
Preface

This lecture notes in physics volume mainly focuses on the semi classical and quantum aspects of percolation and breakdown in disordered, composite or granular systems. The main reason for this undertaking has been the fact that, of late, there have been a lot of (theoretical) work on quantum percolation, but there is not even a (single) published review on the topic (and, of course, no book). Also, there are many theoretical and experimental studies on the nonlinear current-voltage characteristics both away from, as well as one approaches, an electrical breakdown in composite materials. Some of the results are quite intriguing and may broadly be explained utilising a semi classical (if not, fully quantum mechanical) tunnelling between micron or nano-sized metallic islands dispersed separated by thin insulating layers, or in other words, between the dangling ends of small percolation clusters. There have also been several (theoretical) studies of Zener breakdown in Mott or Anderson insulators. Again, there is no review available, connecting them in any coherent fashion. A compendium volume connecting these experimental and theoretical studies should be unique and very timely, and hence this volume.

The book is organised as follows. For completeness, we have started with a short and concise introduction on classical percolation. In the first chapter, D. Stauffer reviews the scaling theory of classical percolation emphasizing (biased) diffusion, without any quantum effects.

The next chapter by A.K. Sen deals with the physics between the classical and the quantum effects. It gives a detailed, pedagogic description of the genesis and development of a semi classical (or, semi quantum) percolation model as a driven network, named as a Random Resistor cum Tunnelling-bond Network, or RRTN in short. The chapter deals with both the semi classical percolation transition and its breakdown characteristics. On the way, it covers nonlinear dc and ac response, low-temperature variable range hopping (VRH) conductance and some dynamical aspects such as two early-stage power-law relaxations, predictability of the breakthrough (breakdown process starting at one end and crossing to the other) time in terms of a relaxation time, etc. Each of the topics covered has been introduced with a variety of observations and the theoretical results have been contrasted with experiments. It has been emphasized that a large variety of *non-electrical* phenomena of Nature,

where the ‘tunnelling’ response may be replaced by some other *non-linear response with threshold* (e.g., capillary forces of fluids in porous media, frictional forces in mechanical systems, viscous drags in some fluids, forces due to random ‘pinning’ of vortices in sliding charge density wave type of systems, etc.), may be modelled using the RRTN or its variant.

A. Mookerjee, T. Saha-Dasgupta and I. Dasgupta start out the Chapter 3 with a concise review of the models and techniques to study quantum percolation and conclude from numerical studies that it belongs to the same universality class as the single electron (i.e. no e-e interaction) Anderson (impurity) model. Their numerical approach, using vector recursion method alongwith finite size scaling analysis, can not resolve the controversy whether quantum percolation exists in two dimensions, and seems to indicate that there is not localised to extended states transition (i.e. no q-percolation) in 2D. More precisely, they did not find any fool-proof answer to the question of whether, even for a very weak (non-zero) disorder in 2D, all the states are exponentially localised or some power-law-localised states exist in the vicinity of the band centre.

The chapter by H. Nakanishi and M.F. Islam describes the subject of quantum percolation in two dimensions, a topic still controversial with decades of studies regarding the nature of localisation and presence or absence of phase transitions in its transmission behaviour. They show that the results of some recent numerical studies of the transmission coefficient of a hopping Hamiltonian are most consistent with an interpretation that (a) there are two different localisation regimes, exponentially localised and algebraically localised, (b) there is a delocalised regime, and (c) characteristic power-laws in the algebraically localised regime vary with dilution and energy.

The chapter by G. Schubert and H. Fehske presents a large-scale numerical study of localisation effects in 2D and 3D quantum site percolation. Combining exact diagonalisation, Chebyshev expansion and local distribution approaches, the authors analyse the mean and typical densities of states, but also dynamical properties such as the time evolution and recurrence probability of a quantum particle on the spanning cluster. The results throw a new light on the existence of a quantum percolation threshold and may have implications for percolative transport scenarios in novel materials.

The next chapter by C. Sohrmann, J. Oswald and R.A. Römer, addresses the issue of quantum percolation in the presence of perpendicular electric and magnetic fields, especially when the system is in its (well-known) integral quantum Hall effect (IQHE) regime. The authors introduce the topics in a lucid, pedagogical style using single particle (electronic) wave functions and quantised Landau levels in a magnetic field. Then they review some of the most prominent network models for the IQHE. Screening effects due to the e-e exchange interaction are considered at the Hartree-Fock (HF) level. The IQHE phenomenon is finally described in terms of quantum transport (or, percolation) across the ensuing effective potentials.

P. Majumdar concisely reviews next the percolative quantum effects due to the coexistence of competing phases in the manganites. A summary of the key experiments in this area throwing light on spatial clustering and transport is followed by a

description of the two microscopic models carrying the essence of *correlated* quantum percolation involving both ‘site’ and ‘bond’ randomness. The distribution function involves the iterative solution of a Schrodinger equation. The microscopic results are compared with predictions from phenomenological resistor network theory and experimental results.

The chapter by D. Samanta, B.K. Chakrabarti and P. Ray reviews first the classical breakdown properties and their statistics for random fuse-conductor networks and random dielectric-conductor networks. The results are for both the dilute limit as well as near the percolation point and in lattice as well as continuum systems. It discusses next the Zener Breakdown problem in quantum percolation or in Anderson insulators (in particular in three dimension).

A microscopic quantum theory of nonequilibrium (open q-system) insulator to metal transition (quantum dielectric breakdown) in the presence of *strong e-e interaction* (correlation) is discussed in the next chapter by T. Oka and H. Aoki. The equilibrium version (closed q-system) of this is called the Mott transition. In the open q-system, the ground state is separated from the first excited state by the Mott gap, and *quantum breakdown* occurs through non-adiabatic, *Landau-Zener tunnelling* transitions, with *concomitant nonlinear transport*. In retrospect, it is interesting to note how some of these important elements had already been incorporated (at a minimal level) in some phenomenological models, for example, the semi-quantum RRTN model (see Asok K. Sen, *Nonlinear Response, Semi-classical Percolation and Breakdown in the RRTN Model*). Oka and Aoki next discuss the preservation of quantum coherence at sufficiently low temperatures, giving rise to dynamical localisation that saturates the creation process (of excitation pairs) and eventually leads to a nonequilibrium stable state.

The last chapter by K. Kieling and J. Eisert on *quantum computation and communication* is the sole representative of a very important and fast growing field of research. Here, one encounters a ‘percolative type situation’ originating from the statistical nature of quantum measurement or quantum state preparation processes. So, at a mental level, one is dealing with *engineered quantum systems* and the disorder is due to probabilistic *quantum gates*, or to Mott defects arising from the preparation of a cluster state. As such, the authors point out that the percolative disorder here is *statistically correlated* (not fully random). This general theme of correlated percolation is also of essence in the topics of two previous chapters: one on the RRTN model and the other on colossal magneto resistance in manganites. In the present chapter, methods of renormalisation are useful to overcome the probabilistic aspects of manipulating various quantum states. As the authors emphasize, one of the most important issues is to understand how far renormalisation over mixed quantum states could be of use to develop fault-tolerance, error correcting codes etc. for the use of quantum computation.

It gives us great pleasure to acknowledge all the contributors for their sincere support and patience while interacting with us on their individual chapters. Without the enthusiasm towards keeping their contributions up-to-date, this lecture notes volume would not be what it is today. We expect that all these combined bits of informations contained within its pages would have some usefulness to the relevant research

communities alongwith its overlapping areas, and to the newcomers who would like to be introduced to the topics covered here. Our efforts would be worthwhile and we would certainly hope so, if this book gives rise to further research in these and related areas. Finally, we must acknowledge the editors of Springer for their warm acceptance of this book project when it was proposed last year, and very helpful support throughout the process of publication.

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