

## Preface

Peter Fishburn has had a splendidly productive career that led to path-breaking contributions in a remarkable variety of areas of research. His contributions have been published in a vast literature, ranging through journals of social choice and welfare, decision theory, operations research, economic theory, political science, mathematical psychology, and discrete mathematics. This work was done both on an individual basis and with a very long list of coauthors.

The contributions that Fishburn made can roughly be divided into three major topical areas, and contributions to each of these areas are identified by sections of this monograph. Section 1 deals with topics that are included in the general areas of utility, preference, individual choice, subjective probability, and measurement theory. Section 2 covers social choice theory, voting models, and social welfare. Section 3 deals with more purely mathematical topics that are related to combinatorics, graph theory, and ordered sets. The common theme of Fishburn's contributions to all of these areas is his ability to bring rigorous mathematical analysis to bear on a wide range of difficult problems.

Part 1 covers a variety of topics stemming from several of Fishburn's books: *Decision and Value Theory* [Fishburn (1964)], *Utility Theory for Decision Making* [Fishburn (1970)], *Mathematics of Decision Theory* [Fishburn (1973a)], *The Foundations of Expected Utility Theory* [Fishburn (1982)], and *Nonlinear Preference and Utility Theory* [Fishburn (1988)]. Fishburn has made cutting-edge contributions to the theory of utility, including work on nontransitive preference, stochastic utility, and decision theory, broadly speaking. He has contributed greatly to the theory of expected utility, including important work on axioms for expected utility, the study of multiattribute expected utility, behavioral models of risk taking, and the study of dominance relations, as well as fundamental contributions to the understanding of subjective expected utility. He has also contributed to nonlinear utility theory, with contributions dealing with risk and with uncertainty. Fishburn's work on choice has dealt with choice probability, choice functions, and nonprobabilistic preference and utility. His work on measurement theory has concentrated on uniqueness of representations, as well as on additive and on nondecomposable representations.

The contributions in Part 1 reflect different facets of the aforementioned research. They start with three papers that are related to the general topic of utility theory. Luce, Marley and Ng (Entropy-Related Measures of the Utility of Gambling) develop utility models to explain individual behavior in gambling situations by adding an additional ‘entropy’ term to the individual’s utility function to account for an individual’s preference, or aversion, for gambling in specific situations. Bell and Keeney (Altruistic Utility Functions for Joint Decisions) consider situations in which two or more individuals are involved in selecting some alternative in a decision making situation. The specific situation that is considered describes scenarios in which each of the decision makers has an underlying interest in selecting an alternative that will please the other decision makers. Nakamura (SSB Preferences: Nonseparable Utilities or Nonseparable Beliefs) extends aspects of Fishburn’s Skew-symmetric Bilinear Utility model to the case of decision making under uncertainty.

Four contributions are on the general topic of decision theory. The paper by Jia and Dyer (Decision Making Based on Risk-Value Tradeoffs) starts this section with a survey of risk-value decision models that have been developed in the last decade. This paper merges two streams of research, the modeling of individual preference and the modeling of risk judgment, in an effort to develop a more descriptively powerful risk-value model. Bodurtha and Shen (Normally Distributed Admissible Choices are Optimal) consider one particular aspect of risk-value models by examining mean-variance analysis to determine the characteristics of optimal solutions for decisions involving both mutually exclusive investments and financial portfolios of investments with normally distributed returns. Their analysis shows that these optimal solutions meet the conditions that are described by Fishburn’s definitions of stochastic dominance of admissible choices. Bouyssou, Marchant and Pirlot (A Conjoint Measurement Approach to the Discrete Suengo Integral) extend Fishburn’s work on subjective expected utility in multiple criteria decision making by showing conditions under which a noncompensatory multiple-criteria decision-making model is equivalent to a model that is based on the discrete Suengo integral. Slinko (Additive Representability of Finite Measurement Structures) presents a survey of recent developments that are related to Fishburn’s work on the additive representation of finite measurement structures, work relating closely to the classical measurement-theoretic topic of additive conjoint measurement. The paper highlights the remaining open problems that Fishburn formulated in this area.

Part 2 mirrors Fishburn’s interest in voting and social-choice theory that he developed in two major books: *The Theory of Social Choice* [Fishburn (1973b)] and *Approval Voting* [Brams and Fishburn (1983)]. He has made pioneering contributions to the understanding of social choice functions, which includes work on anonymity conditions, paradoxes of preferential voting, and Borda’s rule and Condorcet’s principle. His research on scoring-rule sensitivity and scoring vectors is also significant. Fishburn has been a leader in developing and analyzing new voting rules, with his analysis of approval voting being an important case in point. His contributions to the comparison of voting methods are also noteworthy, including work on two-stage voting systems, single transferable vote, and positional voting rules. He has

also undertaken important studies of majority choice, including finding conditions on preferences that guarantee a simple majority winner and a location theorem for single-peaked preferences. He has also studied Condorcet proportions and probabilities, social-choice lotteries, and impossibility theorems. Finally, his work includes beautiful results on social welfare and equity, including equity axioms for public risks and fair-cost allocation schemes.

Four contributions in this part are connected to Fishburn's work on identifying conditions that require the existence of a Condorcet winner in an election and the associated probabilities of observing events in election outcomes. Monjardet (*Acyclic Domains of Linear Orders*) presents a survey of work that has focused on the identification of domains of voters' preferences that require an 'acyclic set' or transitive majority rule relationship; he develops intriguing connections between some of these domains. Saari (*Condorcet Domains: A Geometric Perspective*) addresses the same topic, taking a geometric approach that offers intuitive insight to the problem. Gehrlein (*Condorcet's Paradox with Three Candidates*) analyzes the probability that a Condorcet winner will exist and shows that this probability is quite large for a small number of candidates when voters have preferences that are at all close to being mutually coherent (according to any of several possible measures of mutual coherence in group preferences). Feix, Lepelley, Merlin, and Rouet (*On the Probability to Act in the European Union*) extend some of Fishburn's work on probabilities of election outcomes to analyze the probability that the voting rules used by the European Union will produce deadlock.

Two contributions consider properties of voting rules. Brams and Sanver (*Voting Systems that Combine Approval and Preference*) provide an extensive analysis of two hybrid voting systems that combine approval voting with voting procedures that require either a complete ranking of candidates or a partial ranking of only the candidates in the approved subset. Zwicker (*Anonymous Voting with Abstention: Weighted Voting*) considers an extension of the standard case of yes-no legislative voting in which abstention is viewed as being a voter preference position somewhere between a yes and a no vote. Characterizations are provided in which a specified set of weighted scores are linked to voter responses of yes, abstain or no.

Two contributions address the general topic of social choice. Campbell and Kelly (*Social Welfare Functions that Satisfy Pareto, Anonymity and Neutrality, but not IIA: Countably Many Alternatives*) extend earlier work that showed that in the presence of the conditions of Pareto, non-dictatorship, full domain, and transitivity, an extremely weak independence condition is incompatible with anonymity and neutrality for a finite number of alternatives; here they consider the case of countably many alternatives.

Hopkins and Jones (*Bruhat Orders and the Sequential Selection of Indivisible Items*) extend some of Fishburn's work on fair division by considering the case in which two players sequentially make selections from a set of indivisible items. Necessary and sufficient conditions are found under which players receive their most preferred and least preferred outcomes.

Part 3 explores fundamental mathematical constructs that arise in the more applied work, described in Parts 1 and 2, through the study of binary relations, partial

orders, graphs and networks, combinatorics, number theory, linear programming, inequalities, and coding theory. Fishburn's partial order work includes foundational introductions to the theory of partial-order dimension, linear extensions of partial orders, the FKG property, and so on, as well as research on geometric partial orders such as angle orders and circle orders. Interval orders and semiorders are important special classes of partial orders that arise in problems in economics, psychology, biology, scheduling, and so on. Fishburn has made both theoretical and applied contributions to the understanding of such orders, highlighted in his book, *Interval Graphs and Interval Orders* [Fishburn (1985)].

Graph theory topics are the subject of a wide variety of Fishburn's papers. His research in that area includes important contributions to such topics as niche overlap graphs (arising in ecology), tolerance graphs (arising in psychology and operations research), and  $L(2,1)$ -colorings (arising in communications) as well as the design of various kinds of communication and other networks. Combinatorial geometry involves the study of various configurations, and Fishburn's work here has included the study of convex  $n$ -gons, planar sets, partial set covering, and a wide variety of related topics.

In addition, coding problems often can be analyzed using combinatorial and related algebraic methods. Fishburn's contributions to a variety of coding problems have included important results on sequence-based methods for data transmission and source compression, binary convolutional codes, and related lattice concepts.

Much of Fishburn's work involves counting, enumeration, and asymptotic behavior of structures, including posets and graphs, but also sequences arising in number theory, solutions to inequalities, and types of geometries. This work falls at the interface among combinatorics, probability, number theory, and a number of other sub-disciplines and often intersects ideas of convexity, linear programming, and so on.

Seven contributions tie into Fishburn's work on posets, graphs, and networks. There are two different representations for interval orders and semiorders. The basic definitions of interval orders and semiorders both relate a poset to a set of intervals on the number line. A second representation describes interval orders as the subset of posets that do not include a  $2 + 2$  configuration; it describes semiorders as the subset of interval orders that do not include a  $3 + 1$  configuration. Shuchat, Shull and Trenk (Fractional Weak Discrepancy of Posets and Certain Forbidden Configurations) find the range of possible values of fractional weak discrepancy for the subset of posets that contain a  $3 + 1$  but no  $2 + 2$ . Isaak (Interval Order Representation via Shortest Paths) develops an alternative proof of the second representation for interval orders and semiorders by showing that they are special instances of existence results that are related to the measure of potentials in digraphs. Brown and Langley (Probe Interval Orders) investigate probe interval graphs that arise in molecular biology and are obtained with a variation of the model by which interval orders are determined by intervals on the number line; they also consider restrictions that must be placed on these intervals such that the resulting probe interval graph is a probe interval order. Falmagne and Ovchinnikov (Mediatic Graphs) discuss the concept of mediatic graphs that trace their study to "stochastic token theory" in mathematical psychology. They show that the sets of all interval orders and semiorders on a finite

set each can be represented as mediatic graphs. Poljak and Roberts (An Application of Stahl's Conjecture About the  $k$ -tuple Chromatic Numbers of Kneser Graphs) analyze the chromatic number in graph coloring problems and apply known results about Stahl's Conjecture to answer two open questions about the relation between the  $n$ -tuple chromatic number of a graph and  $n$  times the size of the largest clique in the graph. Hwang and Dou (Optimal Reservation-Scheme Routing for Two-Rate Wide-sense Nonblocking Three-Stage Clos Networks) study interconnection networks that are widely used in data communications and parallel computing. In particular, they are interested in using these networks for different media to communicate. By using reservation-scheme routing, they show that such networks can require much less hardware. Sahi (The Harris Inequality for Partially Ordered Algebras) deals with inequalities concerning increasing functions on a distributive lattice. Partially ordered algebras are associative algebras over the reals with a nonempty subset closed under addition, multiplication and multiplication by positive real numbers. In special cases, the results relate to the Harris inequality that arises in percolation on random graphs and to the more general FKG inequality, both topics on which Fishburn has made important contributions.

Three papers tie in to a variety of issues at the intersection among combinatorics, probability, number theory, and linear programming. Lagarias, Rains and Vanderbei (The Kruskal Count) analyze a well-known (at least among mathematicians) card trick that relies on the high likelihood that two processes with different starts (one chosen by the subject, one by the magician) will converge before the deck runs out, enabling the latter to appear clairvoyant. The trick is modeled by a Markov chain; two different value distributions (geometric and uniform) are studied, the second for the first time; and then the results are compared to MC simulations of a real deck. Applegate, LeBrun, and Sloane (Descending Dungeons and Iterated Base-Changing) study the special sequences where each term arises from interpreting the previous term in a different base. These iterated base changes (dungeons) are distinguished from iterated exponentiation (or towers). They prove a theorem about the asymptotic value of the  $n$ th term in such a sequence. Shepp (Updating Hardy, Littlewood and Polya with Linear Programming) discusses ideas dating back to the famous 1934 book, *Inequalities*, by the authors named in his title. He studies inequalities that can be proven using linear programming or convexity arguments.

No tribute to Peter Fishburn would be complete without saying something about him as a person. The three of us have collaborated with him over many years on a variety of topics. Peter is not only conscientious and responsible to a fault, but he is also a delight to work with, always doing more than his fair share quickly and efficiently. We marvel at his ability to come up with new ideas, develop extensions of old ones, and demonstrate linkages—all cheerfully, with no fuss and bother. We have great admiration for this brilliant scholar, and we take enormous pleasure in having had the opportunity to work with him on so many exciting projects and to interact with him as a colleague and a friend.

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