Quantum Gravity (General) and Applications

Claus Kiefer

What is Quantum Gravity?

Quantum theory is a general theoretical framework to describe states and interactions in Nature. It does so successfully for the strong, weak, and electromagnetic interactions. Gravity is, however, still described by a classical theory – Einstein's theory of general relativity, also called geometrodynamics. So far, general relativity seems to accommodate all observations which include gravity; there exist some phenomena which could in principle need a more general theory for their explanation (Dark Matter, Dark Energy, Pioneer Anomaly), but this is an open issue.

Quantum gravity would ultimately be a physical theory, both mathematically consistent and experimentally tested, that accommodates the gravitational interaction into the quantum framework. Such a theory is not yet available. Therefore, one calls quantum gravity all approaches which are candidates for such a theory or suitable approximations thereof. The following sections will first focus on the general motivation for constructing such a theory, and then introduce the approaches which at the moment look most promising.

Why Quantum Gravity?

Q

No experiment or observation is known which definitely needs a quantum theory of gravity for its explanation. There exist, however, various theoretical reasons which indicate that the current theoretical framework of physics is incomplete and that one needs quantum gravity for its completion. Here is a list of such reasons:

- *Singularity theorems*: Under general conditions, it follows from mathematical theorems that spacetime singularities are unavoidable in general relativity. The theory thus predicts its own breakdown. The two most relevant singularities are the initial cosmological singularity ('Big Bang') and the singularity inside black holes. Since the classical theory is then no longer applicable, a more comprehensive theory must be found – the general expectation is that this is a *quantum* theory of gravity.
- *Initial conditions in cosmology*: This is related to the first point. Cosmology as such is incomplete if its beginning cannot be described in physical terms. According to modern cosmological theories, the Universe underwent an era of exponential expansion in its early phase called inflation. While inflation gives a satisfactory explanation for issues such as structure formation, it cannot give, by itself, an account of how the Universe began. Nor is it clear how likely inflation

indeed is. A thorough understanding of initial conditions should shed some light on this as well as on the origin of irreversibility, that is, on the arrow of time.

- *Evolution of black holes*: Black holes radiate with a temperature proportional to $\hbar$, the Hawking temperature, see below. For the final evaporation, a full theory of quantum gravity is needed since the semi-classical approximation leading to the Hawking temperature then breaks down. This final phase could be of astrophysical relevance, provided small relic black holes from the early Universe ('primordial black holes') exist.
- *Unification of all interactions*: All nongravitational interactions have so far been successfully accommodated into the quantum framework. Gravity couples universally to all forms of energy. One would therefore expect that in a unified theory of all interactions, gravity is described in quantum terms, too.
- *Inconsistency of an exact semi-classical theory*: All attempts to construct a fundamental theory where a classical gravitational field is coupled to quantum fields have failed up to now. Such a framework is here called an 'exact semi-classical theory'; it corresponds to the limit where the quantum fields propagate on a classical background spacetime.
- *Avoidance of divergences*: It has long been speculated that quantum gravity may lead to a theory devoid of the ubiquitous divergences arising in quantum field theory. This may happen, for example, through the emergence of a natural cutoff at small distances (large momenta). In fact, modern approaches such as string theory or loop quantum gravity (see below) provide indications for a discrete structure at small scales.

Quantum gravity is supposed to be a fundamental theory which is valid at all scales. There exists, however, a distinguished scale where one would expect that typical quantum-gravity effects can never be neglected. This scale is found if one combines the gravitational constant (G), the speed of light (c), and the quantum of action ($\hbar$) into units of length, time, mass (and energy). In honour of Max Planck, who presented these units first in 1899, they are called *Planck units*. Explicitly, they read

$$l_{\rm P} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33}\,\text{cm}\,, \tag{1}$$

$$t_{\rm P} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\,\text{s}\,, \tag{2}$$

$$m_{\rm P} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\,\text{g} \approx 1.22 \times 10^{19}\,\text{GeV}/c^2\,. \tag{3}$$

They are called Planck length, Planck time, and Planck mass, respectively.

Structures in the Universe usually occur at scales which are simple powers of the gravitational 'fine-structure constant'

$$\alpha_{\rm g} = \frac{G m_{\rm pr}^2}{\hbar c} = \left(\frac{m_{\rm pr}}{m_{\rm P}}\right)^2 \approx 5.91 \times 10^{-39}\,, \tag{4}$$

where m_{pr} denotes the proton mass. Stellar masses and stellar lifetimes can be derived, in an order-of-magnitude estimate, from this number. Its smallness is responsible for the irrelevance of quantum gravity in usual astrophysical considerations.

Structural Issues of Quantum Gravity

Quantization of gravity means quantization of geometry ► quantization. But which structures should be quantized, that is, to which structures should one apply the ► superposition principle? Following Chris Isham, one can do this at each order of the following hierarchy of structures:

Point set of events → topological structure → differentiable manifold → causal structure → Lorentzian structure.

Those structures that are not quantized remain as absolute (nondynamical) entities in the formalism. One would expect that in a fundamental theory no absolute structure remains. This is referred to as *background independence* of the theory. Still, however, most of the approaches to quantum gravity contain at least the first three structures as classical entities.

A particular aspect of background independence is the 'problem of time,' which arises in any approach to quantum gravity. On the one hand, time is external in ordinary quantum theory; the parameter t in the ► Schrödinger equation is identical to Newton's absolute time – it is *not* turned into an operator and is presumed to be prescribed from the outside. This is true also in special relativity where the absolute time t is replaced by Minkowski spacetime, which is again an absolute structure. On the other hand, time in general relativity is dynamical because it is part of the spacetime described by Einstein's equations. Both concepts cannot be fundamentally true, so a theory of quantum gravity would entail important changes for our understanding of time.

Experimental Status

One of the main problems in searching for a theory of quantum gravity is the lack of a direct experimental hint. For example, in order to probe the Planck scale directly, present-day accelerators would have to be of galactic size. Direct tests are therefore expected to arise from astrophysical or cosmological observations. However, some speculative theories with higher dimensions allow for the possibility of an experimental test at the Large Hadron Collider (LHC), which starts to operate at CERN in 2009.

Experiments are available only for the level of external Newtonian gravity interacting with micro- or mesoscopic systems (► Mesoscopic Quantum Phenomena). Examples are neutron and atom interferometry. On the level of quantum field theory on a curved spacetime, a definite, but not yet tested prediction was made: black holes emit thermal radiation. This is the Hawking effect, named after the physicist Stephen Hawking (*1942) who derived it in 1974. For a Schwarzschild black hole of mass M, the temperature is

$$T_{\rm BH} = \frac{\hbar c^3}{8\pi k_{\rm B} G M} \approx 6.17 \times 10^{-8} \left(\frac{M_\odot}{M}\right) \text{K}\,, \tag{5}$$

where $k_{\rm B}$ denotes Boltzmann's constant. The black hole shrinks due to Hawking radiation and possesses a finite lifetime. The final phase, where γ-radiation is being emitted, could be observable. The temperature (5) is unobservably small for black holes that result from stellar collapse. One would need primordial black holes produced in the early Universe because they could possess a sufficiently low mass. For example, black holes with an initial mass of 5×10^{14} g would evaporate at the present age of the Universe. In spite of several attempts, no experimental hint for black-hole evaporation has been found.

Since black holes radiate thermally, they also possess an entropy, the 'Bekenstein–Hawking entropy,' which is given by the expression

$$S_{\rm BH} = \frac{k_{\rm B} c^3 A}{4 G \hbar} = k_{\rm B} \frac{A}{4 l_{\rm P}^2}, \tag{6}$$

where A is the surface area of the event horizon. For a Schwarzschild black hole with mass M, this reads

$$S_{\rm BH} \approx 1.07 \times 10^{77} k_{\rm B} \left(\frac{M}{M_\odot}\right)^2 . \tag{7}$$

Since the Sun has an entropy of about $10^{57} k_{\rm B}$, this means that a black hole resulting from the collapse of a star with a few solar masses would experience an increase in entropy by twenty orders of magnitude during its collapse. It is one of the challenges of any theory of quantum gravity to provide a microscopic explanation for this entropy, that is, to derive (6) from a counting of microscopic quantum gravitational states.

Due to the equivalence principle, there exists an effect related to (5) in flat Minkowski space. An observer with uniform acceleration a experiences the standard Minkowski vacuum not as empty, but as filled with thermal radiation with temperature

$$T_{\rm DU} = \frac{\hbar a}{2\pi k_{\rm B} c} \approx 4.05 \times 10^{-23}\, a \left[\frac{\text{cm}}{\text{s}^2}\right] \text{K}. \tag{8}$$

This temperature is often called the 'Davies–Unruh temperature,' named after the physicists Paul Davies (*1946) and William Unruh (*1945). It, too, has not yet been experimentally tested, but efforts are being made in this direction.

What are the Main Approaches?

The main present approaches to find a theory of quantum gravity can be classified according to the following scheme.

- *Quantum general relativity*: The most straightforward attempt, both conceptually and historically, is the application of 'quantization rules' to classical general relativity. One further distinguishes the following subapproaches:
  - *Covariant approaches*: These are approaches that employ four-dimensional covariance at some stage of the formalism. Examples include perturbation theory, effective field theories, renormalization-group approaches, and path integral methods (such as Regge calculus or dynamical triangulation). For example, in the path integral one sums over all suitable four-dimensional metrics in order to arrive at a quantum gravitational Green function or wave functional. Pioneers of the covariant approach include Léon Rosenfeld, Matvei Bronstein, and Bryce DeWitt.
  - *Canonical approaches*: Here one makes use of a Hamiltonian formalism and identifies appropriate canonical variables and conjugate momenta. Examples include quantum geometrodynamics (where gravity is described in metric form) and loop quantum gravity (where gravity is described by a connection integrated around a closed loop). They are characterized by a constraint equation of the form
$$H\Psi = 0\,, \tag{9}$$
    where H denotes the full Hamilton operator for the gravitational field as well as all nongravitational fields; Ψ is the full wave functional for these degrees of freedom. In the geometrodynamical approach, this equation is called the *Wheeler–DeWitt equation*, in honour of the physicists John Archibald Wheeler (1911–2008) and Bryce DeWitt (1923–2004), who first discussed this equation in detail. The loop approach goes mainly back to work by Abhay Ashtekar (*1949), Lee Smolin (*1955), and Carlo Rovelli (*1956).

    As can be recognized from the stationary form of equation (9), these theories are explicitly timeless, that is, devoid of any classical time parameter. They thus solve the 'problem of time' by getting rid of time at the fundamental level. This should happen in the other approaches, too, but the situation is there much less clear.

- *String theory*: This is the main approach to construct a unifying quantum framework of all interactions. The quantum aspect of the gravitational field only emerges in a certain limit in which the different interactions can be distinguished from each other. All particles have their origin in excitations of fundamental strings. The fundamental scale is given by the string length; it is supposed to be of the order of the Planck length, although the Planck length is here a derived quantity.

  String theory was originally developed as a theory of hadrons. While its unsuitability in this field became soon clear, it was later devised as a theory for the physics at the Planck scale. Among the pioneers who introduced string theory in the gravitational context are Joël Scherk and John Schwarz.

- Other attempts such as the quantization of topology or the theory of causal sets.

In perturbation theory, the important concept of the *graviton* emerges. In this approximation one decomposes the metric, $g_{\mu\nu}$, into a background part, $\bar{g}_{\mu\nu}$, and a 'small' perturbation, $f_{\mu\nu}$,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{\frac{32\pi G}{c^4}} f_{\mu\nu}. \tag{10}$$

Only the perturbation is being quantized. The important assumption is the presence of an (approximate) *background* with respect to which standard perturbation theory (formulation of Feynman rules, etc.) can be applied. In this approximate framework the quantum aspects of gravity are encoded in a spin-2 particle propagating on the background – the *graviton*, which arises from $f_{\mu\nu}$. The ensuing perturbation theory is, however, *nonrenormalizable*: at each order in the expansion with respect to G, new types of divergences occur which have to be absorbed into appropriate parameters that in turn have to be fixed by measurement. Nevertheless, one can derive in the low-energy limit concrete effects from perturbation theory. One is the quantum gravitational correction to the Newtonian potential between two masses m_1 and m_2,

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + 3\frac{G(m_1+m_2)}{2rc^2} + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3}\right). \tag{11}$$

Another is the decay rate of excited states in atomic physics through emission of gravitons; for example, the decay rate in hydrogen from the 3d level to the ground state is

$$\Gamma_{\mathrm{g}} = \frac{Gm_{\mathrm{e}}^3 c\alpha^6}{360\hbar^2} \approx 5.7 \times 10^{-40}\ \mathrm{s}^{-1}, \tag{12}$$

where α is the fine-structure constant and m_{e} the electron mass. This corresponds to a life-time of

$$\tau_{\mathrm{g}} \approx 5.6 \times 10^{31}\ \text{years}, \tag{13}$$

which is too large to be measurable. The problem of nonrenormalizability in perturbation theory is avoided by string theory.

Quantum general relativity as well as string theory have found applications for quantum black holes and for quantum cosmology. Both approaches have, for restricted situations, proposed a microscopic explanation for the black-hole entropy (6). The corresponding microscopic states are either those of spin networks (in loop quantum gravity) or D-branes (in string theory). On the other hand, a clear picture of black-hole evaporation is elusive, although there is strong evidence in all approaches that there is no fundamental loss of information during this process. As for quantum cosmology, preliminary results exist for a wide range of topics: singularity avoidance, initial conditions, origin of structure, and the arrow of time. Direct effects may be seen in the anisotropy spectrum of the cosmic background radiation, but the situation is presently unclear. It should also be mentioned that both string theory and loop quantum gravity predict that space is discrete at very small scales (near the string length or the Planck length, respectively), with possible observational relevance.

A central issue is also the recovery of established physical theories as approximations from quantum gravity. Quantum geometrodynamics gives at least on the formal level a picture of how a semi-classical time parameter and the limit of quantum field theory in a background spacetime emerge as approximations (using a type of scheme similar to the Born–Oppenheimer approximation in molecular physics). This includes the classical behavior of spacetime due to decoherence (► decoherence, experimental observation of decoherence; time in quantum mechanics). The situation in loop quantum gravity is not yet fully clear. As for string theory, it has not yet succeeded to achieve one of its major goals – the recovery of the Standard Model of ► particle physics.

Primary Literature

1. L. Rosenfeld, *Über die Gravitationswirkungen des Lichtes*. Zeitschrift für Physik, **65**, 589–599 (1930).
2. M. Bronstein, *Quantentheorie schwacher Gravitationsfelder*. Physikalische Zeitschrift der Sowjetunion, **9**, 140–157 (1936).
3. B. S. DeWitt, *Quantum theory of gravity. I. The canonical theory*. Physical Review, **160**, 1113–1148 (1967).
4. B. S. Dewitt, *Quantum theory of gravity. II. The manifestly covariant theory*. Physical Review, **162**, 1195–1239 (1967).
5. B. S. Dewitt, *Quantum theory of gravity. III. Applications of the covariant theory*. Physical Review, **162**, 1239–1262 (1967).
6. J. Scherk and J. H. Schwarz, *Dual models for non-hadrons*. Nuclear Physics B, **81**, 118–144 (1974).
7. A. Ashtekar, *New variables for classical and quantum gravity*. Physical Review Letters, **57**, 2244–2247 (1986).

General Introductions

8. S. Carlip, *Quantum gravity: a progress report*. Reports on Progress in Physics, **64**, 885–942 (2001). A related online version is http://arxiv.org/abs/gr-qc/0108040.
9. C. Kiefer, *Quantum Gravity. Second Edition* (Oxford University Press, Oxford 2007).
10. C. Kiefer, *Quantum gravity: general introduction and recent developments*. Annalen der Physik, **15**, 129–148 (2006). A related online version is http:arxiv.org/abs/gr-qc/0508120.

Special Topics

11. D. Coule, *Quantum cosmological models*. Classical and Quantum Gravity, **22**, R125–R166 (2005). A related online version is http://arxiv.org/abs/gr-qc/0412026.
12. C. Kiefer and C. Weber, *On the interaction of mesoscopic quantum systems with gravity*. Annalen der Physik, **14**, 253–278 (2005). A related online version is http://arxiv.org/abs/quant-ph/0408010.

13. C. Rovelli, *Quantum Gravity* (Cambridge University Press, Cambridge, 2004).
14. H. D. Zeh, *The Physical Basis of the Direction of Time*. Fifth Edition (Springer, Berlin, 2007). See also http://www.time-direction.de.
15. B. Zwiebach, *A First Course in String Theory* (Cambridge University Press, Cambridge, 2004).
16. Review articles by various authors in the online journal Living Reviews in Relativity, see http://relativity.livingreviews.org/

Collections of Essays on General and Special Topics

17. *Quantum Theory of Gravity*, edited by S. M. Christensen (Adam Hilger, Bristol, 1984).
18. *Quantum Concepts in Space and Time*, edited by R. Penrose and C. J. Isham (Clarendon Press, Oxford, 1986).
19. *Conceptual Problems of Quantum Gravity*, edited by A. Ashtekar and J. Stachel (Birkhäuser, Boston, 1991).
20. *Quantum Gravity: From Theory to Experimental Search*, edited by D. Giulini, C. Kiefer, and C. Lämmerzahl. Lecture Notes in Physics **631** (Springer, Berlin, 2003).
21. *The Structural Foundations of Quantum Gravity*, edited by D. Rickles, S. French, and J. T. Saatsi (Clarendon Press, Oxford, 2006).
22. *Quantum Gravity: Mathematical Models and Experimental Bounds*, edited by B. Fauser, J. Tolksdorf, and E. Zeidler (Birkhäuser, Basel, 2007).

Popular Books

23. J. B. Barbour, *The End of Time* (Oxford University Press, Oxford, 2000).
24. B. Greene, *The Elegant Universe* (Vintage, London, 2005).
25. L. Smolin, *Three Roads to Quantum Gravity* (Perseus Books Group, 2002).

Quantum Hall Effect*

Rolf R. Gerhardts, Jürgen Weis, and Klaus von Klitzing

In 1980, Klaus von Klitzing made the unexpected discovery that, at sufficiently low temperatures and high magnetic fields, the Hall resistance of a two-dimensional electron system assumes quantized values, which turned out to depend only on fundamental constants and integer numbers. For this discovery, which nowadays is used to reproduce the unit of the electrical resistance with an unprecedented accuracy, he was honored in 1985 with the Nobel Prize in Physics. A coherent explanation of the

* This contribution is based on an original German article by Klaus von Klitzing, Rolf Gerhardts, Juergen Weis, '25 Jahre Quanten Hall-Effekt', in Physik Journal (June 2005, pp. 37–44). It was translated by Rolf Gerhardts and is reprinted by permission of the authors and Physik Journal.

Springer

http://www.springer.com/978-3-540-70622-9

Compendium of Quantum Physics
Concepts, Experiments, History and Philosophy
Greenberger, D.; Hentschel, K.; Weinert, F. (Eds.)
2009, XVI, 901 p., Hardcover
ISBN: 978-3-540-70622-9