What Is Coming: Issues Raised from Observation of the Shape of the Sun

J.-P. Rozelot

Université de Nice-Sophia-Antipolis, CNRS –UMR 6525, Observatoire de la Côte d'Azur (Dpt H. Fizeau), Av. Copernic, 06130 Grasse, France
Jean-Pierre.Rozelot@obs-azur.fr

Abstract Variations of the diameter, shape and irradiance are ultimately related to solar activity, but a further investigation of how a weak magnetic field might cause variations in the irradiance amplitude and phase, combined with a shrinking or an expanding shape, is still needed. Indeed, accurate measurements of the solar diameter started by Jean Picard showed that the solar diameter might be greater during the Maunder minimum of the solar activity. After Jean Picard (and some other heirs), there has been a lot of other measurements, ground based or from space. In this chapter we will review the question, extending diameter variability to shape changes. We will show how helioseismology results allow us to look at the variations below the surface, where changes are not uniform, and putting in evidence a new shallow layer, the *leptocline*. This layer is the seat of solar asphericities, radius variations with the 11 year cycle and probably also the cradle of sub-layers where act complex physical processes such as partial ionization of the light elements, opacities changes, superadiabaticity, strong gradient of rotation and pressure. We will base our discussion on physical grounds and show why it is important to get accurate measurements from space (SDO – Solar Dynamics Observatory or DynaMICCS/GOLF-NG). Such measurements will provide us a unique opportunity to study in detail changes of the global solar properties and their relationship to changes in the Sun's interior.

1 Introduction

Since the highest Antiquity the determination of the value of the solar diameter has been a subject widely debated. A number of historical books already treated this question and the topic could be considered as ended. By opening a book on astronomy, such as the *Astrophysical Quantities* [3], one may find the value

$$R_{\odot} = (6.955\,08\ \pm\ 0.000\,26) \times 10^{8}\ \ \mathrm{m}$$

which appears as the best measure up to date of the solar radius. This estimate could be even considered as "definitive" and is thus widely used. However,

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looking carefully at this question, it is not so obvious. First, an absolute value is not yet determined. Just as an example, a discussion of measurements of the solar diameter made during the nineteenth century by Wittmann [89] yields $R_\odot = 696\,265 \pm 65\,\text{km}$ (without any temporal trend), whereas measurements made by these authors at Izaña during the years 1990–2000 yield 960.63 ± 0.02 (arc second, at 1 AU), always without any significant cycle-dependence variations in excess of about 400 km [90, 91].[1] Such values are different from that adopted by Allen.[2] But giving a value of the solar diameter requires a definition, as the Sun is not a spherical solid. Several expressions can be given. The most commonly accepted is the diameter defined as the distance taken between the two opposite inflection points of the limb intensity profile, at a given wavelength. But other definitions can be used. For instance, an equipotential level of gravity (to a constant) perfectly defines the outer shape. Second, the Sun is a fluid body in rotation. It follows to first order an oblateness of the whole figure, and to other orders, deviations to sphericity. The diameter D under consideration must be thus identified. The semi-diameter R (radius, more frequently used) is referred to as equatorial, R_{eq}, or polar, R_{pol}, for which values are as follow [61]:

(a) if the Sun can be considered (for instance in stellar structure models) as a body rotating at a uniform rotation speed rate

$$R_{eq} = 6.95991756 \times 10^8 \text{ m and } R_{pol} = 6.95985961 \times 10^8 \text{ m (uniform rotation)}$$

and

(b) under a (surface) differential rotation speed rate

$$R_{eq} = 6.95991756 \times 10^8 \text{ m and } R_{pol} = 6.9598438 \times 10^8 \text{m (non-uniform rotation)}$$

At last, on a pure physical point of view, as the distribution of matter is not uniform inside the Sun (from the core to the surface), as well as the distribution of the velocity rates, the outer shape shows distortions which are linked to the successive gravitational moments. Hence, the solar radius, $R(\theta)$, must be a function of the latitude (θ). As a consequence, all layers that constitute the Sun are not spherical (Fig. 1). This has been already recognized for instance for the tachocline [30], which is prolate.

The knowledge of the value of the solar radius, once the definition is stated, is a key parameter not only in stellar physics but also in solar models. Indeed, the solar radius is a function of time. On very long-term evolution (several millenia), this has been recognized as the paradox of the faint young Sun.[3] On shorter term

[1] An attempt of solving discrepancies has been made by Habereitter et al. (ApJ., **675**, L53–L56, 2008) while this paper was in press.

[2] See also Table I in paper [41].

[3] The "faint young Sun paradox" was first pointed out by Carl Sagan and George Mullen [14]: it postulates that stars similar to the Sun should gradually brighten over their life time (excluding a very bright phase just after formation). This prediction is supported by the observation of lower brightness in young stars of solar type. It is generally acknowledged that the early Sun (the faint young Sun) had only 70% of the energy output that it has today. This would mean that the Earth would have been entirely frozen (no liquid water) in its early history, in contradiction with geological

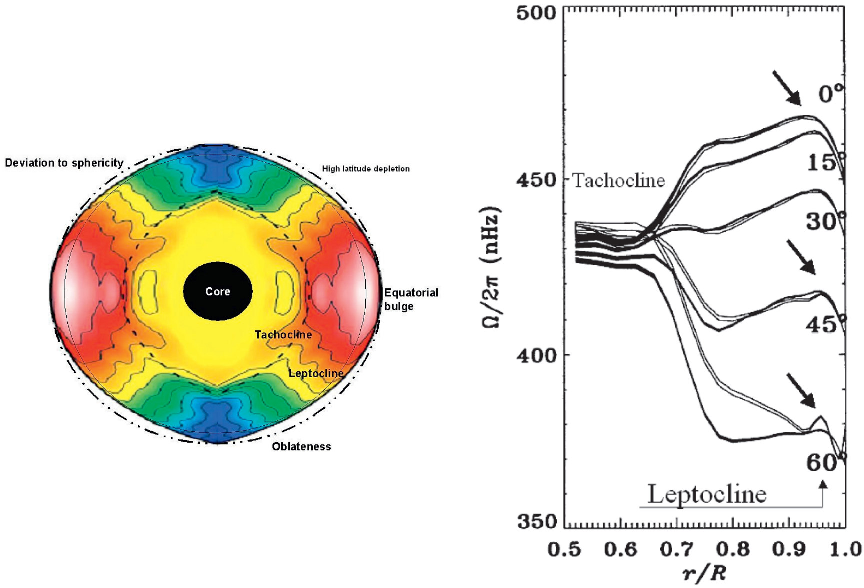

Fig. 1. *Left*: Due to the non-uniform distribution of mass and velocity rates inside the Sun (isocontours are shown), the resulting outer shape is not spherical and shows deviations to sphericity (exaggerated size here). However, the global shape remains oblate. The *inner dot line* (– –) shows the prolate *tachocline* and the *thin line* (—), the *leptocline*. *Right*: Several profiles of the rotation rate are plotted according to different heliographic latitudes. The changes indicated by an *arrow* show the seat of the *leptocline*

(since around 1600 up to now), it has been shown that the solar radius may also evolve with time (see Fig. 2 in [55], Fig. 3 in [67], or Figs. 2 and 3 in [68], all upgraded from [82]), likely on a very large periodic modulation, of about 110–120 years (from one extrema to the other one), the amplitude being not yet accurately determined.[4] On shorter periods of time (ranging over some solar cycles), the temporal variability has remained unclear for a long time, but it has been shown recently that it is in antiphase with the solar cycle for layers lying

observations of sedimentary rocks, which required the presence of flowing liquid water to form. (The case for Mars is even more extreme due to its greater distance from the Sun.) Does this paradox – between the icehouse that one would expect based on stellar evolution models and the geologic evidence for copious amounts of liquid water – indicate a problem with our stellar evolution models? Or is there another way around this conundrum?

[4] A significant 110 yr period in solar activity is not fully recognized. However Damon and Jirikovic [18, 19] identified two powerful harmonics considered as fundamental at 211.5 and 88.1 yr – the Suess and the Gleissberg cycles – which modulate the Schwabe 11 yr period and produce periods of maxima and minima in solar activity: the around 110–120 yr period could be a sub-harmonic.

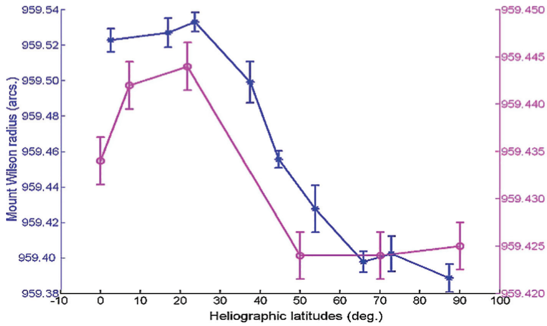

Fig. 2. Comparison between results obtained deduced from the Mount Wilson data, over 30 years of analysis (left scale) [43, 44], and those of the Pic du Midi, obtained on September, 1–4, 2001, where exceptional conditions of seeing were encountered (right scale) [72]. The observed solar limb contour does not follow an ellipsoidal shape and shows deviations to sphericity, as theory states [46]. However, the excess of asphericity found in the Mount Wilson data (120 mas around 20° of heliographic latitude with respect to 70° latitude) can be interpreted by the spectral domain; a contribution of the chromosphere can be suspected, as it was found that the chromosphere maybe oblate (Auchère et al., 36, L57 – L60, 1998)

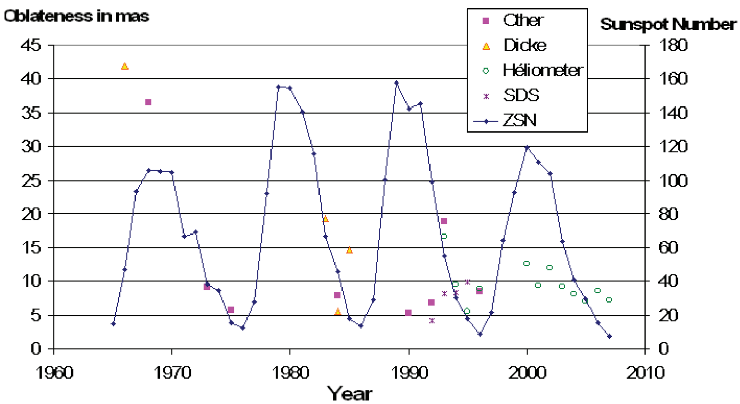

Fig. 3. The values of the difference ΔR between the equatorial and polar radii, according to several authors (see a list of estimates in [32] and in this chapter), plotted versus the international sunspot number. The oblateness seems to be in phase with the solar activity index. However the mean radius and the radius of the equatorial regions are out of phase. This result can be interpreted by means of the combined effects of the two first multipolar gravitational moments: the quadrupolar J_2 term is prevailing during lower activity periods of time, albeit the contribution of the hexadecapolar J_4 term is predominant during higher solar activity (and J_2 weaker)

at the very near surface of the Sun, and in phase for layers seated most deeper inside [45, 48, 49] (see Sect. 5.3).

The relevance of precise measurements of the Sun's shape can be summarized as follows:

- If R(t) is known over a long period of time, ranging over several centuries, and even on undecennial cycles, then luminosity variations can be tackled (solar luminosity has increased over the life time of the Sun). By contrast, we do not know yet how radius variations on time scales ranging from seconds to hours, if they exist, may play a role in the luminosity variations. Hence, determination, in real time, of the so-called asphericity-luminosity parameter [44]
$$w = \partial ln(R)/\partial ln(L)$$
is required. A table summarizing the estimated values of w is given in [25, 26]. Note also that the knowledge of this parameter is of high importance for the study of the Earth's upper atmosphere.
- If $R(\theta)$ is known, then asphericities coefficients c_n can be deduced, leading in principle to a determination of the solar gravitational moments J_n. The knowledge of these parameters is relevant to celestial mechanics and is required to set up precise ephemeris (due to the relation between J_n and the inclination of the orbits of planets, i.e., spin–orbit couplings) in a general relativistic description [59, 61].
- If $R(t)$ and $R(\theta)$ are known, then the solar core dynamics can be inferred. This can be achieved either through ground-based observations where the Fried parameter is larger than 15–20 cm or through dedicated space missions, such as SDO (Solar Dynamics Observatory) [39] or DynaMICCS/GOLF-NG (Dynamics and Magnetism from the Inner Core to the Corona of the Sun) [83, 84], expected to be launched by the end of 2008 (SDO) and 2012–2015 (DynaMICCS).

2 Observations of the Solar Shape

The solar shape is very difficult to observe and hence very difficult to measure because it requires an astrometric accuracy. If Dicke [21] can be considered as a pioneer in this task, his first attempts at Princeton were not convincing. Several other measurements, made between 1974 and 1994 (see a review in [59] or [65]), lead to more reliable results. Up to the 1990s, only the oblateness was searched for. To summarize, it was shown that, if the Sun were rotating at a uniform velocity rate, the oblateness is[5]

$$\Delta R = (R_{eq} - R_{pol}) = 6187 \quad \text{m} \quad \text{or} \quad 8.53 \quad \text{mas}. \tag{1}$$

[5] "*mas*" stands for milliarcsecond.

Note that ΔR (Eq. (2)) is upper bounded by 10.54 ± 0.25 mas as a maximum and 6.39 ± 1.31 mas as a minimum, according to the value adopted for the velocity rate (at the surface). See Sect. 4.

However, taking the differential rotation into account, the oblateness becomes

$$\Delta R = (R_{eq} - R_{pol}) = 7370 \quad \text{m} \quad \text{or} \quad 10.15 \quad \text{mas.} \tag{2}$$

It must be noted that the differential rotation increases the oblateness, in apparent contradiction with the theory of rotating stars. This can be explained by a change in the radial velocity rate near 45° latitude: $(d\omega/dr) = 0$ at this latitude, $(d\omega/dr) > 0$ at higher latitudes and $(d\omega/dr) < 0$ at lower latitudes.

Today, the best results concerning estimates of ΔR are given through three main different techniques. The results of the first one, deduced through balloon flights (in limited number) and the so-called SDS experiment, can be found in [79]. The second one, still into operation, has been developed at Mount Wilson Observatory (USA) [87]. Observations are based on a spectrographic analysis of the neutral iron line Fe I at 525 nm. Measurements have been recently reanalyzed by Lefebvre et al. [43, 46]. The third one is developed at the Pic du Midi Observatory by means of the scanning heliometer, initially conceived by J. Rösch. A full description of the apparatus can be found in [62] and the observational dependence of the solar radius with heliographic latitude is presented in [72]. A comparison of the results obtained by these two last techniques is given here in Fig. 2, and an analysis can be found in [43, 46]. Departures from a pure sphere are clearly seen: a bulge extends at the equator, up to around (30–40°), followed by a depression, the polar shape remaining oblate. Lastly, as measurements extend in time, it has been possible to study the temporal shape variations. SDS experiments yield an oblateness in phase opposition with the solar cycle, in contradiction with all other results: ground-based observations, (heliometer) [62] or space observations, either through SOHO-MDI [24] analysis or through recent RHESSI [28, 37] space experiments. Other results of the solar oblateness (mainly performed by Dicke and collaborators) can be found in [62] and are summarized in Table II given by Pireaux [59].

The variability of the solar oblateness with time can be briefly interpreted by the contribution of the two terms J_2 and J_4. Emilio et al. [24] reported a solar shape distortion using the Michelson Doppler Imager (MDI) aboard the Solar and Heliospheric Observatory (SOHO) satellite after correcting measurements for bright contamination. It was found that the shape distortion is nearly a pure oblateness term in 2001, while 1997 has a significant hexadecapolar (J_4) shape contribution. However, due to the fact that the hexadecapolar term might be of the same order of magnitude than the quadrupolar term, but obviously in opposite sign, it results that the equatorial radius at the surface is in antiphase with the solar cycle, which is consistent with the results deduced from the f-mode analysis.

Dicke and Rösch can be considered as precursors in the field of the solar shape. The first one has undeniably set the basis for the underlying physics of the oblateness (see also [67]). Even if his papers were often examined critically, they triggered a great amount of ideas which have moved astrophysics forward. The second one carefully examined the conditions of solar diameter observations, such as blurring effects or displacement of the inflection point toward the inner part of the disk (see also Hill and Oleson [36]). He defined also the *helioid* as the whole outer solar shape, in an analogy with the Earth's *geoid*.

Finally, a recent analysis of the data obtained at the Pic du Midi Observatory shows that the maximum value of the departures of the solar shape from a sphere does not exceed 20 mas; the oblateness varies slowly in time, in phase with the solar cycle (see figures in [62, 73, 74]). Figure 3 shows the variations with time of the oblateness as deduced from our own measurements (circles) and compared with those of Dicke (triangles), SDS experiments (cross) and other measurements (squares) found in the literature (see [32]).

One of the first attempts to understand theoretically solar surface distortions was made by Lefebvre [44] who showed that the thermal wind effect is one of the contributors at the solar surface. Note that the thermal wind (which is not the solar wind) is due to the difference in temperature between the pole and the equator and is the equivalent to the geostrophic effect, well studied by (Earth) meteorologists.

3 How Large Are the Temporal Variations of the Solar Diameter?

On physical grounds, temporal variability of the solar diameter cannot exceed 10 mas peak to peak in amplitude. Callebaut et al. [12] were certainly the first to point out that changes in solar gravitational energy, in the upper layers, necessarily involve limited variations in the size of the envelope. The mechanism can be described as follows, assuming hydrostatic equilibrium. Bearing in mind the definition of the energy $E_g = -\int Gm/r\ dm$, a thin shell of radius dr (or dm) in equilibrium under the gravitational force and the pressure gradient will expand or contract if any perturbation to these forces occurred. In Fazel et al. [25, 27], the authors improved the method and showed that any variations of the size of the solar envelope must be less than some 12 km of amplitude over a solar cycle, a value in agreement with those deduced from inversion of the helioseismic modes, or from space observations through the SOHO-MDI data analysis [40, 41].

Any other larger values are not consistent with astrophysical observations of other solar phenomena. For example, observed temporal irradiance changes, which are observed at a level of $\approx 1/00$ over the solar cycle, could be explained by a ≈ 200 mas changes in the solar diameter, if this mechanism ought to play a unique role. Such a large value is not realistic, as it would automatically cancel all other physical explanations and among them, the magnetism of the surface, which is known to explain most (but not all) of the irradiance variations (unless unknown physical mechanisms play a role at the extreme border of the limb).

As another example, consider the multipolar gravitational moments of the Sun. The injection of larger values of $\Delta R(t)$ in models which are tested in other respects (such as for the inclination of planetary orbits, theory of lunar motion, general relativity) would lead to major impossibilities. In the theory of lunar motion case, the inclusion of J_n estimates in a spin-orbital motion theory can be accurately confronted with observed lunar physical librations. As these librations are known to a few milliarcseconds of precision, it results that $\Delta R(t)$ is inevitably upper bounded [10, 66] by some 10–15 mas.

The next question the reader may ask is why solar astrolabes, distributed around the Earth (in France, Chile, Brazil and Turkey), are still measuring a diameter variability over the solar cycle of about 100–300 mas (sometimes more). A recent careful analysis, based on a statistical variographic analysis [8, 17], showed that measurements made by astrolabes may represent the fluctuations of the upper Earth atmosphere, i.e., the UTLS (Upper Troposphere–Lower Stratosphere) region, and *not*, as it is often claimed, the fluctuations of the lower atmosphere alone (the turbulence). As the UTLS region is modulated by solar activity [16], it results that astrolabes measure a part of the solar signal, but only a small part of it, as the singular spectrum analysis (SSA) shows and as it was suggested earlier ([55, 77]). Figure 4 shows the results obtained in the case of the French solar astrolabe data, but they are the same for other astrolabes. Only two eingenvalues are detected (i.e., the trend and a cyclic modulation), the remaining being noise. In fact, one can say that astrolabes are powerful instruments to measure the stratospheric variability (an amplification of the solar signal may also be produced in this zone): this point is not so trivial.

The last issue is the *phase*: Is the weak solar diameter variability in phase or not with magnetic activity? This question would deserve to be more widely debated, but let us jump to our conclusion, based on papers dealing with three different approaches: the original papers by Godier and Rozelot [31, 32], the papers by Fazel et al. [25, 27] and the papers by Lefebvre et al. [45, 49].

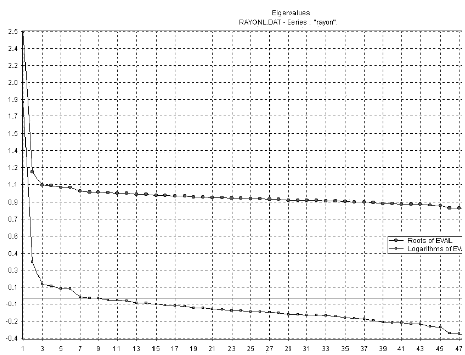

Fig. 4. Singular spectrum analysis applied to the French solar astrolabe data. It can be clearly seen that only two eigenvalues are detected (*upper curve*; the *lower* one is in log unit), the main signal being noise [55, 77]. However, variographic analysis of the same data shows a clear correlation with the stratospheric signal coming from the UTLS region ([8, 17])

– The first quoted papers describe asphericities in the subsurface layers (through the so-called *Theory of Figures*): one asphericity is located around $0.7R_\odot$ (which is identified as the tachocline), and another one is located between 0.982 and 0.993 $R_\odot$, with two dips, at 0.986 $R_\odot$ and at 0.992 $R_\odot$; this last layer constitutes the *leptocline* (Fig. 2 in [33]).
– The second set of papers is based on the assumption that the effective temperature of the surface is nearly immutable, as suggested from observations made by Livingston at Kitt Peak [50, 51]. It is shown that to model the remaining part of the irradiance variations (i.e., the part which is not coming from surface magnetism phenomena), there may exist a phase shift in the $[dT, dR]$ plane, with a $dT(dR)$ curve separating solar variations in antiphase (for temperature values below 0.08 K), and in phase (for temperature values greater than 0.08 K) with solar irradiance variations [27]. It must be pointed out that in this case, the nonvariability of dT over the solar cycle could be explained by the flux tubes passing between the granules, without interaction; due to the magnetic pressure, one would expect a change in the mean size of the granules that would be thus shifted toward the smaller sizes, as magnetic activity is increasing. Such observations were already made at the Pic du Midi Observatory since 1997. As a consequence, the whole size of the Sun would decrease (anticorrelation with solar magnetism).
– The third set of papers reports changes of the Sun's subsurface stratification inferred from helioseismic inversions (see Sect. 5.3), for which a clear phase changing with depth is shown.

4 Solar Shape and Rotation

The study of the rotation of stars is not trivial. In theory, the problem is exceedingly simple and can be formulated as follows. Let us consider a single star that rotates along a fixed direction in space, with an angular velocity ω, and first assume that, for $\omega = 0$, the star is a gaseous body in gravitational equilibrium. The problem is to determine the outer shape of the star when the initial sphere is set rotating at an angular velocity ω. Such studies were conducted for the first time by Milne [54], then fully achieved by Chandrasekhar [15]. If the body is in uniform rotation ω and the density of the form $\rho = r^{-n}$ then the oblateness is given by

$$\varepsilon = (0.5 + 0.856\rho_c/\rho_m)\omega^2 R/g,$$

where ρ_m is the mean density of the star, ρ_c the density of the core, R the radius of the initial sphere and g the gravity at the surface. Applied to the Sun and to first order, this nice formula gives ($g = 2.7\times 10^4 cm/s^2$, $\rho_c/\rho_m = 108.3$)[6]

$$\varepsilon = 1.10 \times 10^{-5} \pm\ 2.62 \times 10^{-7},$$

[6] According to the different values the ratio of central to mean density may take, ε lies between 0.504 and 0.513 $\omega^2 R/g$.

for a constant equatorial velocity rate (of 14°34 ± 0°17/day); ε decreases to 9.03 $\times 10^{-6}$ ± 4.26 $\times 10^{-7}$ for a Sun rotating at a constant rate which is the 45° latitude velocity rate (13°00 ± 0°31/day). At the pole, the uncertainty is a bit higher: ε is 6.7 $\times 10^{-6}$ ± 1.4 $\times 10^{-6}$ (mean latitude velocity rate: 11°10 ± 1°2/day). Such values lead to a mean difference between the equatorial and the polar radii of respectively 10.54 ± 0.25 mas (equator), 8.67 ± 0.41 mas (45°) and 6.39 ± 1.31 mas (90°). In a metric scale, this amounts to a difference of respectively 7.6 ± 0.2 km, 6.3 ± 0.3 km and 4.6 ± 0.9 km. Note that in this case, the difference between R_{eq} and R_{sp} is 3.5 mas ($R_{eq} = 959.63''$). A ponderous process (the best estimate weighted by each error) yields

$$f = 8.33 \times 10^{-6} \pm 1.87 \times 10^{-6}.$$

The second point is to understand what happens if ω is not constant, not only in latitude (differential rotation) but also throughout the body, from the surface to the core. We are faced today with such problems, not only in the solar case but also for stars. With the advent of sophisticated techniques such as interferometry, one is now able to accurately determine the geometrical shape of the free boundary of stars, such as Altair or Achernar for which observations of the geometrical envelope have been made for the first time, respectively, by van Belle et al. [88] and Domiciano de Souza et al. [22].[7] But it would be of little or no interest to observe the geometric shape of a star – or that of the Sun – if one would not be able to infer some information on stellar – solar – physics. With such an approach, the purpose of theoreticians is to enumerate all the possible angular velocity distributions (from the center to the surface) that are compatible with the observed stellar – solar – surface. For stars, Maeder [53] examined the effects of rotation and among them, he described the equation of the surface with a rotation law which is differential, but only in the surface layer.

In other words, the knowledge of the angular velocity distribution from the core to the surface, together with the knowledge of the density function (related to the pressure function), completely determines the outer shape of the stars. Different techniques exist to observe such a figure. Once accurately determined, one would be able to go back to the physical properties of the body. This approach is called *Theory of Figures.*

This theory has been widely used in geophysics and is still used in specific cases, such as for the planet Mars, with an incredible accuracy ($J_{2\ Mars} = 1.860718 \times 10^{-3}$ according to [92], from a 75th degree and order model). Curiously, nothing or very little was done in the solar case, until 2001 by Rozelot et al. [67, 71]. The complexity of the rotation profile (Fig. 1) will highly infer on the photospheric shape: the outer figure is highly sensitive to the interior structure. Thus, in principle, accurate measurements of the limb shape distortions,

[7] Outer shape of other stars has been determined since then; see [45].

which are called "asphericities" (i.e., departures from the "helioid", the reference equilibrium surface of the Sun), combined with an accurate determination of the solar rotation provide useful constraints on the internal layers of the Sun (density, shear zones, surface circulation of the plasma, etc.). Figure 5 shows such asphericities that can be seen at a given spatial resolution. Alternatively, theoretical upper bounds could be inferred for the flattening which may exclude incorrect/biased observations. Lastly, another approach consists in analyzing the effects of rotation on stellar/solar p-mode frequencies, as the rotation of the bodies affects their oscillation frequencies [35].

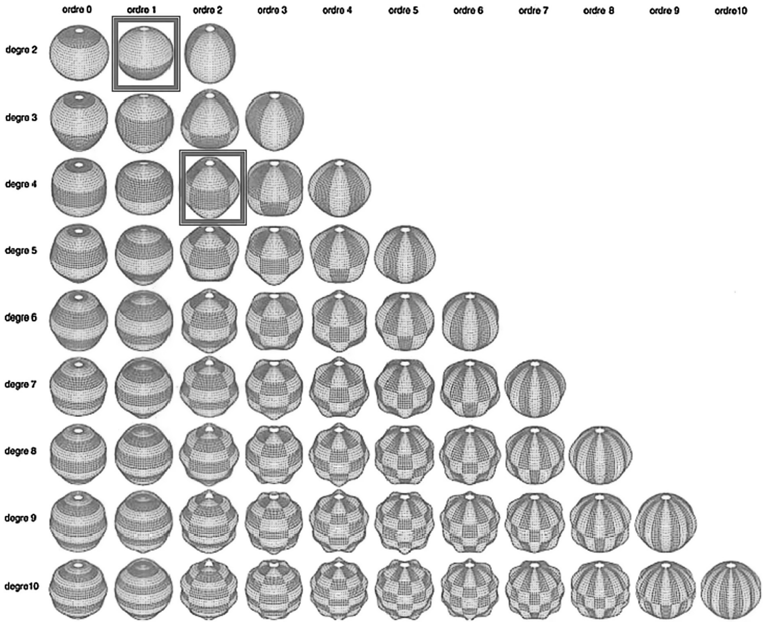

Fig. 5. Laplace spherical harmonics showing the surface distortions of a rotating fluid body. The solar oblateness ($n = 1$, $l = 2$) and the quadrupole moment ($n = 2$, $l = 4$) are illustrated in the two boxes (after R. Biancale, – personal communication)

5 The Solar Shape and Fundamental Physics

Of all the fundamental parameters of the Sun (diameter, mass, temperature, etc.), the successive gravitational moments that determine the solar moments of inertia are still poorly known. However, these moments have a physical meaning: they tell us how much the Sun's material contents deviate from a purely spherical distribution and how much the velocity rate differs from a uniform distribution. Thus, their precise determinations give indications on the solar internal structure.

The dynamic study of the gravitational moments until now is mainly based on solar observations (mainly through helioseismology but also through astrometric observations of the solar diameter) and solar models of rotation and density. Various methods (stellar structure equations coupled to a model of differential rotation, theory of the Figure of the Sun, helioseismology) lead to different estimates of J_n, which, if they agree on the order of magnitude, still diverge for their precise values [60, 73, 75].

5.1 Solar Asphericities

Asphericities, as defined before, can be computed according to the degree l and order n in the development of Laplace spherical harmonics in the general case of a rotating fluid body. Let ρ be the density (function of the radius r) and denote with a subscript 0, the lowest order l, spherically symmetric. Asphericities, described as [7]

$$c_l = -\rho_l / d\rho^{(0)}/dr \quad \text{(density)}, \tag{3}$$
$$s_l = -p_l / dp^{(0)}/dr \quad \text{(pressure)}, \tag{4}$$

measure the perturbation (nonspherically symmetric) and are usually expressed in terms of the normalized potential defined by $J_l = K\phi_l$, where $K = R_\odot/GM_\odot$ at the solar surface. The different gravitational moments can be written as

$$J_{2l} = \frac{R_\odot}{GM_\odot}\ \phi_{2l}(R_\odot), \tag{5}$$

where $\phi_{2l} = 0$ at the surface $r = R_\odot$. The function ϕ_{2l} is the solution to a differential equation requiring the knowledge of $\rho(r)$ and $\omega(r,\ \theta)$, where θ is the colatitude. A complete expression of ϕ_2 and ϕ_4 was provided by Armstrong and

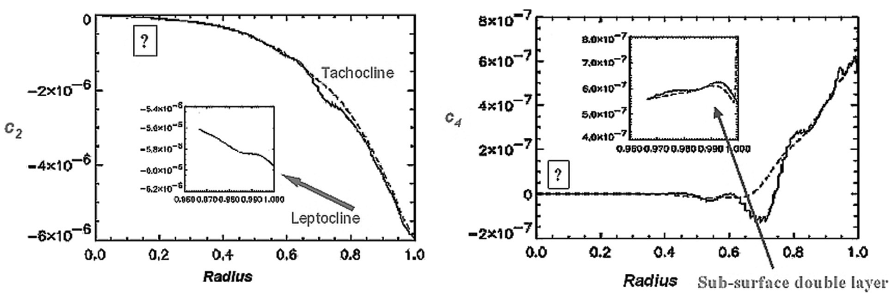

Fig. 6. Asphericity coefficient of degree 2 (*left*) and 4 (*right*), with respect to solar fractional radius ($r/R_\odot$): *solid* and *dashed lines*, respectively, density – Eq. (3) – and pressure asphericities – Eq. (4), after [7]. The dip in the curve at 0.7 $R_\odot$ locates the tachocline and indicates that this layer is prolate (instead of oblate for a bump). The near surface anomaly locates a new double subsurface layer, the "leptocline", that does not seem to be noticed before. See also [33]

Kuhn [7], using the solar standard rotation law $[\omega(\theta) = \omega_0 + \omega_2 \cos^2(\theta)]$, which permits to deduce c_l:

$$\frac{d^2\Phi_2}{dr^2} + \frac{2}{r}\frac{d\Phi_2}{dr} - 6\frac{\Phi_2}{r^2} = \frac{4\pi r^2}{M_r}\left[\Phi_2 \frac{d\rho_0}{dr} - \frac{8}{21}\frac{2\omega_2}{\omega_0} r\rho_0\omega_0^2 - \frac{r^2}{3}\frac{d}{dr}\left(\rho_0\omega_0^2 + \frac{3}{7}\rho_0\frac{2\omega_2}{\omega_0}\omega_0^2\right)\right], \tag{6}$$

$$\frac{d^2\Phi_4}{dr^2} + \frac{2}{r}\frac{d\Phi_4}{dr} - \frac{20}{r^2}\Phi_4 = \frac{4\pi r^2}{M_r}\left[\Phi_4 \frac{d\rho_0}{dr} + \frac{4}{35}\frac{2\omega_2}{\omega_0} r\rho_0\omega_0^2 - \frac{2}{35}r^2\frac{d}{dr}\left(\frac{2\omega_2}{\omega_0}\rho_0\omega_0^2\right)\right]. \tag{7}$$

Results are shown in Fig. 6 for $l = 2$ and $l = 4$ (shape coefficients are expressed in units of the solar radius): a clear signature of the tachocline appears, at $r = 0.7\ R_\odot$, which is prolate (a dip of c_2 plotted as a function of the fractional radius $r/R_\odot$ and a bump of c_4 plotted as a function of $r/R_\odot$). However, the authors seem not to have noticed another asphericity in the c_n curves near the surface, i.e., an oblate layer, determined by a bump of c_2 and a dip of c_4, which is the signature at $r = 0.99\ R_\odot$ of another distorted shell, of different physical properties, the *leptocline* (Sect. 5.3)

5.2 Solar Gravitational Multipole Moments

Observations allow to constrain analytical rotation models, in colatitude θ and depth r. The first attempt to derive an analytical rotation law from helioseismic data has been made by Kosovichev [38]. Using such a law, several authors computed the gravitational moments (see Table 1), but discrepancies appeared, mainly for J_4. The discrepancy between the values obtained through different methods and authors can be explained by the use of different density models and by the way the differential equation Eq. (5) is integrated. It can be seen that the method using helioseimic data leads to multipole moment values lower than those obtained by other methods. However, the octopole moment, J_4, is much more sensitive than the quadrupole moment, J_2, to the presence of latitudinal and radial rotations in the convective zone. Ajabshirizadeh et al. [1] showed that the surface magnetism may reconcile the different approaches, between the Theory of Figures and numerical integration of Eq. (5): J_4 obtained by the first theory seems better matching observations. If we can adopt $(2.4\ \pm 0.4)\times 10^{-7}$ as a good estimate for J_2, it remains that J_4 is very sensitive to the subsurface gradient of rotation: an estimate of $(4–7)\times 10^{-7}$ seems in better agreement with the observations. Finally, we can point out the formula linking J_2 and f in the presence of a magnetic field, as deduced by Ajabshrizadeh et al. ([1], see also Fig. 1):

$$J_2 = (2/3)f(1 - m') - (1/3)m$$

where m takes into account the velocity rate and m' is directly related to the magnetic moment of the rotating body. Kosovichev [38] noted that a subsurface shear layer results when the obtained helioseismically internal rotation is matched with the surface rotation. Hence, we suspected that the shallow layer near the surface may play an important role.

Table 1. Some solar gravitational multipole moments quoted from different authors and methods. The absolute order of magnitude of J_2 is 2.4×10^{-7}; J_4 is very sensitive to the subsurface gradient of rotation: an estimate of $(4\text{–}7)\times10^{-7}$ seems better match the observations

References	Method	J_2	J_4	J_6	Others
Ulrich & Hawkins [84]	SSE + spots rotation law	$(10-15)\times10^{-8}$	$(0.2-0.5)\times10^{-8}$		
Gough [32]	First determination of helioseismic rot. rates	36×10^{-7}			
Campbell & Moffat [3]	Planetary orbits	$(5.5\pm1.3)\times10^{-6}$			
Landgraf [40]	Astrometry of minor planets	$(0.6\pm5.8)\times10^{-6}$			
Lydon & Sofia [50]	SDS experiment	1.84×10^{-7}	9.83×10^{-7}	4×10^{-8}	$J_8=-4\times10^{-9}$ $J_{10}=-2\times10^{-10}$
Paternò et al. [54]	SSE + empirical rotation law and SDS	2.22×10^{-7}			
Pijpers [56]	SSE + GONG and SOI/MIDI data Weighted value	$(2.14\pm0.09)\times10^{-7}$ $(2.23\pm0.09)\times10^{-7}$ $(2.18\pm0.06)\times10^{-7}$			
Armstrong & Kuhn [7]	Vect. spher. harm. numerical error	-0.222×10^{-6} 0.002×10^{-6}	3.84×10^{-9} 0.4×10^{-9}		
Godier & Rozelot [29]	SSE + Kosovichev law	1.6×10^{-7}			
Roxburgh [61]	SSE + two models of rotation law	2.208×10^{-7} 2.206×10^{-7}	-4.46×10^{-9} -4.44×10^{-9}	-2.80×10^{-10} -2.79×10^{-10}	$J_8=1.49\times10^{-11}$ $J_8=1.48\times10^{-11}$
Rozelot et al. [67]	Theory of Figures	$-(6.13\pm2.52)\ 10^{-7}$ Note 3	3.4×10^{-7} Note 4		
Rozelot & Lefebvre [70]	Theory of Figures	-6.52×10^{-7}	4.20×10^{-7}	-9.46×10^{-9}	$J_8=2.94\times10^{-13}$

Table 1. (continued)

References	Method	J_2	J_4	J_6	Others
Rozelot et al. [71, 73, 74]	SSE + subsurface gradient of rotation (SGR)	$-2.28 \times 10^{-7} \pm 15\%$	Very sensitive to SGR Range: $\pm$ 20%		
Ajabshirizadeh et al. [2]	With magnetic field	-2.613×10^{-7}	$+6.29 \times 10^{-7}$	-1.42×10^{-8}	$J_8 = +5.05 \times 10^{-13}$
	No magnetic field	-2.300×10^{-7}	$+6.29 \times 10^{-7}$	-1.42×10^{-8}	$J_8 = +5.05 \times 10^{-13}$

Note 1: SDS stands for solar disk sextant.
Note 2: SSE stands for stellar structure.
Note 3: The apparent large error comes from the fact that the value is a weighted average of several rotation rates.
Note 4: A mistake has been made in [33]: the second term of J_4 (i.e., mA_4) was incorrectly multiplied by "f" in the computations.

5.3 The Leptocline

Analyzing the temporal variation of f-mode frequencies for 1996–2004, Lefebvre and Kosovichev [45] have shown changes in the Sun's subsurface stratification. They have found a variability of the "helioseismic" radius in antiphase with the solar activity, the strongest variations of the stratification being just below the surface, around 0.995 $R_\odot$. On the other hand, the radius of the deeper layers of the Sun, between 0.975 and 0.99 $R_\odot$, changes in phase with the 11 year cycle (Fig. 7). A more careful analysis of these f-modes shows variations in the even-a coefficients, of nonnegligible amplitude, with both the frequency and the cycle, that imply the existence of asphericities in the subsurface layers [9, 47]. The conclusion is that this interface layer corresponding to the border between the interior of the Sun and its atmosphere is the seat of strong physical phenomena (in addition to non-homologous radius changes in time and depth), such as shearing, disturbance of the turbulent pressure, constraints upon the magnetic field, processes of ionization and, likely, inversion of the radial gradient of rotation and some tiny variations of the luminosity (Fig. 8).

Even if this layer is maybe more complex, involving another shell of some oblateness at the very near surface (unreachable for the moment to the f-modes), located at around 0.999 $R_\odot$, the proof is now made that the *leptocline* is a new and crucial zone that cannot be avoided in investigating the global properties of the Sun and its evolution on time scales of the order of months or years.

5.4 Solar Radius and Gravitational Energy Variations

As previously mentioned, the gravitational energy can be computed in the case of a nonspherical Sun, which leads to a variation of the solar luminosity L with

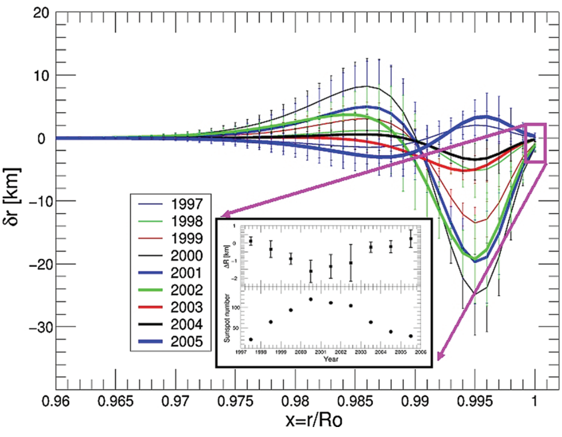

Fig. 7. Nonmonotonic variations of the radius in the most outer layers of the Sun. Below 0.99 $R_\odot$, the radius is varying in phase with the solar cycle with a maximal amplitude of about 10 km. Above 0.99 $R_\odot$ the radius is varying in antiphase with the solar cycle, which implies a compression with a maximal amplitude of about 30 km peak to peak amplitude and 2–3 km at the surface. After [45, 49]

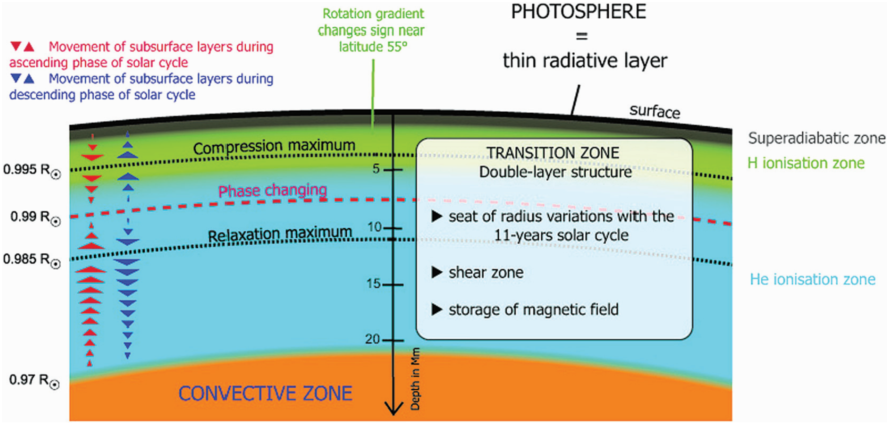

Fig. 8. A schematic view of the outer layers of the Sun shaping the leptocline. (After [47]; see also [33].) The most interesting feature is the changing phase near 0.99 $R_\odot$. This region is the seat of many phenomena: an oscillation phase of the seismic radius, together with a nonmonotonic expansion of this radius with depth, a change in the turbulent pressure, likely an inversion in the radial gradient of the rotation velocity rate at about $(45–50)^\circ$ in latitude ($d\omega/dr$ is positive beyond these latitudes, negative below), opacities changes, superadiabicity, the cradle of hydrogen and helium ionization processes and probably the seat of in situ magnetic fields

dR according to a development of order n (see [26]). The authors made two computations, one with n =1 (monotonic expansion with radius) and the other with n =2 (non monotonic expansion).

Table 2 taken from [27], gives the results for two values of $\Delta L/L$. The first one is the usual adopted value, 0.0011, using TSI composite data from 1987 to 2001 [20]; mean value $L_\odot$ = 1366.495 W/m^2. The second one is 0.00073, determined through a re-analysis of the composite TSI data over the period of time 1978–2004 [29]; mean value $L_\odot$ = 1365.993 W/m^2. For n = 2 (the most likely case consistent with recent other results), the estimate of ΔR is smaller than the 8.9 km obtained in the case of a spherical Sun by Callebaut et al. [12]. However the $\Delta R/R$ agrees with that of [5], i.e., $\Delta R/R = 3\times10^{-6}$, that

Table 2. Variations of the solar radius computed in two cases, monotonic ($n = 1$) and non monotonic ($n = 2$) expansion, and for two mean values of $L_\odot$. The case $n = 2$ is the most likely. The sign (−) indicates a shrinking, i.e., an anticorrelation of ΔR and L in this layer

$\Delta L/L = 0.0011$ [20]		$\Delta L/L = 0.00073$ [29]	
$\Delta R/R = -1.70\times10^{-5}$ (or ΔR = 11.8 km)	($n = 1$)	$\Delta R/R = -1.13\times10^{-5}$ (or ΔR = 7.86 km)	($n = 1$)
$\Delta R/R = -8.38\times10^{-6}$ (or ΔR = 5.83 km)	($n = 2$)	$\Delta R/R = -5.56\times10^{-6}$ (or ΔR = 3.87 km)	($n = 2$)

used f-mode frequency data sets from MDI (from May 1996 to August 2002) to estimate the solar seismic radius with an accuracy of about 0.6 km (see also among other authors [4, 78], for such a determination of the solar seismic radius to a high accuracy). It results that the asphericity-luminosity parameter w is -1.55×10^{-2} $(n = 1)$ and -7.61×10^{-3} $(n = 2)$.

Two points result from the analysis of the data. The first one concerns the "helioseismic radius" which does not coincide with the photospheric one, the photospheric estimate always being larger by about 300 km [11]. This point would require more specific attention in the future.

The second issue addresses the shrinking of the Sun with magnetic activity as pointed out by Dziembowski et al. [23], using f-mode data from the MDI instrument on board SOHO, from May 1996 to June 2000. They found a contraction of the Sun's outer layers during the rising phase of the solar cycle and inferred a total shrinkage of no more than 18 km. Using a larger database of 8 years and the same technique, Antia and Basu [6] set an upper limit of about 1 km on possible radius variations (using data sets from MDI, covering the period of May 1996 to March 2004). However, they demonstrated that the use of f-mode frequencies for $l < 120$ seems unreliable.

It results from the above discussion that the luminosity changes are likely produced in the outer shallow layer of the Sun. It thus appears that the leptocline might be the seat of the observed 1/₀₀ variations of the irradiance.

A recent application of the virial theorem to the radius changes in the Sun induced by magnetic variations [81] shows that the radius decreases around the time of maximum magnetic field strength. If we may have confidence in this result, by satisfying the conversion of total energy, it remains to explain why the observations show that the oblateness is in phase with the magnetic activity. This may be due to the reversal of the radial gradient of rotation near 45° of latitude, as explained before.

5.5 Results from Ground-Based Observations and Space Missions

Indeed, solar asphericities, encoded mainly by the first two coefficients c_2 and c_4, can be observed. An estimate of these two coefficients derived from SOHO-MDI space-based observations is at the surface [7]:

$$c_2 = (-5.27 \pm 0.38) \times 10^{-6} \quad \text{and} \quad c_4 = (+1.3 \pm 0.51) \times 10^{-6}. \tag{8}$$

These results were obtained by measuring small displacements of the solar limb darkening function (details are given in [40]), and the c_n coefficients are comparable to an isodensity surface level (see Eq. (3)). From Earth-based observations at the scanning heliometer of the Pic du Midi Observatory, we also obtained estimates of c_2 and c_4 coefficients. The mean ponderated values, computed over three years (values are given in [70]),[8] are

$$c_2 = (-6.56 \pm 0.18) \times 10^{-6} \quad \text{and} \quad c_4 = (+2.7 \pm 0.6) \times 10^{-6} \tag{9}$$

[8] A complete re-analysis of all the data, from 1996 to 2007, is under consideration.

that indicates (Sect. 2) a slight bulge extending from the equator to the edge of the royal zones (about 40° of latitude), with a depression beyond (at the pole itself, the ellipsoidal figure prevails).

Such a distorted shape, not exceeding some 20 mas of amplitude,[9] can be interpreted through the combination of the quadrupole and octopole terms, which, as shown previously, directly reflect the non-uniform velocity rate in surface (and depth). Moreover, this distribution implies a thermal wind effect, from the poles toward the equator [44]. The observed value of c_2, -6.6×10^{-6}, is not too far from the theoretical one, $\approx -(2/3)f \approx -5.9 \times 10^{-6}$ with $f = 8.9 \times 10^{-6}$ based on a solar model with a differential rotation law. It agrees also with the SOHO-MDI observations (see values given in (8)) and the theoretical estimate deduced from a vector harmonics solution [7], -5.87×10^{-6}. The coefficient c_4 remains difficult to match with the theory, which predicts $+(12/35)f^2$ for a uniform rotation law, and 0.616×10^{-6} for a differential one. The most likely explanation is that the shape coefficient c_4 is very sensitive to surface phenomena and differential rotation (as for J_4).

Only space-dedicated satellites will be able to definitively provide an answer to these questions.

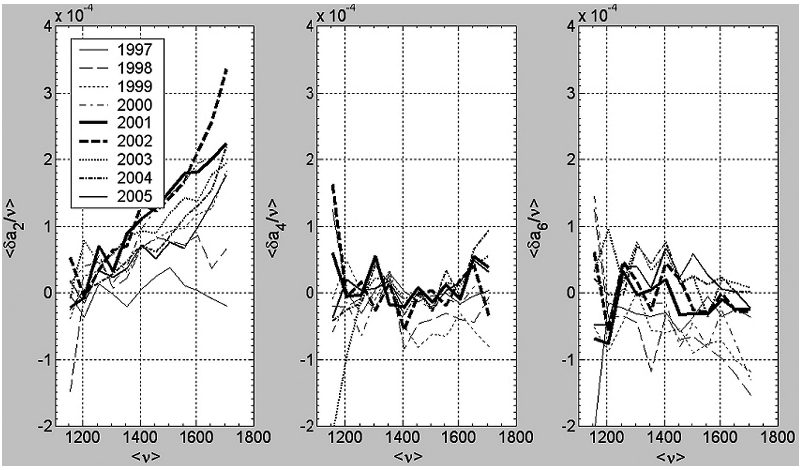

Fig. 9. Three first asphericities parameters γ_k, directly related to the even-a coefficients of f-modes. The first graph is significant and shows an antiphase correlation with the solar cycle (after [9])

[9] A first explanation (based on a temporal average) of such a result lies in the proposal made by Pecker [57]: "in the royal zones the existence of spots diminishes the solar brightness and thus the measured radius; but this effect is compensated and reinforced by the existence of faculae, which extends higher in latitudes. The measured radius is globally greater at the latitudes where faculae are statistically more numerous. At highest latitudes, no spots and faculae appear any longer, and the measured radius is consequently reduced".

5.6 Temporal Variations of the Asphericities Coefficients of the Solar Shape

If we are beginning to understand the significant physical character of the *leptocline*, we are far to know if the asphericities are variable with time. However, it can be reasonably thought that a temporal variability of such parameters might be due to the temporal variation of the internal structure. Lefebvre et al. [48] and Bedding et al. [9] reported first analysis concerning the temporal dependence of even-a coefficients. The available data (SOHO f-modes) permits to have access to the first 18 even-a coefficients, but for a sake of clarity, only the first three (γ_k) were computed (Fig. 9).

Each curve is an average difference over a specific year computed by reference to the minimum year of activity, 1996 (the error bars are not shown). The first graph on the left (γ_1) shows a negative trend, a frequency behavior almost flat and a clear behavior with the cycle in antiphase. The second graph dedicated to the variation of γ_2 shows a slight increase with the frequency, a change of sign at high frequency and no clear variations with the cycle. As far as the variation of γ_3 is concerned, the dispersion is too big to say anything.

In such a way, the outer shape would be time dependent, and this could explain also some tiny fluctuations of the irradiance. It is thus of interest to explore the whole chain, starting from the core up to the surface, to well understand the mechanisms of solar activity, then to get a better prediction, and to understand how the solar output may influence the atmosphere of our planet. One can judge such an investigation as ambitious, but we are today compelled to carefully examine all the sources of the solar variability to get a scientific opinion on the solar forcing – and even if it is to reject one of the processes.

5.7 Solar Shape and General Relativity

If, from a physical point of view, the multipolar moments lead to distortions of the solar surface (asphericities), they have also a dynamic role in the light deflection or in celestial mechanics. In the ephemerids computation, the determination of J_2 is strongly correlated with the determination of the Post-Newtonian Parameters (PPN) characterizing the relativistic theories of the gravitation. Lastly, the ignorance of J_2 is also a barrier to the determination of models of evolution of the solar system on the long term.

The relativistic aspects are crucial in the dynamic approach of the solar parameters and open interesting prospects for the future. In this context, it is interesting to obtain a dynamic constraint of J_2, independent of the solar models of rotation and density, being used thereafter to force the solar models. Such a study is relevant in the scope of space missions such as BeppiColombo (better determination of the PPN; possible measurement of the precession of the apside line of Mercury as a function of J_2), GAIA (better determination of the PPN, possible decorrelation PPN–J_2 thanks to the relativistic advance of the perihelion of planets and minor planets) and obviously *GOLF-NG* (precise determination of the rotation of the core where the quasi-totality of the mass is concentrated). Another key parameter of the solar models, which could also be constrained in a dynamic way is its spin, from the spin–orbit couplings which is introduced in celestial mechanics. From present solar system experiments (Lunar Laser Ranging, Cassini Doppler experiments, etc., see [59]), it turns out that general relativity is not excluded by those, as shown in the most up-to-date values in Fig. 10a. However, general relativity would be incompatible with the Mercury perihelion advance test if $J_2 = 0$ was assumed. But with a non zero J_2, general relativity agrees with this latter test, and there is still room for an alternative theory too (see Fig. 10b). *Space missions such as SDO or DynaMICCS-GOLF-NG* should provide the necessary J_n measurements.

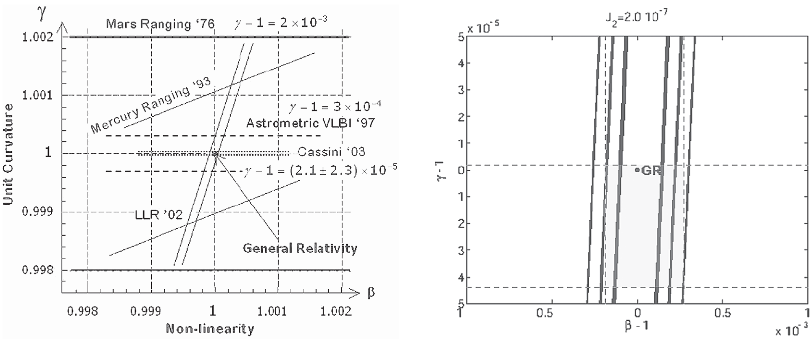

Fig. 10. *Left* (**a**) Thirty years of testing general relativity from space. *Right* (**b**) A given value of J_2 constitutes a test of the PN parameters β and γ. In the β and γ planes, the 1σ (the *smallest*), 2σ and 3σ (the *largest*) confidence level ellipses are plotted. Those are based on the values for the observed perihelion advance of Mercury, Δ_{wobs}, given in the literature and summarized in [59]. General relativity is still in the 3σ contours for the allowed theoretical values of J_2 quoted on the upper part of the chart

Regarding the solar core dynamics, the subject is of high priority for new investigations. Here again, space-dedicated missions, such as DynaMICCS/GOLF-NG in a joint effort with SDO (Solar Dynamics Observatory), scheduled in a next future, should provide a new insight into the question [85].

6 Conclusion

From an historical point of view, the question of whether the diameter of the Sun evolves with time, or not, is very fertile. On time scales of the order of the millennium, the question of the solar luminosity can be tackled. On ranging time going from the medieval era up to now, the debate is not really closed. Wittmann [90] claimed, from Tobias measurements, that no secular solar diameter decrease can be inferred. We are more in favor of a long-term modulation, the Sun being bigger during periods of lower activity, such as during the Maunder minimum, and smaller in periods of more intense activity such as presently. On smaller time scales (over few cycles), models are still needed, but new input will come, paradoxically, from a better knowledge of the temporal evolution of the limb fluctuations (including solar diameter variations). On time scales of the order of months (or years), the variability is upper bounded by some 10 mas (and to 15 mas as an upper bound). Such estimates, deduced on physical grounds, are incompatible and irreconcilable with solar astrolabe measurements, which lead to one, even two orders or magnitude greater. The astrophysical consequences of such large variations would have been detected in indirect effects, such as lunar librations, which is not the case [65]. A possible explanation of the detected variations is through feedbacks mechanisms in the Earth UTLS zone [17]. From this point of view, it seems that the model proposed by Sofia et al. [80] (build to try to explain large variations of astrolabe data by the effect of magnetic field at the surface), which show an increase of the solar radius by a factor of approximately 1000 from a depth of 5 Mm to the solar surface, is not consistent (with the observations of the f-modes at the limb), in spite of its achieved formalism including magnetic field [49].

The question of the phase of the solar radius with activity depends on the depth, the diameter being in antiphase at the surface to progressively go to a phasing below 0.99 $R_\odot$ [45]. According to the analysis of f-mode frequencies, the Sun seems to be bigger in periods of lower activity (to a few km -30 as a maximum). Such an expansion could be due to magnetic fluxes passing through the gap between granules without interacting with them, the photospheric effective temperature playing a key role [51], confirmed by the gravitational energy variations in the upper layers [27]. Such an analysis leads to a phase shift of the solar luminosity with the solar cycle [25, 81].

The study of asphericities, directly linked with solar gravitational moments, is crucial not only for solar physics but also for astrometry (when computing light deflection in the vicinity of the Sun), celestial mechanics (relativistic precession of planets, planetary orbit inclination[10] and spin–orbit couplings) and future tests of alternative theories of gravitation (correlation of J_n with Post-Newtonian parameters): [59, 61].

Another issue is the solar changing shape. It has been shown that the outer solar shape significantly differs from a sphere, with a bulge at the equator, and a depressed zone at higher latitudes (the change being around 45°, due to the reversal of the radial velocity rate); the whole shape remains oblate at the pole. The Pic du Midi observations show *a variability of the whole oblate shape in phase with solar activity* [65] which *is not incompatible* with the above-mentioned long-term solar diameter modulation. Such in-phase dependence of the oblateness with the solar cycle has been confirmed through space by the RHESSI mission [37], at least an excess of the apparent oblateness with an equator to pole radius difference of 13.72 ± 0.44 mas (i.e., of the same order of magnitude that the mean value found at the Pic du Midi for the years 1993–1996: 11.5 ± 3.4 mas, see Table

[10] In the solar case, the potential expanded up to its quadrupole moment is given by $\Phi = -GM/r + J_2 GMR_{Eq}^2(3\sin^2\eta - 1)$, where η is the angle relative to the equatorial plane.

1 in [66] or [64], the larger error coming from ground-based observations; note that the variations reflect temporal changes). Figure 11 shows the last results obtained at the Pic du Midi (F) Observatory (over a solar cycle), by means of the heliometer located in the so-called coupole J. Rösch. It is difficult to deny a phased variation with the cycle. If one wants to be very purist, one will be able to say that the flatness is 8.5 ± 3.5 mas. In such a case, the validity of the observational and analysis process is justified, as the theory gives 8.2 mas.

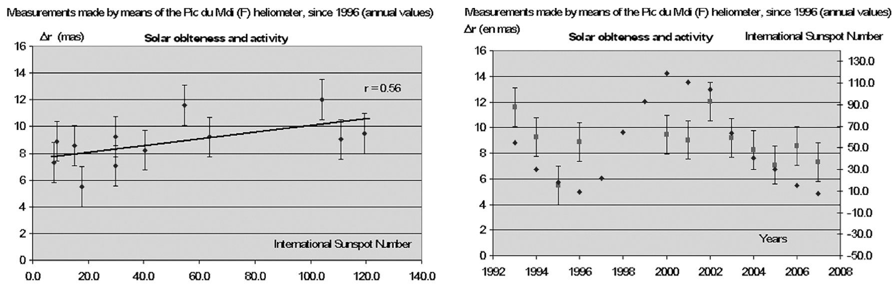

Fig. 11. Oblateness deduced from measurements made at the Pic du Midi Observatory by means of the heliometer since 1996

At last, due to the fact that the hexadecapolar term might be of the same order of magnitude than the quadrupolar one, but obviously in opposite sign, it results that the equatorial radius at the surface is in antiphase with the solar cycle, which is consistent with the results deduced from the f-mode analysis. We would like to emphasize again the key role of the leptocline in probing the sub-surface. To our mind, the inversion of the radial gradient of rotation at 50° contributes to solar asphericities, the whole shape remaining oblate. In period of lower activity, the equatorial diameter slightly increases, J_2 is predominant, J_4 has no influence and ε increases. In period of higher activity, the equatorial diameter slightly decreases under the influence of J_4 which is predominant, J_2 has no influence, so that ε decreases.

Accurate measurements from space observations are needed. They can be achieved by next generation of satellites, such as DynaMICCS/GOLF-NG [83], SDO, or even balloon flights [79]. On a longer term, GAIA, which is expected to flight by the end of 2012, will allow to estimate the perihelion precession of Mercury and other small planets such as Icarus, Talos and Phaeton. In this case, it will be possible to separate the relativistic and the solar contributions in the perihelion advance, so that gravitational moments could be directly determined from dynamics, without the need of a solar model. Note also that presently dynamical estimates of J_2 are strongly correlated with the estimate of the Post-Newtonian parameter β, which, together with other PN parameters, characterizes relativistic theories of gravitation in observational tests. However, future PPN testing space missions, as well as non dedicated missions like GAIA, might help solve the problem.

According to the temporal variation of the f-mode frequencies, the very near solar surface is stratified in a thin double layer, interfacing the convective zone and the surface. This "leptocline" is the seat of many phenomena: an oscillation phase of the seismic radius, together with a nonmonotonic expansion of this radius with depth, a change in the turbulent pressure, likely an inversion in the radial gradient of the rotation velocity rate at about 45° in latitude, opacities changes, superadiabicity, the cradle of hydrogen and helium ionization processes and probably the seat of in situ magnetic fields [48]. Figure 8 shows a schematic view of the complex physics in this shear zone.

The last point could be an interrogation. Why helioseismology always leads to smaller values of the parameters under investigations? The rotation velocity rate at the surface is smaller than those obtained through other techniques (at the surface, see Table 1 given in [44]), as well as the quadrupole moment estimates and the radius itself. To disentangle all these points, we need to wait for space results: first, from the SDO (Solar Dynamics Observatory) mission, already accepted for a flight at the horizon of 2009–2010 with precise resolved velocity oscillation measures (HMI/SDO instrument); second, from GOLF-NG, the successor of GOLF/SOHO, as proposed in a future mission like DynaMICCS, for which the final aim is to reveal the complete 3D vision of the Sun[11].

The problem of determining the temporal diameter evolution of the Sun is still rich and fascinating. We hope to interest a broader community to deeply investigate this field.

Acknowledgments

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[11] The microsatelitte Picard, scheduled to flight by 2009 is now conceived for other purposes than solar radius measurements –space wheather purposes and UV atmospheric images–, as the design –1 pixel per second of arc – cannot permit to achieved an astrometric precision.

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