$$\times \left[-t|t|^{1/3}|\Psi|^2 + \frac{(1-M)|t|^{2/3}}{2}|\Psi|^4 + \frac{M}{3}|\Psi|^6 + \frac{\hbar^2}{2m}|\nabla\Psi|^2 \right].$$

Here $t = T_\lambda - T$, ΔC_p is the jump of specific heat determined by (2.53), M is the constant introduced in the theory, $\Psi = \Psi/\Psi_{00}$, $\Psi_{00} = \sqrt{1.43\rho_\lambda/m}$, $\rho_s = 1.43\rho_\lambda(T_\lambda - T)^{2/3}$. In the simplest version of the theory, we have $M = 0$ and, irrespective of this fact, the reduced order parameter Ψ is sometimes (for instance, in the vicinity of the axis of a vortex line) rather small and the term $|\Psi|^6$ in (2.63) can be ignored. A comparison with an experiment for Helium II leads to the estimate $M = 0.5 \pm 0.3$ (see [112]). The transition is second order for $M < 1$ and first order for $M > 1$.

For a shift of the λ -transition temperature in a film (for $M < 1$), we have

$$\Delta T_\lambda = T_\lambda - T_\lambda(d) = 2.53 \times 10^{-11} \left(\frac{3+M}{3} \right) d^{-3/2} \text{ (K)} \quad (2.63)$$

which generalizes (2.59) and corresponds to experimental data, and for a capillary with diameter d , the coefficient 2.53 in (2.63) is replaced by 4.76. Expressions for a number of other quantities (density, specific heat, etc.) are obtained and the effect of the external (gravitational, electric) fields, as well as Van der Waals forces, are taken into account. The behavior of ions in Helium II, the dependence of the density ρ_s on velocity v_s , and the vortex line structure are considered [119]. Furthermore, the theory is extended to the case of the presence of a flow of the normal part of a liquid (density ρ_n , velocity \mathbf{v}_n) and the presence of dissipation and relaxation (for a non-stationary flow; for the initial Ψ -theory, this was done partially in [120]). The problem of vortex creation in a superfluid liquid (see [110] where the corresponding literature is cited) is very interesting. We note that, somewhat unexpectedly, this question proved to be of interest for simulating the process of creation of so-called topological defects in cosmology [121]. I believe that in an analysis of corresponding experiments the Ψ -theory of superfluidity may turn out to provide quite suitable methods.

The generalized Ψ -theory of superfluidity was not developed ‘from first principles’ or on the basis of a certain reliable microtheory (as in the situation with the Ψ -theory of superconductivity). This is a phenomenological theory that rests on the general theory of second-order phase transitions (Landau theory and scaling theory) and on experimental data [110, 111]. Such data is unfortunately quite insufficient for drawing a vivid conclusion concerning the region of applicability of the Ψ -theory. In the papers [122, 123], we find rather pessimistic judgements in this respect but Sobyenin was of the opinion that such a criticism is groundless. I do not hold any particular viewpoint here but my intuition suggests a very positive role of both the initial [96] and the generalized [108–118] Ψ -theories of superfluidity. In any case, clarification of

the precision and the role of the Ψ -theory of superfluidity is currently pressing because experimental studies of superfluidity in Helium II are continued (see, for example, [124, 125]; see also comments 4*, 9*).

2.5 Thermoelectric Phenomena in Superconductors

Different papers have their own fate. My first paper [19] on superconductivity now seems dull to me and this is all from a bygone time. And, what concerns the second paper [22] accomplished in the same year, 1943, remains topical up to the present date. It was devoted to thermoelectric phenomena in superconductors. Before that, thermoelectric effects had been considered (see, for example, [58, 126]) to disappear completely in the superconducting state. Specifically, when a superconducting current passes through a seal of two superconductors, the Peltier effect is absent, the same as a noticeable thermoelectric current is absent upon heating one of the seals of a circuit consisting of two superconductors. But as a matter of fact, thermoelectric phenomena in superconductors do not vanish completely, although they can manifest themselves only under special conditions [22, 24]. The point is that in a superconductor one should take into account the possibility of the appearance of two currents – superconducting (the density \mathbf{j}_s) and normal (the density \mathbf{j}_n). In a non-superconducting (normal) state in a metal, there may flow only one current \mathbf{j} , Ohm's law $\mathbf{j} = \sigma \mathbf{E}$ holding in the simplest case. If there exists a gradient of chemical potential μ of electrons in a metal and a temperature gradient, then

$$\mathbf{j} = \sigma \left(\mathbf{E} - \frac{\nabla \mu}{e_0} \right) + b \nabla T. \quad (2.64)$$

In the superconducting state, as can readily be seen (see, for example, [128]), for the normal current, we have

$$\mathbf{j}_n = \sigma_n \left(\mathbf{E} - \frac{\nabla \mu}{e_0} \right) + b_n \nabla T \quad (2.65)$$

instead of (2.3), and in the Londons' approximation (2.4) is preserved; instead of (2.5), we obtain

$$\frac{\partial(\Lambda \mathbf{j}_s)}{\partial t} = \mathbf{E} - \frac{\nabla \mu}{e_0} + \nabla \frac{\Lambda j_s^2}{2\rho_e} \quad (2.66)$$

where μ is the chemical potential of electrons and $\rho_e = e_0 n_s$, n_s is the concentration of 'superconducting electrons' ($\mathbf{j}_s = e_0 n_s \mathbf{v}_s$). Here, we omit the detail connected with the necessity of introducing different chemical potentials μ_n and μ_s in non-equilibrium conditions for a normal and superconducting electron subsystems (see [128]). Note that the last term on the right-hand side of (2.66) is of a hydrodynamic character [see (2.6)] and, in (2.5), it was omitted

because of its small magnitude. However, the contribution of this term can be observed experimentally (see [128, 243, 246, 270–272] and references therein). Forgetting again about the last term in (2.66) in the stationary case for a superconductor, we have

$$\mathbf{E} - \frac{\nabla\mu}{e_0} = 0 \quad (2.67)$$

from which it follows that [see (2.65)]:

$$\mathbf{j}_n = b_n(T)\nabla T. \quad (2.68)$$

Thus, in a superconductor, the thermoelectric current \mathbf{j}_n does not vanish completely. Why then is it not observed? As has already been mentioned, under particularly simple conditions, a normal current is totally compensated for by a superconducting current, i.e.,

$$\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n = 0, \quad \mathbf{j}_s = -\mathbf{j}_n. \quad (2.69)$$

By ‘particularly simple conditions’, we understand a homogeneous and isotropic superconductor, say, a non-closed small cylinder (a wire) on one end of which the temperature is T_1 and on the other end T_2 (we assume that $T_{1,2}$ is less than T_c).⁹ In such a specimen, in the normal state (for $T_{1,2} > T_c$), we certainly have $\mathbf{j} = 0$ and $\mathbf{E} = \nabla\mu/e_0 - b\nabla T/\sigma$ [see (2.64)]; in the superconducting state, we of course also have $\mathbf{j} = 0$ but [see (2.68) and (2.69)]:

$$\mathbf{j}_s = -\mathbf{j}_n = -b_n\nabla T, \quad \mathbf{E} - \frac{\nabla\mu}{e_0} = 0. \quad (2.70)$$

If a superconductor is inhomogeneous and (or) anisotropic, then, generally speaking, the total compensation (2.69) does not occur and a certain, although weak, thermoelectric current must be [22] and is, in fact, observed [128, 129, 213]. But, one should not think that in the simple case considered earlier, when $\mathbf{j} = 0$, all thermoelectric effects disappear. Indeed, the thermoelectric current \mathbf{j}_n must be associated with some heat transfer, i.e., in superconductors, there must occur an additional (say, circulatory or convective) heat transfer mechanism similar to the one that exists in a superfluid liquid.¹⁰ This analogy was, properly speaking, the starting point for me in paper [22]. However, in [22], I made no estimate of the additional (circulatory) thermal conductivity. Later I decomposed [64] the total heat conductivity κ involved into the heat transfer equation $\mathbf{q} = -\kappa\nabla T$ (\mathbf{q} is the heat flux) into three parts: $\kappa = \kappa_{ph} + \kappa_e + \kappa_c$. Here κ_{ph} stands for the contribution due to phonons (the

⁹ I did not want to place figures in this paper, although perhaps they would not be out of place here. But all the necessary illustrations concerning thermoeffects can be found in the readily available papers [128, 129, 213] and also in my Nobel Lecture [264], which is also published in this book.

¹⁰ Such heat transfer is also possible in semiconductors that possess the corresponding electron and hole conductivities simultaneously (see [131]).

lattice), κ_e is due to electron motion such that there is no circulation (i.e., under the condition $\mathbf{j}_n = 0$), and κ_c is due to circulation (convection). The estimates done in [64] indicated that κ_c must be negligibly small compared to κ_e but now, unfortunately, I do not understand these estimates.

After the BCS theory was created, it became possible to carry out a microscopic evaluation of κ_e and κ_c . According to [130], at $T \sim T_c$,

$$\frac{\kappa_c}{\kappa_e} \sim \frac{k_B T_c}{E_F} \quad (2.71)$$

where E_F is the Fermi energy of electrons in a given metal.

This estimation was obtained earlier [222] on the basis of the two-fluid model and some assumptions. Finally, I also came to result (2.71) by estimating the thermal flux (heat transfer) due to creation of Cooper pairs in the colder end of a sample and their decay in the hot end [129,132].¹¹ I had some doubts of whether the heat flux calculated on the basis of the kinetic equation [130] and an allowance of the effect on the boundaries [129,132] should be summed up. Such an assumption is, however, erroneous: whenever the kinetic equation holds (i.e., the free path of ‘normal electrons’ is small compared to the sample length), the kinetic calculation, and the allowance for pair creation and breakdown on the boundaries are equivalent. However, the doubts in the validity of estimate (2.71) appeared to be useful since a more consistent estimation gave another result [213]:

$$\frac{\kappa_c}{\kappa_e} \sim \left(\frac{k_B T_c}{E_F} \right)^2. \quad (2.72)$$

Apparently, kinetic calculations in [130] contained an error. The previously mentioned referred to isotropic superconductors but, in this case, the Seebeck coefficient $S = b/\sigma$ is known to be underestimated for the well-known reason by a quantity of the order of $E_F/k_B T$ (see [133,134,223]). Hence, for anisotropic and unconventional superconductors, estimate (2.71) is likely to be reasonable. For conventional isotropic superconductors at $T_c \sim (1-10)$ K and $E_F \sim (3-10)$ eV, the convective thermal conductivity is quite negligible according to (2.72) because $\kappa_c/\kappa_e \lesssim 10^{-7}$. But for high-temperature superconductors at $T_c \sim 100$ K and $E_F \sim 0.1$ eV, we already have $\kappa_c/\kappa_e \sim 0.1$ according to (2.71). The roughness of the estimate allows the suggestion that, in some cases, convective thermal conductivity may be appreciable. Therefore, I tried to explain [132] in this way the observed peak of thermal conductivity coefficient in HTSCs at $T \sim T_c/2$ (see [135–138]). However, this effect can also be explained by the corresponding temperature dependence of the coefficients κ_{ph} and κ_c . This issue was discussed in the literature. Observation of the Righi–Leduc effect, also referred to as the thermal Hall effect [224], led to the

¹¹ This result was also presented in Sect. 5 of the paper *Phys.–Usp.* **40**, 407 (1997) before its modification.

conclusion [225] that the phonon part of thermal conductivity makes no case here (i.e., the contribution of the coefficient κ_{ph} is insignificant; see [138, 224]). At the same time, it is impossible to separate directly the contributions from κ_e and κ_c and I am not aware whether it can generally be done (an analysis is needed that would involve the role of anisotropy and the external magnetic field; see [213]).¹²

I have dwelt on the convective thermal conductivity (heat transfer) in superconductors at such length because I feel somewhat particularly unsatisfied in this respect. I have never properly investigated the microtheory or, as it is more often called, the electron theory of metals. That is why I was unable, and never even tried, to construct a consistent microtheory of convective heat transfer. Now it is certainly too late for me. But I hope that someone will investigate this problem sooner or later.

If a superconductor is not homogeneous and isotropic, as has already been mentioned, no complete compensation of currents \mathbf{j}_n and \mathbf{j}_s occurs, and some thermal currents must generally flow. The simplest cases are as follows: an isotropic but inhomogeneous superconductor and a homogeneous but anisotropic superconductor (monocrystal). More than 50 years ago (!), when paper [22] was written, alloys and generally inhomogeneous superconductors were thought of as something ‘polluted’, and it was not even clear whether the Londons’ equations can be used in these conditions. For this reason, the case of inhomogeneous superconductors was only slightly touched upon in [22]. Concretely, it was pointed out that if in a bimetallic plate (say, different superconductors sealed or welded to each other), there is a temperature gradient perpendicular to the seal plane, an uncompensated current \mathbf{j} is excited along the seal line, which runs around the seal: this generates a magnetic field perpendicular to both the plate and the seal line (see Fig. 3a in [128] and Fig. 3 in [129]). As I have said, such a version does not seem interesting. Attention was, therefore, given to a monocrystal with non-cubic symmetry when the tensor A_{ik} does not degenerate into a scalar (for cubic and isotropic superconductors $A_{ik} = A\delta_{ik}$). If, in such a plate-like crystal, the temperature gradient ∇T is not directed along the symmetry axis, a current \mathbf{j} flowing round the plate is excited and a magnetic field H_T , proportional to $|\nabla T|^2$, is generated perpendicular to the plate. This field can easily be measured using modern methods. For details, see [22, 128, 129, 140]. Unfortunately, attempts to observe the thermoelectric effect in question were made only in [141], the results of which remain ambiguous [128, 140].

As it turned out, the thermoeffect for inhomogeneous isotropic superconductors is easier to analyze and easier to observe. For this purpose, it is most convenient to consider not a bimetallic plate but rather a superconducting ring (a circuit) consisting of two superconductors (with one seal at a temperature T_2 and the other at a temperature $T_1 < T_2$; see Fig. 3b in [128], Fig. 7 in [129], or Fig. 3 in [213]; see also my Nobel Lecture in this book). The pertinence of

¹² See also recent papers [244, 245] devoted to heat transfer and related problems.

the choice of this particular version was indicated in [142, 143]. Paper [142] argued that this effect was quite different from that considered in [22] but this was a misunderstanding [128, 144]. Indeed, a bimetallic plate and a circuit of two superconductors differ topologically because of the presence of a hole in the latter case, which leads to the possibility of the appearance of a quantized magnetic-field flux through the hole (see Fig. 3 in [128]). A simple calculation (see [128, 129, 142–145]) shows that the flux through the indicated hole is equal to

$$\Phi = k\Phi_0 + \Phi_T, \quad \Phi_T = \frac{4\pi}{c} \int_{T_1}^{T_2} (b_{n,II} \delta_{II}^2 - b_{n,I} \delta_I^2) dT$$

$$\Phi_0 = \frac{hc}{2e_0} = 2 \times 10^{-7} \text{ G cm}^2, \quad k = 0, 1, 2, \dots \quad (2.73)$$

where the indices I and II refer to metals I and II forming the superconducting circuit, respectively, and $\delta \equiv \delta_0$ is the penetration depth: for $k = 0$, we obtain the result for a bimetallic plate. If we assume for simplicity that $(b_n \delta^2)_{II} \gg (b_n \delta^2)_I$ and $\delta_{II}^2 = \delta_{II}^2(0)(1 - T/T_{c,II})^{-1}$, then from (2.73), we obtain ($T_c = T_{c,II}$)

$$\Phi_T = \frac{4\pi}{c} b_{n,II} \delta_{II}^2(0) T_c \ln \left(\frac{T_c - T_1}{T_c - T_2} \right). \quad (2.74)$$

Estimates for tin ($b_n(T_c) \sim 10^{11}\text{--}10^{12}\text{CGSE}$, $\delta(0) \approx 2.5 \times 10^{-6}\text{cm}$) when $(T_c - T_2) \sim 10^{-2}\text{K}$, $(T_c - T_1) \sim 0.1\text{K}$, and generally $\ln[(T_c - T_1)/(T_c - T_2)] \sim 1$ lead to the value $\Phi_T \sim 10^{-2}\Phi_0$. Such a flux can readily be measured, and this was done in a number of papers as far back as 20 years ago (for the references, see [128, 145]). Here I will only refer explicitly to [146], which also confirmed the result (2.74).

As far as the thermoelectric current in a superconducting circuit is concerned, everything seems to be clear in principle, but this is not so. The point is that for a sufficiently massive and closed toroidal-type circuit (a hollow cylinder made of two superconductors), the measured flux $\Phi(T)$ appeared [145] to be several orders of magnitude higher than the flux (2.74) and, moreover, to possess a different temperature dependence. The origin of such an ‘enormous’ thermoeffect in superconductors has not yet been clarified. A probable explanation was suggested by R.M. Arutyunyan and G.F. Zharkov [147], although it has not yet been confirmed by an experiment. There are other explanations [246] of the results obtained in [145].¹³ In this case, the measured flux through the hole is equal to $\Phi_T + k\Phi_0$ rather than Φ_T . As the critical temperature of the hottest seal T_2 approaches the temperature T_c of one of the

¹³ In the recent paper [271] it is stated that the abovementioned ambiguity in the problem of thermocurrent is clarified if the so-called Bernoulli term $\nabla(Aj_s^2/2\rho_e)$ is taken into account in (2.66). One may hope that this issue will be made clear in the near future. The other new publications devoted to thermal effects are [270, 272].

superconductors, the resulting increase in thermoelectric current increases the entrapped flux $k\Phi_0$, i.e., a growth of k , energetically advantageous. This question was discussed in a number of papers [?, ?, 149, 150] but the mechanism responsible for the increase in the flux $\Phi(T)$ still remained unclear, and no new experiments have been carried out. The mechanism of vortex formation in the walls of a superconducting cylinder that leads to an increase of an entrapped flux with increasing thermoelectric current has been proposed only recently in [151].

It should be noted that (2.73), which also implies (2.74), is derived on the assumption that the total current density $\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n$ is zero throughout the entire circuit thickness. Meanwhile, near T_c , when the field penetration depth δ increases (more than this, $\delta \rightarrow \infty$ as $T \rightarrow T_c$), the current density \mathbf{j} tends to the value corresponding to the current in the normal state (i.e., at $T > T_c$). Clearly, the flux Φ must then increase. Under such conditions, allowance should be made for the appearance in a superconductor of some charges (the so-called charge imbalance effect; see [145, 226] and other literature cited in [145, 213, 226]). It is only an allowance for the role of these charges that provides continuity for the transition from the normal to the superconducting state. By the way, near T_c , particularly when the coherent length ξ is small, the fluctuation effects also deserve attention. The influence of the charge imbalance effect upon the temperature dependence of the flux Φ in a superconducting ring was discussed in [227], where the effect was found to be small but the physical meaning of this result is not clear to me. I believe, in particular, that the allowance for flux entrapment (i.e., an increase in the number of trapped quanta of the flux with temperature) should be analyzed simultaneously with the allowance for the charge imbalance effect. The latter would also provide a clear insight (which in my opinion has not yet been attained) into the character and the results of measurements of thermal e.m.f. in the circuit upon a superconducting transition of one and then both of its units.

I turned [129, 132, 139] to the convective mechanism of thermal conductivity in superconductors many times and could not then understand why this issue was being ignored. Now (after paper [213]), the most probable explanation seems to be the fact that, within a correct kinetic calculation, the convective mechanism is involved automatically. Therefore, the contributions of κ_e and κ_c need not be separated from the observed coefficient of the electron component of thermal conductivity $\kappa_e^{\text{tot}} = \kappa_e + \kappa_c$. But is it always (when anisotropy and the action of external forces are involved) impossible to separate κ_e and κ_c ? This remains unclear to me. Furthermore, the coefficient κ_e can perhaps be determined by measuring the conductivity σ_n according to the Wiedemann–Franz law. Then, the coefficient κ_c will be determined as the difference $\kappa_e^{\text{tot}} - \kappa_e$.

I hope, although not very much, that thermoeffects in superconductors (in a superconducting state) will no longer be ignored and there will finally appear corresponding experiments involving, in particular, HTSCs. In my opinion, it

is nevertheless conceivable that the convective heat conduction mechanism plays a part in some cases.^{11*}

Concluding this section, I would like to emphasize that, in accordance with the general context of this paper, I have only concentrated on those thermoelectric phenomena in superconductors which I investigated myself. Nevertheless, there exist some other related aspects of the problem. In this respect, I shall restrict myself by referring the reader to reviews [128, 129, 145] and the references therein, as well as to the books [40, 228] and the papers [152–154, 225, 270–272].^{14*}

2.6 Miscellanea (Superfluidity, Astrophysics, and Other Things)

As mentioned in Sect. 2.1, my first work [23] in the field of low-temperature physics, which was accomplished at the beginning of 1943, was devoted to light scattering in Helium II. This question was rather topical at that time because, when comparing the transition in helium and the Bose–Einstein gas condensation, one might expect very strong scattering near the λ -point. At the same time, the Landau theory [4] suggested no anomaly. But this was, so to say, a trivial result. The most interesting thing is that the scattering spectrum must consist not of the central line and a Mandelstam–Brillouin doublet, as in usual liquids, but of two doublets. Indeed, the Mandelstam–Brillouin doublet is associated with scattering on sound (or, more precisely, hypersonic) waves, while the central line is associated with scattering on entropy (isobaric) fluctuations. In the case of Helium II, and generally superfluid liquids, entropy fluctuations propagate (or, more precisely, dissipate) in the form of a second sound. This is the reason why, instead of a central peak, a doublet must be observed that corresponds to scattering on second sound waves also. In paper [23], I noted, however, that ‘the inner anomalous doublet cannot be actually observed because on the one hand the corresponding splitting is too small ($\Delta\omega_2/\omega_2 \sim u_2/c \lesssim 10^{-7}$) and on the other hand, and this is particularly important, the intensity of this doublet relative to Mandelstam–Brillouin doublet is quite moderate’. Indeed, the inner-to-outer doublet intensity ratio is $I_2/I_1 \approx C_p/C_V - 1$ ($C_{p,V}$ is the specific heat at a constant pressure or for a constant volume). Even near the λ point, at low pressure in Helium II, we have $C_p/C_V = 1.008$. However, as in many other cases in physics, the pessimistic prediction did not prove to be correct. Firstly, the intensity of the inner doublet increases greatly with pressure and, secondly, and this is especially significant, the use of lasers promoted great progress in the study of light scattering. As a result, the inner doublet could be observed and investigated (see [155] and review [156], p. 830).

I have already mentioned papers [49, 106] devoted to superfluidity, to say nothing of papers [96, 108–112, 119] on the Ψ -theory of superfluidity. I would like also to mention the notes [157, 158] whose titles cast light on their contents.

Finally, I shall dwell on the thermomechanical circulation effect in a superfluid liquid [144, 159]. In a ring-shaped vessel filled with a superfluid liquid (concretely, Helium II was discussed) and having two ‘weak links’ (for example, narrow capillaries), in the presence of a temperature gradient, there must occur a superfluid flow spreading to the entire vessel. Curiously, the conclusion concerning the existence of such an effect was suggested [144] by analogy with the thermoelectric effect in a superconducting circuit. At the same time, the conclusion was drawn concerning the existence of thermoelectric effects in superconductors [22], in turn, by analogy with the ‘inner convection’ occurring in Helium II in the presence of temperature gradient.

The effect under discussion was observed [160] but the accuracy of measurements of the velocity v_s was not enough to fix the jumps of circulation in superfluid helium (the circulation quantum is $2\pi\hbar/m_{\text{He}} \approx 10^{-3}\text{cm}^2\text{s}^{-1}$) which had been predicted by the theory [159]. Meanwhile, there exist interesting possibilities of observing not only jumps of circulation of a superfluid flow but also peculiar quantum interference phenomena (to this end, ‘Josephson contacts’ must be present in the ‘circuit,’ for example, narrow-slit diaphragms). In my opinion, the circulation effect in a non-uniformly heated ring-shaped vessel is fairly interesting, and not only for ^4He or solutions of ^4He with ^3He , but perhaps also in the case of the superfluidity of pure ^3He . Considering an extensive front of research in the field of superfluidity all over the world, I cannot understand why this effect is totally neglected. I do not know whether this is a matter of fashion, a lack of information, or something else.¹⁴

To save space in the other sections of the present paper, I shall mention here the works [114, 161–163]. The first of them [114] stresses the fairly obvious fact that molecular hydrogen H_2 does not become superfluid only for the reason that, at a temperature T_m exceeding the λ -transition temperature T_λ , it solidifies. As is well known, for H_2 the temperature T_m is 14 K, whereas by estimation T_λ should be nearly 6 K. Perhaps liquid hydrogen may be supercooled, for example, by way of expansion (a negative pressure), application of some fields and the use of films on different substrates as well as in the dynamical regime.

The possibility of observing the secondary sound and convective heat transfer in superconductors, in the first place accounting for exciton-type excitations (we mean bosons) was considered in [161]. I should say that paper [161] was written in 1961 and I am unaware of the present state of the questions discussed in it.

In 1978, there appeared reports on the observation of a very strong diamagnetism (superdiamagnetism) in CuCl , when the magnetic susceptibility χ is negative, and $|\chi| \sim 1/4\pi$ (of course, $|\chi| < 1/4\pi$ because $\chi = -1/4\pi$ corresponds to an ideal diamagnetism). After that (in 1980), there appeared

¹⁴ A.A. Sobyanin has pointed out the interesting possibility of ‘spinning-up’ the normal component of Helium II inside a vessel by means of electric and magnetic fields acting on the helium ions.^{3*}

indications of the existence of superdiamagnetism in CdS, too. What it was that was actually observed in the corresponding experiments (for references see [162]) remains still unclear and this question was somehow ‘drawn in the sand’.^{5*} Many physicists believe that the measurements were merely erroneous. In any case, attempts were made to associate the observations with the possibility of the existence of superdiamagnetics other than superconductors.¹⁵

The last study in this direction in which I took part was reported in [162]. Further on, the question of superdiamagnetism somehow ‘faded away’ (see, however, [164]) and I am unacquainted with the progress in this field. When seeking ways of explaining superdiamagnetism, I made an attempt to generalize the Ψ -theory of superconductivity [163]. It is unknown to me whether this paper is of any value now.

Concluding this section, I shall dwell on an astrophysical problem, namely the possibility of the existence of superconductivity and superfluidity in space. It seems to me that a small digression will not lead us beyond the scope of the general context of the paper. When I was young and then middle-aged, I used to entertain myself by doing an exercise which I called then a brainstorming (I wrote about it in my book [1], p. 305).^{6*} The procedure of the ‘attack’ was as follows: looking at my watch, I set myself a task to think up some effect within a certain time interval, say, within 15–30 min. Here is a concrete example. If I am not mistaken, it was 1962, and I was travelling by train from Kislovodsk to Moscow. I was alone in the compartment with no book to read and so decided to conceive of something. I had been engaged in low-temperature physics and astrophysics for a number of years and, therefore, a natural question for me was where and under what conditions superfluidity and superconductivity could be observed in space. To formulate a question is frequently equivalent to doing half the work. It actually took me no more than the prescribed time to think that the existence of superfluidity is possible in neutron stars and superconductivity in the atmosphere of white dwarfs and that there may exist superfluidity of the neutrino ‘sea.’ On returning to Moscow, I took up all three problems – the first two together with D.A. Kirzhnits [165, 166] and the third in collaboration with G.F. Zharkov [167].

The interaction between neutrons with antiparallel spins in the s-state corresponds to attraction and, therefore, in a degenerate neutron gas, there will appear pairing in the spirit of BCS theory. For the gap width $\Delta(0) \sim k_B T_c$, we obtained the estimate $\Delta(0) \sim (1-20)$ MeV, i.e., in the center of a neutron star (for a density $\rho \sim 10^{14}-10^{15} \text{ g cm}^{-3}$) we obtained $T_c \sim 10^{10}-10^{11} \text{ K}$, while, on the neutron phase boundary (for $\rho \sim 10^{11} \text{ g cm}^{-3}$), we had $T_c \sim 10^7 \text{ K}$. It

¹⁵ In these experiments, a very strong diamagnetism was observed but the conductivity of the samples was not at all anomalously large. Such a situation is also possible for superconductors in the case of superconducting seeds (granules) which are separated by non-superconducting layers. The question, however, arose as to whether or not superdiamagnetism can be observed in dielectrics and non-superconductors in general.

was also indicated that the rotation of a neutron star results in the formation of vortex lines. The fact that, in nuclear matter, superfluidity may occur had actually been known before but applied to neutron stars (at that time, in 1964, they had not yet been discovered), as far as I know, our paper was pioneering. Incidentally, in [168], where I summarized my activity in the field of superfluidity and superconductivity in space, I also pointed to a possible superconductivity of nuclei-bosons (for example, α -particles) in the interior of white dwarfs and to the superconductivity of protons which are present in a certain amount in neutron stars.

The possibility of the existence of superconductivity in some surface layer of the cold stars—white dwarfs was discussed in papers [166,168]. The estimates give little hope. For example, for a density $\rho \sim 1 \text{ g cm}^{-3}$ the temperature is $T_c \sim 200 \text{ K}$ and, as the density increases, T_c falls rapidly. Somewhat more interesting is the possibility of the superconductivity of metallic hydrogen in the depths of large planets – Jupiter and Saturn [168]. The estimates of the critical temperature T_c for metallic hydrogen, which are known from the literature, reach 100–300 K but the temperature in the depth of the planets is unknown. I am unacquainted with the present-day state of the problem but it seems to me that the existence of superconductivity in stars and large planets is hardly probable. The possibility of the appearance of superfluidity in the degenerate neutrino ‘sea,’ whose existence at the early stages of cosmological evolution was discussed in some papers, was considered in note [167] (see also [168]). Such a possibility, as applied to neutrinos or some hypothetical particles now involved in the astrophysical arsenal, is currently of no particular interest, but nevertheless it is reasonable to bear in mind.

2.7 High-Temperature Superconductivity

Beginning in 1964, I started investigating high-temperature superconductivity (HTSC) and from that time this problem remained, and remains, at the center of my attention, although I was interested in many other things as well. My story about this work should, however, begin with quite a different question that concerns surface superconductivity. This question is as follows: can two-dimensional superconductors in which the electrons (or holes) participating in superconductivity are concentrated near the boundary of, say, a metal or a dielectric with a vacuum, on the boundary between, e.g., twins (i.e., on the boundary of twinning), etc. exist? It seems to me that surface superconductivity might be particularly well-pronounced for electrons on surface levels which were first considered by I.E. Tamm as far back as 1932 [169]. The possibility of this particular superconductivity was discussed in paper [170]. The answer was affirmative – the Cooper pairing and the whole BCS scheme works in the two-dimensional case as well. The following possibility was also pointed out: electrons are located at volume-type levels but their attraction, which leads to superconductivity, takes place only near the body surface (or on the twinning

boundary). Note that surface ordering, although absent in the volume, may certainly take place not only in the case of superconductivity; it is also possible, for example, for ferro- and antiferromagnetics, and ferroelectrics [171]. I subsequently saw experimental research testifying to the realistic character of such situations. But I did not follow the appearance of the corresponding literature and cannot, therefore, give any references. Besides, this is not the subject of the present paper. As to surface superconductivity, it was emphasized in 1967 that long-range superconducting order is impossible in two dimensions [172]. At the same time, as distinguished from the one-dimensional case, in two dimensions (the case of a surface) the fluctuations that destroy the order increase with the surface size L only logarithmically. Accordingly, even for surfaces of macroscopic size ($L \gg a$, where a is atomic size), the fluctuations may not be so large [173]. An even more important circumstance is that, in a two-dimensional system, there may occur a quasi-long-range order under which superfluidity and superconductivity are preserved. This is an extensive issue and I, therefore, restrict ourselves to mentioning paper [174] and the monograph [175] (Chap. 1, Sect. 5 and Chap. 6, Sect. 5), where one can find the corresponding citations.^{6*} Briefly speaking, superconductivity may well exist in two-dimensional systems. From an electrodynamic point of view, surface superconductors must behave as very thin superconducting films [176, 177]. In a certain sense, surface superconductivity is realized. For instance, superconductivity is observed in a NbSe₂ film with a thickness of only two atomic layers [178]. It would be more interesting to obtain surface superconductors on the Tamm (surface) levels [170]. It is obvious how interesting, and probably important from the point of view of applications, would be a dielectric possessing surface superconductivity. I am not, however, definitely sure that such a version may be thought of as radically different from a dielectric covered itself by a superthin superconducting film. But, after all, the difference does exist. The problem of surface superconductivity seems to be demanding and significant, irrespective of the corresponding value of the critical temperature T_c .

The fates decreed, however, that surface superconductivity was to be associated with the problem of HTSC. To be more precise, the association appeared in my own work.

Before clarifying the matter, I shall make several remarks (henceforth, I shall sometimes use the text of my paper [179] which may prove to be unavailable to the reader).

For a full 65 years, the science of superconductivity was part of low-temperature physics, i.e., temperatures of liquid helium (and, in some cases, liquid hydrogen). Thus, for example, the critical temperature of the first known superconductor, mercury, discovered in 1911, is $T_c = 4.15$ K, and the critical temperature of lead, whose superconductivity was discovered in 1913, is $T_c = 7.2$ K. If I am not mistaken, higher T_c values were not achieved until 1930, although it was definitely understood that higher T_c were desirable. The next important step on this way was the synthesis of the compound

Nb_3Sn with $T_c = 18.1\text{ K}$ in 1954. Despite a great effort, it was not until 1973 that the compound Nb_3Ge with $T_c = 23.2\text{--}24\text{ K}$ was synthesized. Subsequent attempts to raise T_c were unsuccessful until 1986, which saw the first indications (soon confirmed) of superconductivity in the La-Ba-Cu-O system with $T_c \sim 35\text{ K}$ [180]. Finally, in early 1987, a truly HTSC $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ with $T_c = 80\text{--}90\text{ K}$ was created [181]. (This statement reflects my opinion that the term ‘high-temperature’ is appropriate only for superconductors with $T_c > T_{b, \text{N}_2} = 77.4\text{ K}$, where, obviously, T_{b, N_2} is the boiling nitrogen temperature at atmospheric pressure.)

The discovery of HTSCs became a sensation and gave rise to a real boom. One of the indicators of this boom is the number of publications. For example, in the period of 1989–1991, about 15,000 papers devoted to HTSC appeared, i.e., on average, approximately 15 papers a day.^{7*} For comparison, one of the reference books states that, in the 60 years from 1911 to 1970, about 7,000 papers in total were devoted to superconductivity. Another indicator is the scale of conferences devoted to HTSC. Thus, at the conference M²HTSC III in Kanazawa (Japan, July 1991) there were approximately 1,500 presentations and the conference proceedings occupied four volumes with a total size of over 2,700 pages (see [182]). Undoubtedly, such a scale of research is, to a large extent, explained by the high expectations for HTSC applications in technology. These expectations, by the way, from the very beginning, appeared to me to be somewhat exaggerated, and this was later confirmed in practice. But, of course, the potential importance of HTSC for technology, medicine (nuclear magnetic resonance tomograph), and physics itself leaves no doubts. Nevertheless, I still do not completely understand such a hyperactive reaction from the scientific community and the general public to the discovery of HTSC: it is some sort of social phenomenon.

Another phenomenon that may be attributed either to sociology or to psychology is the complete oblivion to which HTSC researchers, who began working successfully in 1986, consigned their predecessors. Indeed, the problem of HTSC was born not in 1986 but at least 22 years earlier – in its current form, this problem was first stated by W.A. Little in 1964 [183]. Firstly, Little posed the question: why was the critical temperature of the superconductors known at the time not so high? Secondly, he pointed out a possible way of raising T_c to the level of room temperature, or even higher. Specifically, Little proposed replacing the electron–phonon interaction, responsible for superconductivity in the Bardeen, Cooper, and Schrieffer (BCS) model [18], by the interaction of conduction electrons with bound electrons or, in a different terminology which Little did not use, with excitons. In terms of the well-known BCS formula for the critical temperature

$$T_c = \theta \exp\left(-\frac{1}{\lambda_{\text{eff}}}\right) \quad (2.75)$$

the meaning of the exciton mechanism is that the region of attraction between conduction electrons θ is set to be $\theta \sim \theta_{\text{ex}}$, where $k_B\theta_{\text{ex}}$ is the characteristic

exciton energy. In contrast, for the electron–phonon mechanism of attraction in (2.75), we have $\theta \sim \theta_D$, where θ_D is the Debye temperature of the metal. Since the situation in which $\theta_{\text{ex}} \gg \theta_D$ is quite possible and even typical, it follows that, for the same value of the effective dimensionless interaction parameter λ_{eff} , for the exciton mechanism T_c is $\theta_{\text{ex}}/\theta_D$ times higher than for phonons. Concretely, Little proposed to create an ‘excitonic superconductor’ on the basis of organic compounds by designing a long conducting (metallic) organic molecule (a ‘spine’) surrounded by side ‘polarizers’ – other organic molecules [183].

It is not appropriate to go into details here. Let me just point out that Little’s work did not remain unnoticed. Quite the opposite: it attracted a lot of attention. In particular, I also followed up Little’s work by suggesting a somewhat different version: roughly speaking, replacing the quasi-one-dimensional conducting thread in Little’s model with a quasi-two-dimensional structure (‘sandwich’), i.e., with a conducting thin film placed between two ‘polarizers’ (dielectric plates) [184]. More precisely, in paper [184], with a reference to the paper [170] on surface superconductivity, it was assumed that T_c may be raised with the help of some dielectric coverings of metallic surfaces. It was emphasized that quasi-two-dimensional structures are much more advantageous than quasi-one-dimensional structures [183] because of the considerably smaller role of fluctuations (this argument was worked out in [173]). Later on, I became engaged in earnest in the HTSC problem and concentrated on ‘sandwiches’, i.e., thin metallic films in dielectric and semiconducting ‘coatings’ and on layered superconducting compounds – these kind of ‘files’ of sandwiches [175, 185–189].

I should say that I write rather easily and, moreover, I even feel the necessity of expressing my thoughts in written form. As a result, during the 32 years in which I have been interested in the HTSC problem, I wrote many (probably, too many) papers on the subject, particularly popular papers. I do not think I need to refer to many of them here. Among the published works, special attention is deserved by the monograph [175]. This book was the outcome of the joint efforts undertaken by L.N. Bulaevskii, V.L. Ginzburg, D.I. Khomskii, D.A. Kirzhnits, Y.V. Kopaev, E.G. Maksimov, and G.F. Zharkov (the I.E. Tamm Department of Theoretical Physics of the P.N. Lebedev Physical Institute of the USSR Academy of Sciences, Moscow) who had been ‘attacking’ the HTSC problem for several years. This monograph was published in Russian in 1977 and in an English translation in 1982, and was the first and, up to 1987, the only one devoted to this issue. In [175], a whole spectrum of possible ways of obtaining HTSC was considered.

I shall now dwell on some of the results of our work.

A very important question is whether or not there are some limitations on admissible T_c values in metals, say, due to the requirement of crystal lattice stability. Such limitations are possible in principle and, moreover, in the 1972 paper [190], it was stated that it was the requirement of lattice stability that fully obstructs the possibility of the existence of HTSC. The point is that the dimensionless parameter of the interaction force λ_{eff} in the BCS formula

(2.75) can be written in the form

$$\lambda_{\text{eff}} = \lambda - \mu^* = \lambda - \frac{\mu}{1 + \mu \ln(\theta_F/\theta)}. \quad (2.76)$$

Here λ and μ are, respectively, the dimensionless coupling constants for phonon or exciton attraction and Coulomb repulsion, and $k_B\theta_F = E_F$ is the Fermi energy. At the same time, in the simplest approximation (homogeneity and isotropy of material, and weak coupling), we have

$$\mu - \lambda = \frac{4\pi e^2 N(0)}{q^2 \varepsilon(0, q)} \quad (2.77)$$

where $\varepsilon(\omega, q)$ is the longitudinal permittivity for the frequency ω and for the wavenumber q , and the factor $1/q^2 \varepsilon(0, q)$ should be understood as a certain mean value in \mathbf{q} , and $N(0)$ is the density of states on the Fermi boundary for a metal in the normal state. If, as was assumed in [190], the stability condition has the form

$$\varepsilon(0, q) > 0 \quad (2.78)$$

then, from (2.77), it follows that,

$$\mu > \lambda. \quad (2.79)$$

Both this inequality and (2.76) imply that superconductivity (for which, certainly, $\lambda_{\text{eff}} > 0$) is generally possible only due to the difference between μ^* and μ , the T_c value being not large. It was, however, already known empirically that $\mu < 0.5$ and sometimes $\lambda > 1$ and, thus, that inequality (2.79) is violated. Apart from this and some other arguments already expressed in the early stages [188], it was later shown strictly (see [175, 191, 192] and the literature cited there) that the stability condition (2.78) is invalid and, in fact, the stability condition has the form (for $q \neq 0$)

$$\frac{1}{\varepsilon(0, q)} \leq 1 \quad (2.80)$$

i.e., is satisfied if one of the inequalities

$$\varepsilon(0, q) \geq 1, \quad \varepsilon(0, q) < 0 \quad (2.81)$$

holds. It is interesting that the values $\varepsilon(0, q) < 0$ for large q , important in the theory of superconductivity, are realized in many metals [193, 194]. From the second inequality (2.81) and (2.77), it is obvious that the parameter λ may exceed μ . On the basis of this fact, our group came to the conclusion even before 1977 (I mean in the Russian edition of the book [175]) that the general requirement of stability does not restrict T_c and it is quite possible, for example, that $T_c \lesssim 300$ K.

As has already been mentioned, the idea of the exciton mechanism is connected with the possibility of raising T_c by increasing the temperature θ

in (2.75) which determines the energy range $k_B\theta$ where the electrons attract one another near the Fermi surface and, thus, form pairs. It is assumed that weak coupling takes place here, when

$$\lambda_{\text{eff}} \ll 1. \quad (2.82)$$

It is only under this condition that (2.75) and the BCS model are applicable. But, the BCS theory is, on the whole, more extensive and admits consideration of the case of strong coupling [195], when

$$\lambda_{\text{eff}} \gtrsim 1. \quad (2.83)$$

Under conditions (2.83) for the strong coupling formula, (2.75) is, of course, already invalid although it is clear from it that the temperature T_c rises with increasing λ_{eff} . In the literature, a large number of expressions for T_c are proposed for the case of strong coupling (see [175, 192, 196, 197] and some references therein). The simplest of these expressions is as follows:

$$T_c = \theta \exp\left(-\frac{1 + \lambda}{\lambda - \mu^*}\right). \quad (2.84)$$

Exactly as it should be under weak coupling conditions (2.82) or, more precisely, under the condition $\lambda \ll 1$, formula (2.84), of course, becomes (2.75). If, in (2.84), we set $\mu^* = 0.1$ then, for example, for $\lambda = 3$ the temperature is $T_c = 0.25\theta$. Therefore, for the value $\theta = \theta_D = 400$ K, which is readily admissible for the phonon mechanism, we already have $T_c = 100$ K. More accurate formulae also suggest that, for strong coupling (2.83), the phonon mechanism can already allow temperatures $T_c \sim 100$ K and even $T_c \sim 200$ K. But, the analysis carried out in [175] and later showed that for ‘conventional’ superconductors with strong coupling, the temperature T_c is rather small. For example, for lead we have $\theta_D = 96$ K and, therefore, in spite of the high value $\lambda = 1.55$, the critical temperature is $T_c = 7.2$ K. For such a conclusion, i.e., that θ_D falls with increasing λ , there also exist theoretical arguments (see [175], Chap. 4). That was the reason why we (or, at least, I) did not hope for the creation of HTSCs at the expense of strong coupling but possessing the phonon mechanism. In any case, as I have already mentioned, in [175], a versatile and unprejudiced approach to the HTSC problem prevailed. Here I cite the last part of Chap. 1 written by myself for the book [175]:

‘On the basis of general theoretical considerations, we believe at present that the most reasonable estimate is $T_c \lesssim 300$ K, this estimate being, of course, for materials and systems under more or less normal conditions (equilibrium or quasi-equilibrium metallic systems in the absence of pressure or under relatively low pressures, etc.). In this case, if we exclude from consideration metallic hydrogen and, perhaps, organic metals, as well as semimetals in states near the region of electronic phase transitions, then it is suggested that we should use the exciton mechanism of attraction between the conduction electrons.

In this scheme, the most promising materials from the point of view of the possibility of raising T_c , are apparently layered compounds and dielectric-metal-dielectric sandwiches. However, the state of the theory, let alone the experiment, is still far from being such as to allow us to regard as closed other possible directions, in particular, the use of filamentary compounds. Furthermore, for the present state of the problem of high-temperature superconductivity, the most sound and fruitful approach will be one that is not preconceived, in which attempts are made to move forward in the most diverse directions.

The investigation of the problem of high-temperature superconductivity is entering into the second decade of its history (if we are talking about the conscious search for materials with $T_c \gtrsim 90$ K using exciton and other mechanisms). Supposedly, there begins at the same time a new phase of these investigations, which is characterized not only by greater scope and diversity but also by a significantly deeper understanding of the problems that arise. There is still no guarantee whatsoever that the efforts being made will lead to significant success, but a number of new superconducting materials have already been produced and are being investigated. Therefore, it is in any case difficult to doubt that further investigations of the problem of high-temperature superconductivity will yield many interesting results for physics and technology, even if materials that will remain superconducting at room (or even liquid-nitrogen) temperatures will not be produced. However, as has been emphasized, this ultimate aim does not seem to us to have been discredited in any way. As may be inferred, the next decade will be crucial for the problem of high-temperature superconductivity.'

This was written in 1976. Time passed, but the multiple attempts to find a reliable and reproducible way for creating HTSC have been unsuccessful. As a result, after the burst of activity came a slackening which gave cause for me to characterize the situation in a popular paper [198] published in 1984, as follows:

'It somehow happened that research into high-temperature superconductivity became unfashionable (there is good reason to speak of fashion in this context since fashion sometimes plays a significant part in research work and in the scientific community). It is hard to achieve anything by making admonitions. Typically, it is some obvious success (or reports of success, even if erroneous) that can radically and rapidly reverse attitudes. When they sense a 'rich strike,' the former doubters, and even dedicated critics, are capable of turning coat and become ardent supporters of the new work. But this subject belongs to the psychology and sociology of science and technology. In short, the search for high-temperature superconductivity can readily lead to unexpected results and discoveries, especially since the predictions of the existing theory are rather vague.'

I did not expect, of course, that this 'prediction' would come true in two years [180, 181]. It came true not only in the sense that HTSCs with $T_c > T_{b, N_2} = 77.4$ K were obtained but also, so to say, in the social aspect: as I

have already mentioned, a real boom began and a ‘HTSC psychosis’ started. One of the manifestations of the boom and psychosis was an almost total oblivion to everything that had been done before 1986, as if discussion of the HTSC problem had not begun 22 years before [183, 184]. I have already dwelt on this subject and in the papers [179, 196] and would not like to return to it here. I will only note that J. Bardeen, whom I always respected, treated the HTSC problem with understanding both before 1986 and after it (see [199]).

The present situation in solid state theory and, in particular, the theory of superconductivity does not allow us to calculate the temperature T_c or indicate, with sufficient accuracy and certainly, especially in the case of compound materials, what particular compound should be investigated. Therefore, I am of the opinion that theoreticians could not have given experimenters better and more reliable advice as to how and where HTSC could be sought than was done in the book [175]. An exception is perhaps only an insufficient attention to the superconductivity of the $\text{BaPb}_{1-x}\text{Bi}_x\text{O}_3$ (BPBO) oxide discovered in 1974. When $x = 0.25$, for this oxide, we have $T_c = 13\text{ K}$ which is a high value for a T_c when it is estimated in a way similar to that used for conventional superconductors. In the related oxide $\text{Ba}_{0.6}\text{K}_{0.4}\text{BiO}_3$ (BKBO), superconductivity with $T_c \sim 30\text{ K}$ was discovered in 1988. Most importantly, the compound $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$ (LBCO) in which superconductivity with $T_c \sim 30\text{--}40\text{ K}$ was discovered in 1986 [180] and is thought that the discovery of HTSC belongs to the oxides. However even now, 10 years later, one cannot predict, even roughly, the values of T_c for a particular material and, moreover, even the very mechanism of superconductivity in cuprates and, in particular, in the most thoroughly investigated cuprate $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ (YBCO) with $T_c \sim 90\text{ K}$ is not yet clear.

It is inappropriate to dwell here extensively on the current state of the HTSC problem. I shall restrict myself to several remarks. At first glance, HTSC cuprates differ strongly from ‘conventional’ superconductors (see, for example, [53, 182, 200, 214]). This circumstance gave rise to the opinion that HTSC cuprates are something special – either the BCS theory is inapplicable to them or, in any case, a non-phonon mechanism of pairing acts in them. This tendency was very clearly expressed at the 1991 M²HTSC III conference [182].

Indeed, the phonon mechanism has no exclusive rights. In principle, the exciton (electronic) mechanism, the Schafroth mechanism (creation of pairs at $T > T_c$ with a subsequent Bose–Einstein condensation), the spin mechanism (pairing due to exchange of spin waves or, as it is sometimes called, due to spin fluctuations), and some other mechanisms (for some more details and references see, for example, [197, 214]) may all exist. Since I have always been a supporter of the exciton mechanism, I would be only glad if this very mechanism proves to act in HTSC. However, there is not yet any grounded basis for such a statement. In the BKBO oxide and in doped fullerenes (fullerites) of the K_3C_{60} and Rb_3C_{60} type (they all possess a cubic structure) with $T_c \sim 30\text{--}40\text{ K}$, the phonon mechanism obviously prevails. The same relates to superconductivity in MgB_2 at $T_c = 40\text{ K}$ [260, 261] (see Chap. 6 in [2]).

The situation is more complicated with cuprate oxides, which also are highly anisotropic layered compounds. However, E.G. Maksimov, O.V. Dolgov, and their colleagues indicate, I believe, convincingly, that the phonon mechanism may quite possibly also dominate in HTSC cuprates. In any case, omitting the important question of a ‘pseudogap’ [233], HTSC cuprates in the normal state differ from ordinary metals in only a quantitative respect. Formally, a standard electron–phonon interaction with a coupling constant $\lambda \approx 2$ accounts well for the high values $T_c \sim 100\text{--}125\text{ K}$ as being due to the high Debye temperature $\theta_D \sim 600\text{ K}$ (see [197, 201–205, 256] and the literature cited there).¹⁶ The properties of the superconducting state of HTSC cuprates are a more complicated entity. To explain them, it is already insufficient to use a standard isotropic approximation in the model of a strong electron–phonon interaction. However, allowing for the anisotropy of the electron spectra and interelectron interaction, the electron–phonon interaction all the same may play a decisive role in the formation of a superconducting state. As has been shown [215, 216] (see also [206–208]), in the framework of multi-zone models allowing for standard electron–phonon and Coulomb interactions, one can obtain a strongly anisotropic superconducting gap including its sign reversal in the Brillouin zone, which imitates d-pairing. It is also possible that the electron–exciton interaction and peculiarities of the electron spectrum, which are almost insignificant for understanding the properties of the normal state, make their contribution to the formation of the superconducting state. I do not regard myself competent enough to think of such statements as proved. But it is beyond doubt that a general denial of the important role of the phonon mechanism of HTSC (in cuprates) typical of the recent past (see [182]) is already behind us [204, 256, 267, 268] (possibly I only hope so, see [263]).^{10*}

Suppose, for the sake of argument, that in the already known HTSCs the exciton mechanism does not play any role. This is, of course, important and interesting but in no way discredits the very possibility of a manifestation of the exciton mechanism. As has already been mentioned, we are not aware of any evidence contradicting the action of the exciton mechanism. But, it is actually not easy for the exciton mechanism to manifest itself. This will require some special conditions which are not yet clear (see, in particular, [205]).

The highest critical temperature fixed today (for $\text{HgBa}_2\text{Ca}_2\text{Cu}_3\text{O}_{8+x}$ under pressure) reaches 164 K. Such a value can be attained with the phonon mechanism. But if one succeeds in reaching a temperature $T_c > 200\text{ K}$, the phonon mechanism will hardly be sufficient (when $\lambda = 2$, the temperature $T_c = 200\text{ K}$ is obtained for $\theta_D \approx 1000\text{ K}$). As to the exciton mechanism, even room temperature is not a limit for T_c . A search for HTSCs with the highest

¹⁶ I find it necessary to note that the report [201] was, in fact, prepared by E.G. Maksimov alone. My name appeared in [201] only because there was a difficulty with including this report on the agenda and I had, by Maksimov’s consent, to include my name which enabled him to participate in the 1994 M²HTSC IV conference. It is not a pleasure to speak about such morals and manners, but this is the truth.

possible critical temperatures is now being and will, of course, be undertaken. It seems to me, as before, that the most promising in this respect are layered compounds and dielectric–metal–dielectric ‘sandwiches.’¹⁷ It would be natural to use the atomic layer-by-layer synthesis here [209, 218, 247]. The role of a dielectric in such sandwiches can be played by organic compounds in particular. Still and all, the possibilities that may open on the way are virtually boundless. It is, therefore, especially reasonable to be guided by some qualitative consideration (see, for example, [175], Chap. 1).

For 22 long years (from 1964 to 1986), which, however, flew by very quickly, HTSC was a dream for me and to think of it was something like a gamble. Now it is an extensive field of research, tens of thousand papers are devoted to it, and hundreds or even thousands of researchers are engaged in the study of one or another of its aspects. Much has already been done, but much remains to do. Even the mechanism of superconductivity in HTSC cuprates is rather obscure, to say nothing of the myriad particular questions. I think that among these questions the first place belongs to the question of the maximum attainable value of the critical temperature T_c under not very exotic conditions, say, at atmospheric pressure and for a stable material. More concretely, one can pose a question concerning the possibility of creating superconductors with T_c values lying within the range of room temperatures (the problem of room temperature superconductivity (RTSC)). RTSC is, in principle, possible but there is no guarantee in this respect. The problem of RTSC has generally taken the place that had been occupied by HTSC before 1986–1987. I am afraid that I do not see any possibility for myself to do something positive in this direction and it only remains to wait impatiently for coming events (see Chap. 3 in this book).

2.8 Concluding Remarks

By 1943, when I began studying the theory of superconductivity, 32 years had already passed since the discovery of the phenomenon. Nonetheless, at the microscopic level, superconductivity had not yet been understood and had actually been a ‘white spot’ in the theory of metals and, perhaps, in the whole physics of condensed media. The superfluidity of Helium II had been discovered in its explicit form no more than 5 years before that time, and its connection with superconductivity had only been outlined. The world was in a terrible war and I myself hardly understand now why the enigmas of low-temperature physics seemed so tempting to me when I was cold and semi-starving in evacuation in Kazan. But it was so. A poor command of mathematics, an inability to concentrate on one particular task (I was simultaneously engaged in several problems), and difficulties in the exchange of

¹⁷ In addition to intuitive arguments [175, 186, 188, 189], there are also some concrete arguments [201, 205] in favor of such quasi-two-dimensional structures.

scientific information, especially with experimenters, in the war and post-war years obstructed a rapid advance, and it was only in 1950 that something appeared completed (I mean the Ψ -theory of superconductivity). But this completeness is, of course, rather conditional because new questions and problems constantly arose.

At the same time, the character of studies in the field of low-temperature physics, as well as the whole of physics, was changing radically. It is even hard to imagine now that it was only one laboratory that succeeded in obtaining liquid helium between 1908 and 1923. It is hard to imagine that applications of superconductivity in physics, to say nothing of technology, were fairly modest for three decades. And it was not until the 1960s that strong superconducting magnets were created and extensively used. At the present time, superconductivity finds numerous applications (see, for example, [71, 210]). Even the small book [211] intended for schoolchildren presents various applications of superconductivity, including giant superconducting magnets in tokamaks and tomographs. The creation of HTSCs (1986–1987) gave rise to great expectations of the possibility of new applications of superconductivity. These expectations were partly exaggerated but nevertheless now, after 20 years, much has already been done in this direction, even in respect of electric power lines and strong magnets [212], not to mention some other applications [219]. I wrote in Sect. 2.7 about the boom provoked by the creation of HTSC. Many thousands of papers and hundreds or even thousands of researchers – what a contrast with what was observed in, say, 1943 or as recently as 20 years ago!

In the light of the present state of the theory of superconductivity and superfluidity, much of what has been said in this paper is only of historical interest and, in other cases, is somewhere far from the forefront of the current research. At the same time, and this is very important, I have mentioned a large number of questions and problems which still remain unclear. This lack of clarity concerns the development of the Ψ -theory of superconductivity and its application to HTSC, the application of the Ψ -theory of superfluidity, the problem of surface (two-dimensional) superconductivity, the question of thermoeffects in superconductors (and especially their connection with heat transfer, see also [257]), the circulation effect in a non-uniformly heated vessel filled with a superfluid liquid, and some other things, to say nothing of HTSC theory (see [256] for a review) and also Chap. 3 in the present book and references therein); for further investigation of ferromagnet superconductors see, for example, [274]. The aim of the paper will have been attained if it at least helps to draw attention of both theoreticians and experimenters to these problems.^{12*, 13*}

Acknowledgments

Taking the opportunity, I express my gratitude to Y.S. Barash, E.G. Maksimov, L.P. Pitaevskii, A.A. Sobyenin, and G.F. Zharkov for reading the manuscript and remarks.

2.9 Notes

1*. The paper was at first published in *Usp. Fiz. Nauk* **167**, 429 (1997) [*Phys. Usp.* **40**, 407 (1997)].

Some more details concerning the thermoelectric phenomena appeared in *Usp. Fiz. Nauk* **168**, 363 (1998). With account of both these publications, the paper was then included in my book [2] *About Science, Myself, and Others* (Moscow, Fizmatlit, 2003). In the present book the text of the paper is the same, but has some additional small specifications and citations.

2*. In paper [220], the critical field for superheating was calculated to rather a high approximation (for $\varkappa \ll 1$) with the result,

$$\frac{H_{c1}}{H_{cm}} = 2^{-1/4} \varkappa^{-1/2} \left(1 + \frac{15\sqrt{2}}{32} \varkappa + O(\varkappa^2) \right).$$

3*. The talented theoretician physicist A.A. Sobyenin died on 10 June, 1997 at the age of 54. Unfortunately, I am unaware of the fate of his last note mentioned in the footnote on p. 59.

4*. For several years now (beginning from 1995), great attention has been shown in experimental studies of Bose–Einstein condensation (BEC) of rarefied gases at low temperatures. The theoretical analysis has mostly been based on the Gross–Pitaevskii theory (see [221, 234, 237, 240]). The development of this theory, I believe, was significantly influenced by the Ψ -theory of superfluidity. It seems probable that the Ψ -theory of superfluidity [96], both in its original and generalized forms, may also be useful when applied to BEC in gases, particularly in the neighborhood of the λ -point. The thermomechanical circulation effect in superfluids [144, 159, 160] also can be interesting in BEC systems.

5*. In this connection, see the supplement to Chap. 6 in [2].

6*. See also Chap. 19 in [2].

7*. I have seen a statement in the literature that over 50,000 papers have been devoted to HTSC over 10 years.

A number of new interesting and unexpected experimental data concerning high-temperature cuprates have been obtained in recent years [273]. As a result, the mechanism of their superconductivity remains unclear. I hope that

its clarification is a matter of the near future although 20 years have already passed since they were discovered (see Chap. 3 in the present book).

8*. According to [231], in superfluid ^3He the length $\xi(0) \sim 10^{-5}\text{cm}$ while, as I pointed out in the text, for ^4He the length $\xi(0) \sim 10^{-8}\text{cm}$. Clearly, in ^3He and in some other cases, the order parameter is not the scalar function Ψ . So, one considers generalizations of the Ψ -theory with Ψ substituted by the corresponding order parameter.

9*. In the preprint [232], some scheme is elaborated that combines the generalized Ψ -theory of superfluidity with BEC theory. I do not see grounds for such a theory but, nevertheless, its analysis seems interesting. In paper [?], the Ψ -theory is somewhat generalized (by taking into account the Van der Waals forces) and compared with an experiment. Unfortunately, the author's conclusions remain obscure to me.

I have to state with regret that the Ψ -theory of superfluidity has not attracted any attention in the new publications known to me devoted to superfluidity in liquid helium, and it is totally ignored in other cases. In fact, superfluidity in liquids is very rarely studied generally now, which can be understood, in particular, in connection with the enthusiasm for BEC in gases. Nevertheless, studies of liquid ^4He continue and, for example, I think it would be quite relevant to involve the Ψ -theory of superfluidity for analysis in papers [241, 242]. The same relates to studies of superfluidity in ^3He , in neutron stars, and in other cases.

10*. I first of all mean the theory in which the parameter Ψ has several components. A good example is here the Ψ -theory developed in application to the superconductor MgB_2 which has two gaps (in this case, two complex scalar functions Ψ_1 and Ψ_2 serve as the order parameter [275]).

11*. This presentation is based on the assumption that the Fermi-liquid model is applicable to cuprates (when they are considered). If, however, the Fermi-liquid notion is inapplicable to cuprates (and possibly to some other superconductors) (see [262] and references therein and [103, 104, 143] in the reference list to Chap. 6 in [2]), a special investigation (both theoretical and experimental) of thermoelectric phenomena in such materials will be necessary. With this fact in mind, it seems to me that the study of thermoelectric effects in superconductors acquires an additional interest.

12*. The problems I see in the field of superconductivity and superfluidity are also discussed in my preprint [257]. They mostly coincide with the topics discussed in the present paper.

13*. It should be borne in mind that the present paper was published in 1997. In presenting it in this book, we have added only a few notes and references to new literature, with the exception of Sect. 2.5 (which is devoted to thermoelectric phenomena). Of course, this does not make the paper as it would have appeared should it be written anew in 2007. However, this concerns

only the present state of the physics of superconductivity and not the history of its development, to which (though in the autobiographical aspect) this paper is devoted. So, I hope the small changes to the original text, which somewhat violate its just proportions, prove to be justified.

I take the opportunity to make reference to several new papers (in addition to the already mentioned paper [247]) which have drawn my attention: [248] – superconductivity in thin films with tensions, [249, 250] – vortexes in alternating superconducting and ferromagnetic films, [251, 274, 276] – study of ferromagnetic superconductors, [252] – combinational (Raman) scattering of light in HTSC, [269] – λ -transition analysis in ^4He , [260, 261] – superconductivity in MgB_2 and ferromagnetics [274], and superconducting clusters [279]; [263, 278] – discussion of HTSC mechanism, [262] – the elucidation of quasi-particles nature in HTSC material.

14*. In relation to the discussion of thermoelectric effects, I should note that the Ψ -theory [29] was developed under conditions that the electric field \mathbf{E} is either absent or ignored. Thus, the vector potential \mathbf{A} alone was accounted for. When considering thermoelectric effects, in view of necessity of taking into account the field \mathbf{E} there is a need to generalize the Ψ -theory by introducing to it both the vector potential \mathbf{A} and the scalar potential φ .

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¹⁷ Papers written by the present author or those where he is a co-author are given with titles. This was done naturally with only the purpose of providing additional information, because very little is said about some of these papers in the main text.

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