2

Modeling Credit Risk

In this chapter we present some simple approaches to measure credit risk. We start in Section 2.1 with a short overview of the *standardized approach* of the Basel framework for banking supervision. This approach is a representative of the so-called *notional-amount approach.* In this concept, the risk of a portfolio is defined as the sum of the notional values of the individual securities in the portfolio, where each notional value may be weighted by a certain risk factor, representing the riskiness of the asset class to which the security belongs. The advantage of this approach is its apparent simplicity, however, it has several drawbacks as, for example, netting and diversification effects are not taken into account.

One main challenge of credit risk management is to make default risks assessable. For this purpose we present several *risk measures* based on the portfolio loss distributions. These are typically statistical quantities describing the conditional or unconditional loss distribution of the portfolio over some predetermined time horizon. The *expected* and *unexpected loss,* which we present in Section 2.2, are defined as the expectation and standard deviation, respectively, of the portfolio loss variable. Hence, they belong to this class of risk measures. Further representatives are the *Value-at-Risk* (VaR) and the *Expected Shortfall* (ES) which we discuss in Sections 2.3 and 2.4. Based on the expected loss and Value-at-Risk we introduce in Section 2.5 the concept of *economic capital* of a portfolio. All of these risk measures have a lot of advantages as, for example, the aggregation from a single position to the whole portfolio makes sense in this framework. Moreover, diversification effects and netting can be reflected and the loss distributions are comparable across portfolios. However, the problem is that any estimate of the loss distribution is based on past data which are of limited use in predicting future risk. Furthermore, it is in general difficult to estimate the loss distribution accurately, particularly for large portfolios. Models that try to predict the future development of the portfolio loss variable will the studied in later chapters.

2.1 The Regulatory Framework

The First Basel Accord of 1988, also known as Basel I, laid the basis for international minimum capital standard and banks became subject to *regulatory capital requirements,* coordinated by the Basel Committee on Banking Supervision. This committee has been founded by the Central Bank Governors of the Group of Ten (G10) at the end of 1974.

The cause for Basel I was that, in the view of the Central Bank Governors of the Group of Ten, the equity of the most important internationally active banks decreased to a worrisome level. The downfall of Herstatt-Bank underpinned this concern. Equity is used to absorb losses and to assure liquidity. To decrease insolvency risk of banks and to minimize potential costs in the case of a bankruptcy, the target of Basel I was to assure a suitable amount of equity and to create consistent international competitive conditions.

The rules of the Basel Committee do not have any legal force. The supervisory rules are rather intended to provide guidelines for the supervisory authorities of the individual nations such that they can implement them in a suitable way for their banking system.

The main focus of the first Basel Accord was on credit risk as the most important risk in the banking industry. Within Basel I banks are supposed to keep at least 8% equity in relation to their assets. The assets are weighted according to their degree of riskiness where the risk weights are determined for four different borrower categories shown in Table 2.1.

Table 2.1. Risk weights for different borrower categories

Risk Weight in %	0	10	50	100
Borrower Category	State	Bank	Mortgages	Companies and Retail Customers

The required equity can then be computed as

$$\text{Minimal Capital} = \text{Risk Weighted Assets} \times 8\%.$$

Hence the portfolio credit risk is measured as the sum of *risk weighted assets,* that is the sum of notional exposures weighted by a coefficient reflecting the creditworthiness of the counterparty (the risk weight).

Since this approach did not take care of market risk, in 1996 an amendment to Basel I has been released which allows for both a *standardized approach* and a method based on internal Value-at-Risk (VaR) models for *market risk* in larger banks. The main criticism of Basel I, however, remained. Namely, it does not account for methods to decrease risk as, for example, by means of portfolio diversification. Moreover, the approach measures risk in an insufficiently differentiated way since minimal capital requirements are computed independent of the borrower's creditworthiness. These drawbacks lead to the development of the Second Basel Accord from 2001 onwards. In June 2004 the Basel Committee on Banking Supervision released a *Revised Framework on International convergence of capital measurement and capital standards* (in short: Revised Framework or Basel II). The rules officially came into force on January 1st, 2008, in the European Union. However, in practice they have been applied already before that date. The main targets of Basel II are the same as in Basel I as well. However, Basel II focuses not only on *market* and *credit risk* but also puts *operational risk* on the agenda.

Basel II is structured in a *three-pillar framework.* Pillar 1 sets out details for adopting more risk sensitive minimal capital requirements, so-called *regulatory capital,* for banking organizations, Pillar 2 lays out principles for the *supervisory review process* of capital adequacy and Pillar 3 seeks to establish *market discipline* by enhancing transparency in banks' financial reporting.

The former regulation lead banks to reject riskless positions, such as asset-backed transactions, since risk weighted assets for these positions were the same as for more risky and more profitable positions. The main goal of Pillar 1 is to take care of the specific risk of a bank when measuring minimal capital requirements. Pillar 1 therefore accounts for all three types of risk: credit risk, market risk and operational risk.

Concerning credit risk the new accord is more flexible and risk sensitive than the former Basel I accord. Within Basel II banks may opt for the *standard approach* which is quite conservative with respect to capital charge and the more advanced, so-called *Internal Ratings Based* (IRB) approach when calculating regulatory capital for credit risk. In the standard approach, credit risk is measured by means of external ratings provided by certain rating agencies such as Standard&Poor's, Moody's or Fitch Ratings. In the IRB approach, the bank evaluates the risk itself. This approach, however, can only be applied when the supervisory authorities accept it. The bank, therefore, has to prove that certain conditions concerning the method and transparency are fulfilled. Basel II distinguishes between expected loss and unexpected loss. The former directly charges equity whereas for the latter banks have to keep the appropriate capital requirements.

The capital charge for market risk within Basel II is similar to the approach in the amendment of 1996 for Basel I. It is based mainly on VaR approaches that statistically measure the total amount a bank can maximally lose.

A basic innovation of Basel II was the creation of a new risk category, operational risk, which is explicitly taken into account in the new accord.

The supervisory review process of Pillar 2 is achieved by the supervisory authorities which evaluate and audit the compliance of regulations with respect to methods and transparency which are necessary for a bank to be allowed to use internal ratings.

The main target of Pillar 3 is to improve market discipline by means of transparency of information concerning a bank's external accounting. Transparency can, for example, increase the probability of a decline in a bank's own stocks and therefore, motivate the bank to hold appropriate capital for potential losses.

2.2 Expected and Unexpected Loss

Although it is in general not possible to forecast the losses, a bank will suffer in a certain time period, a bank can still predict the average level of credit loss, it can expect to experience for a given portfolio. These losses are referred to as the *expected loss* (EL) and are simply given by the expectation of the portfolio loss variable L defined by equation (1.1). Note that we omit the index N here as the number N of obligors is fixed in this chapter. We will use the index n to refer to quantities specific to obligor n. The expected loss EL_n on a certain obligor n represents a kind of *risk premium* which a bank can charge for taking the risk that obligor n might default. It is defined as

$$\mathrm{EL}_n = \mathbb{E}[L_n] = \mathrm{EAD}_n \cdot \mathrm{ELGD}_n \cdot \mathrm{PD}_n,$$

since the expectation of any Bernoulli random variable is its event probability. The *expected loss reserve* is the collection of risk premiums for all loans in a given credit portfolio. It is defined as the expectation of the portfolio loss L and, by additivity of the expectation operator, it can be expressed as

$$\mathrm{EL} = \sum_{n=1}^{N} \mathrm{EAD}_n \cdot \mathrm{ELGD}_n \cdot \mathrm{PD}_n \,.$$

As one of the main reasons for banks holding capital is to create a protection against peak losses that exceed expected levels, holding only the expected loss reserve might not be appropriate. Peak losses, although occurring quite seldom, can be very large when they occur. Therefore, a bank should also reserve money for so-called *unexpected losses* (UL). The deviation of losses from the EL is usually measured by means of the standard deviation of the loss variable. Therefore, the unexpected loss UL_n on obligor n is defined as

$$\mathrm{UL}_n = \sqrt{\mathbb{V}[L_n]} = \sqrt{\mathbb{V}[\mathrm{EAD}_n \cdot \mathrm{LGD}_n \cdot D_n]}.$$

In case the default indicator D_n, and the LGD variable are uncorrelated (and the EAD is constant), the UL on borrower n is given by

$$\mathrm{UL}_n = \mathrm{EAD}_n \sqrt{\mathrm{VLGD}_n^2 \cdot \mathrm{PD}_n + \mathrm{ELGD}_n^2 \cdot \mathrm{PD}_n(1-\mathrm{PD}_n)},$$

where we used that for Bernoulli random variables D_n the variance is given by $\mathbb{V}[D_n] = \mathrm{PD}_n \cdot (1 - \mathrm{PD}_n)$.

On the portfolio level, additivity holds for the variance UL^2 if the default indicator variables of the obligors in the portfolio are pairwise uncorrelated; due to Bienaymé's Theorem. If they are correlated, additivity is lost. Unfortunately this is the standard case and leads to the important topic of correlation modeling with which we will deal later on. In the correlated case, the unexpected loss of the total portfolio is given by

$$\mathrm{UL} = \sqrt{\mathbb{V}[L]} = \sqrt{\sum_{n=1}^{N}\sum_{k=1}^{N} \mathrm{EAD}_n \cdot \mathrm{EAD}_k \cdot \mathrm{Cov}\left[\mathrm{LGD}_n \cdot D_n; \mathrm{LGD}_k \cdot D_k\right]}$$

and, for constant loss given defaults ELGD_n, this equals

$$\mathrm{UL}^2 = \sum_{n,k=1}^{N} \mathrm{EAD}_n\,\mathrm{EAD}_k\,\mathrm{ELGD}_n\,\mathrm{ELGD}_k\,\varrho_{n,k}\sqrt{\mathrm{PD}_n(1-\mathrm{PD}_n)\,\mathrm{PD}_k(1-\mathrm{PD}_k)}$$

where $\varrho_{n,k} \equiv \mathrm{Corr}[D_n, D_k]$.

2.3 Value-at-Risk

As the probably most widely used risk measure in financial institutions we will briefly discuss *Value-at-Risk* (VaR) in this section. Here and in the next section we mainly follow the derivations in [103], pp. 37-48, to which we also refer for more details.

Value-at-Risk describes the maximally possible loss which is not exceeded in a given time period with a given high probability, the so-called confidence level. A formal definition is the following.[1]

Definition 2.3.1 (Value-at-Risk)
Given some confidence level $q \in (0,1)$. The *Value-at-Risk* (VaR) of a portfolio with loss variable L at the confidence level q is given by the smallest number x such that the probability that L exceeds x is not larger than $(1-q)$. Formally,

$$\mathrm{VaR}_q(L) = \inf\left\{x \in \mathbb{R} : \mathbb{P}\left(L > x\right) \le 1-q\right\} = \inf\left\{x \in \mathbb{R} : F_L(x) \ge q\right\}.$$

Here $F_L(x) = \mathbb{P}(L \le x)$ is the distribution function of the loss variable.

Thus, VaR is simply a quantile of the loss distribution. In general, VaR can be derived for different holding periods and different confidence levels. In credit risk management, however, the holding period is typically one year and typical values for q are 95% or 99%. Today higher values for q are more

[1] Compare [103], Definition 2.10.

and more common. The confidence level q in the Second Basel Accord is e.g. 99.9% whereas in practice a lot of banks even use a 99.98% confidence level. The reason for these high values for q is that banks want to demonstrate external rating agencies a solvency level that corresponds at least to the achieved rating class. A higher confidence level (as well as a longer holding period) leads to a higher VaR.

We often use the alternative notation $\alpha_q(L) := \mathrm{VaR}_q(L)$. If the distribution function F of the loss variable is continuous and strictly increasing, we simply have $\alpha_q(L) = F^{-1}(q)$, where F^{-1} is the ordinary inverse of F.

Example 2.3.2 Suppose the loss variable L is normally distributed with mean μ and variance σ^2. Fix some confidence level $q \in (0,1)$. Then

$$\mathrm{VaR}_q(L) = \mu + \sigma\Phi^{-1}(q)$$

where Φ denotes the standard normal distribution function and $\Phi^{-1}(q)$ the q^{th} quantile of Φ. To prove this, we only have to show that $F_L(\mathrm{VaR}_q(L)) = q$ since F_L is strictly increasing. An easy computation shows the desired property

$$\mathbb{P}(L \le \mathrm{VaR}_q(L)) = \mathbb{P}\left(\frac{L-\mu}{\sigma} \le \Phi^{-1}(q)\right) = \Phi\left(\Phi^{-1}(q)\right) = q.$$

Proposition 2.3.3 *For a deterministic monotonically decreasing function $g(x)$ and a standard normal random variable X the following relation holds*

$$\alpha_q(g(X)) = g(\alpha_{1-q}(X)) = g(\Phi^{-1}(1-q)).$$

Proof. Indeed, we have

$$\begin{aligned}
\alpha_q(g(X)) &= \inf\{x \in \mathbb{R} : \mathbb{P}(g(X) \ge x) \le 1-q\} \\
&= \inf\left\{x \in \mathbb{R} : \mathbb{P}(X \le g^{-1}(x)) \le 1-q\right\} \\
&= \inf\left\{x \in \mathbb{R} : \Phi(g^{-1}(x)) \le 1-q\right\} \\
&= g(\Phi^{-1}(1-q)).
\end{aligned}$$

which proves the assertion. □

By its definition, however, VaR gives no information about the severity of losses which occur with a probability less than $1-q$. If the loss distribution is heavy tailed, this can be quite problematic. This is a major drawback of the concept as a risk measure and also the main intention behind the innovation of the alternative risk measure *Expected Shortfall* (ES) which we will present in the next section. Moreover, VaR is not a *coherent* risk measure since it is not subadditive (see [7], [8]). Non-subadditivity means that, if we have two loss distributions F_{L_1} and F_{L_2} for two portfolios and if we denote the overall loss distribution of the merged portfolio $L = L_1 + L_2$ by F_L, then we do not necessarily have that $\alpha_q(F_L) \le \alpha_q(F_{L_1}) + \alpha_q(F_{L_2})$. Hence, the VaR of the merged portfolio is not necessarily bounded above by the sum of the VaRs of the individual portfolios which contradicts the intuition of diversification benefits associated with merging portfolios.

2.4 Expected Shortfall

Expected Shortfall (ES) is closely related to VaR. Instead of using a fixed confidence level, as in the concept of VaR, one averages VaR over all confidence levels $u \geq q$ for some $q \in (0,1)$. Thus, the tail behavior of the loss distribution is taken into account. Formally, we define ES as follows.[2]

Definition 2.4.1 (Expected Shortfall)
For a loss L with $\mathbb{E}[|L|] < \infty$ and distribution function F_L, the *Expected Shortfall* (ES) at confidence level $q \in (0,1)$ is defined as

$$\mathrm{ES}_q = \frac{1}{1-q} \int_q^1 \mathrm{VaR}_u(L) du.$$

By this definition it is obvious that $\mathrm{ES}_q \geq \mathrm{VaR}_q$. If the loss variable is integrable with continuous distribution function, the following Lemma holds.

Lemma 2.4.2 *For integrable loss variable L with continuous distribution function F_L and any $q \in (0,1)$, we have*

$$\mathrm{ES}_q = \frac{\mathbb{E}\left[L; L \geq \mathrm{VaR}_q(L)\right]}{1-q} = \mathbb{E}\left[L | L \geq \mathrm{VaR}_q(L)\right],$$

where we have used the notation $\mathbb{E}[X; A] \equiv \mathbb{E}[X \mathbf{1}_A]$ for a generic integrable random variable X and a generic set $A \in \mathcal{F}$.

For the proof see [103], page 45. Hence, in this situation expected shortfall can be interpreted as the expected loss that is incurred in the event that VaR is exceeded. In the discontinuous case, a more complicated formula holds

$$\mathrm{ES}_q = \frac{1}{1-q} \left(\mathbb{E}\left[L; L \geq \mathrm{VaR}_q(L)\right] + \mathrm{VaR}_q(L) \cdot (1 - q - \mathbb{P}(L \geq \mathrm{VaR}_q(L))) \right).$$

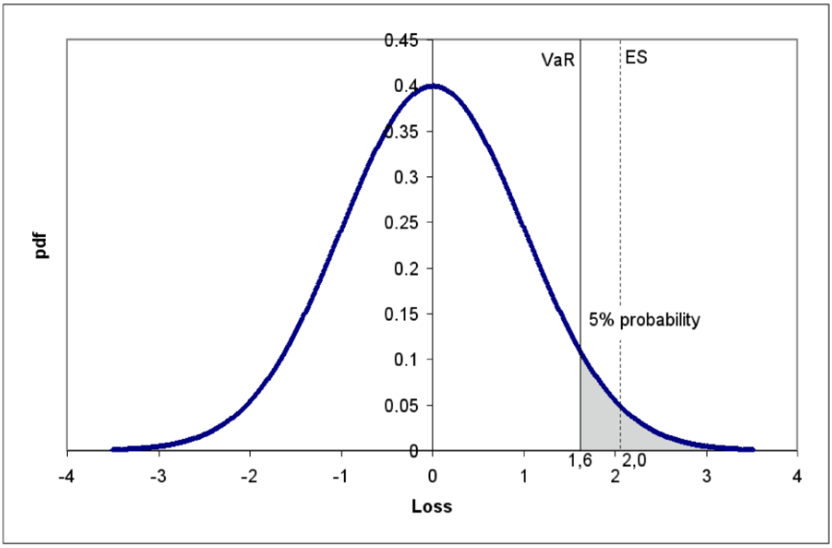
For a proof see Proposition 3.2 of [1].

Example 2.4.3 Suppose the loss distribution F_L is normal with mean μ and variance σ^2. Fix a confidence level $q \in (0,1)$. Then

$$\mathrm{ES}_q = \mu + \sigma \frac{\phi(\Phi^{-1}(q))}{1-q},$$

[2] Compare [103], Definition 2.15.

Fig. 2.1. VaR and ES for standard normal distribution



where ϕ is the density of the standard normal distribution. For the proof, note that

$$\mathrm{ES}_q = \mu + \sigma \mathbb{E}\left[\frac{L-\mu}{\sigma}\middle|\frac{L-\mu}{\sigma} \geq \alpha_q\left(\frac{L-\mu}{\sigma}\right)\right].$$

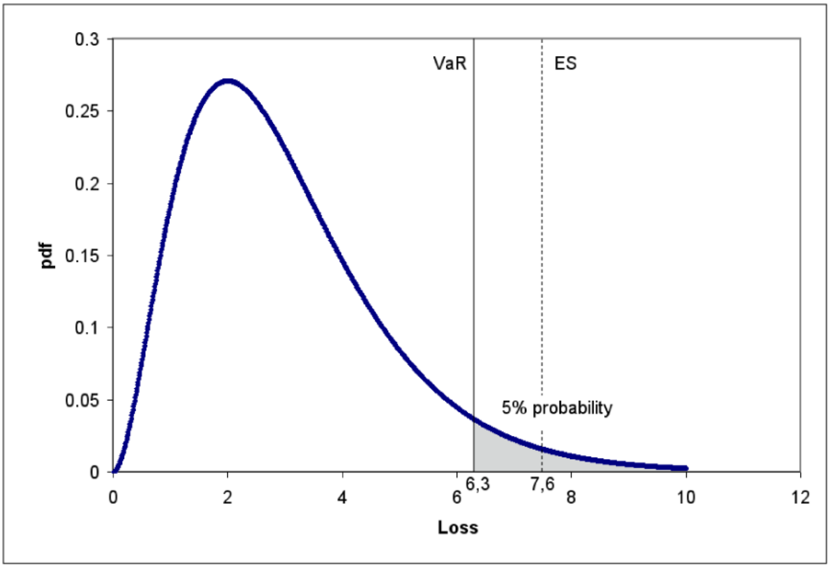
Hence it is sufficient to compute the expected shortfall for the standard normal random variable $\tilde{L} := (L-\mu)/\sigma$. Here we obtain

$$\mathrm{ES}_q(\tilde{L}) = \frac{1}{1-q}\int_{\Phi^{-1}(q)}^{\infty} l\phi(l)dl = \frac{1}{1-q}\left[-\phi(l)\right]_{l=\Phi^{-1}(q)}^{\infty} = \frac{\phi(\Phi^{-1}(q))}{1-q}.$$

Figure 2.1 shows the probability density function of a standard normal random variable. The solid vertical line shows the Value-at-Risk at level 95% which equals 1.6, while the dashed vertical line indicates the Expected Shortfall at level 95% which is equal to 2.0. Hence, the grey area under the distribution function is the amount which will be lost with 5% probability.

For an example to demonstrate the sensitivity to the severity of losses exceeding VaR and its importance see [103], Example 2.2.1, pp. 46–47. In particular for heavy-tailed distributions, the difference between ES and VaR is more pronounced than for normal distributions. Figure 2.2 shows the probability density function of a $\Gamma(3,1)$ distributed random variable with vertical lines at its 95% Value-at-Risk and Expected Shortfall. The grey area under the distribution function is the portion which is lost with 5% probability. In this case, the Value-at-Risk at level 95% equals 6.3 while the Expected Short-

Fig. 2.2. VaR and ES for Gamma distribution



fall at level 95% for the $\Gamma(3, 1)$ distribution equals 7.6.

Figures 2.1 and 2.2 also show that the ES for a distribution is always higher than the Value-at-Risk, a result we already derived theoretically in the above discussion.

2.5 Economic Capital

Since there is a significant likelihood that losses will exceed the portfolio's EL by more than one standard deviation of the portfolio loss, holding the UL of a portfolio as a risk capital for cases of financial distress might not be appropriate. The concept of *economic capital* (EC) is a widely used approach for bank internal credit risk models.

Definition 2.5.1 (Economic Capital)
The *economic capital* EC_q for a given confidence level q is defined as the Value-at-Risk $\alpha_q(L)$ at level q of the portfolio loss L minus the expected loss EL of the portfolio,

$$\mathrm{EC}_q = \alpha_q(L) - \mathrm{EL}\,.$$

For a confidence level $q = 99.98\%$, the EC_q can be interpreted as the (on average) appropriate capital to cover unexpected losses in $9,998$ out of $10,000$ years, where a time horizon of one year is assumed. Hence it represents the capital, a bank should reserve to limit the probability of default to a given confidence level. The VaR is reduced by the EL due to the common decomposition of total risk capital, that is VaR, into a part covering expected losses and a part reserved for unexpected losses.

Suppose a bank wants to include a new loan in its portfolio and, thus, has to adopt its risk measurement. While the EL is independent from the composition of the reference portfolio, the EC strongly depends on the composition of the portfolio in which the new loan will be included. The EC charge for a new loan of an already well diversified portfolio, for example, might be much lower than the EC charge of the same loan when included in a portfolio where the new loan induces some concentration risk. For this reason the EL charges are said to be *portfolio independent,* while the EC charges are *portfolio dependent* which makes the calculation of the contributory EC much more complicated, since the EC always has to be computed based on the decomposition of the complete reference portfolio.

In the worst case, a bank could lose its entire credit portfolio in a given year. Holding capital against such an unlikely event is economically inefficient. As banks want to spend most of their capital for profitable investments, there is a strong incentive to minimize the capital a bank holds. Hence the problem of risk management in a financial institution is to find the balance between holding enough capital to be able to meet all debt obligations also in times of financial distress, on the one hand, and minimizing economic capital to make profits, on the other hand.

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