
Contents

1	Introduction	1
----------	---------------------------	----------

Part I The Solutions of the Dirac Equation in Hydrogenic Atoms

2	The Electromagnetic Fields Created by Time-Sinusoidal Current	5
2.1	Properties of the Electromagnetic Field Emitted by an Electron Bound in an Atom	5
2.2	The Field at Large Distance of a Time-Periodic Current	6
2.3	Source Currents of Time-Sinusoidal Polarized Field	7
2.3.1	Linear Polarization: $\epsilon = 0$	7
2.3.2	Circular Polarizations: $\epsilon = \pm 1$	7
2.4	Flux of the Poynting Vector Through a Sphere of Large Radius	8
2.5	Units	9
3	The Dirac Equation of the Electron in the Real Formalism	11
3.1	Algebraic Preliminaries: A Choice of Formalism	11
3.1.1	Quaternions and Biquaternions	13
3.1.2	The Hamilton Quaternion and the Pauli Spinor	15
3.1.3	The Hestenes Spinor and the Dirac Spinor	16
3.2	The Hestenes Real Form of the Dirac Equation	18
3.3	The Dirac Equation in Real Biquaternion	18
3.4	Notations	18
4	The Solutions of the Dirac Equation for the Central Potential in the Real Formalism	19
4.1	General Approach	19

VIII Contents

4.2	The Biquaternionic Form of the Solutions in Spherical Coordinates	20
4.2.1	A Biquaternionic System	20
4.2.2	The Fundamental Quaternionic Equation	21
4.2.3	The Radial Differential System	21
4.2.4	A General Biquaternionic Solution	22
4.2.5	The Dirac Probability Current and the Conditions of Normalization	23
4.3	The Solution of the Quaternionic Equation	24
4.3.1	The Differential System Implying the Angle Theta	24
4.3.2	Properties of the Solutions of Equation $(\mathbf{r} \wedge \nabla)S = \lambda S$	24
4.3.3	Expression of the Solutions by Means of the Legendre Polynomials	25
4.3.4	Expression of the Solutions by Means of a Recursion Formula	26
4.4	Solutions of the Radial Differential System for the Discrete Spectrum	27
4.4.1	Solutions of the System	27
4.4.2	The Levels of Energy for the Discrete Spectrum	29
4.4.3	Case of the States $1S_{1/2}$, $2P_{1/2}$, and $2P_{3/2}$	29
4.4.4	Note: The Gamma and the Confluent Hypergeometric Functions	30
4.5	Solutions in the Pauli Approximation and for the Schrödinger Equation	31
4.5.1	The Pauli Approximation	31
4.5.2	Solution of the Schrödinger Equation	32
4.5.3	Case of the States $s_{1/2}$, $p_{1/2}$, and $p_{3/2}$	33

Part II Fields Created by the Dirac Transition Currents Between Two States

5	The Dirac Transition Currents Between Two States	37
5.1	Assumptions on the Source Current and the Release of Energy	37
5.1.1	Assumptions on the Source Current	37
5.1.2	Assumptions on the Release of Energy	38
5.2	The Transition Current Between Two States	38
6	The Field at Large Distance Created by the Transition Currents	41
6.1	Polarization of the Emitted Light	41
6.2	The Forbidden and Allowed Transitions	42
6.3	Linear Polarization	42

6.4	Circular Polarizations	43
6.5	Sum Rules for the Intensities of the Emitted Light	45
7	Case of the Transitions $P_{1/2}$-$S_{1/2}$ and $P_{3/2}$-$S_{1/2}$	47
7.1	General Formulas	47
7.2	The Pauli Approximation and the Schrödinger Theory	47
7.3	Spontaneous Emission	49
7.3.1	The Energy Balance	49
7.3.2	Spontaneous Emission in the Transitions $2P_{1/2} - 1S_{1/2}$ and $2P_{3/2} - 1S_{1/2}$ for the Hydrogen Atom	49

Part III Interaction with Radiation

8	Interaction with an Incident Wave: The Retardation	53
8.1	Matrix Element of a Transition	53
8.2	The Retardation and the Dipole Approximation	55
9	Relativistic Expression of the Matrix Elements	57
9.1	Geometrical Construction of the Vectors $\mathbf{T}_j^\perp(\mathbf{k})$	57
9.1.1	Integration in the Frame $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$	58
9.1.2	Necessity of the Integration in the Frame (I, J, K)	58
9.2	Case of the Transitions $S_{1/2}$ - $P_{1/2}$ and $S_{1/2}$ - $P_{3/2}$	59
9.2.1	Expression of the Vectors $\mathbf{T}_j^\perp(\mathbf{k})$	59
9.2.2	The Relativistic Retardation and the Sum Rules	61
9.2.3	Case of the Transitions $1S_{1/2}$ - $2P_{1/2}$ and $1S_{1/2}$ - $2P_{3/2}$	62
9.2.4	Comparison with the Dipole Approximation	62

Part IV The Photoeffect

10	The Radial Functions of the Continuum	67
10.1	Solution of the Radial System	67
10.1.1	General Form of the Solution	67
10.1.2	A Choice of Variable	68
10.1.3	Normalization on the Energy Scale	69
10.2	The Different Approximations of the Radial Functions	71
10.2.1	The Approximation $Z^2\alpha^2 \ll \kappa^2$	71
10.2.2	The Approximation $Z^2\alpha^2 \ll n^2$ or Pauli-Schrödinger Approximation	71
10.2.3	The Schrödinger Approximation	72
10.2.4	Interest and Validity of the Approximations	72

11	Matrix Elements for the Transitions $1S_{1/2}$-Continuum	73
11.1	The transitions $1S_{1/2}$ -Continuum in the Dipole and Schrödinger Approximations	73
11.2	Transitions $1S_{1/2}$ - $P_{1/2}$ in the Dipole Approximation	74
11.3	Transitions $1S_{1/2}$ - $P_{3/2}$ in the Dipole Approximation	75
11.4	Transitions $1s$ - p in the Schrödinger Theory	76
11.5	A recapitulative Verification	77
12	Matrix Elements for the Relativistic Transitions with Retardation $1S_{1/2}$-Continuum	81
12.1	General Formulas	81
12.2	Numerical Calculation of the Formulas	83
12.3	Some Numerical Results	84
12.4	Conclusion	88
13	The Radiative Recombination	89
13.1	Motivations and Definition of Cross Sections	89
13.2	Some Numerical Results	90
<hr/>		
Part V Interaction with a Magnetic Field		
<hr/>		
14	The Zeeman Effect	95
14.1	An Approximation Method for Time-Independent Perturbation	95
14.2	The Margenau Formula: The Landé Factor	96
<hr/>		
Part VI Addendum		
<hr/>		
15	The Contribution of the Discrete Spectrum to the Lamb Shift of the $1S_{1/2}$ State	103
15.1	The Lamb Shift	103
15.2	Nonrelativistic Calculation	104
15.3	Relativistic Calculation	105
15.4	Note	106
<hr/>		
Part VII Appendices		
<hr/>		
A	The Hestenes Spinor and the Pauli and Dirac Spinors	109
A.1	The Pauli Spinor as a Decomposition of the Hamilton Quaternion	109
A.2	The Dirac Spinor as a Decomposition of the Biquaternion	110

A.3	The Hestenes Spinor and the Dirac Matrices	110
A.4	Solution for the Central Potential Expressed by Means of the Dirac Spinors	111
B	The Real Formalism and the Invariant Entities	113
B.1	Properties of the Hestenes Spinor	113
B.2	The Proper Angular Momentum or Bivector Spin	113
B.3	The Energy–Momentum Vector	114
C	The Total Angular Momentum Operator	115
D	The Main Properties of the Real Clifford Algebras	117
E	The Expression of the Transition Current	121
F	Conservation of the Charge Transition Current	123
G	An Approximation Method for Time-Dependent Perturbation	125
H	Perturbation by a Plane Wave	129
	References	131
	Index	133

Relativistic Transitions in the Hydrogenic Atoms
Elementary Theory

Boudet, R.

2009, XII, 136 p., Hardcover

ISBN: 978-3-540-85549-1