

# Preface

Over the past two decades, the breadth and depth of influence of nonlinear science more generally, and of dispersive lattice systems such as the discrete nonlinear Schrödinger (DNLS) equation more specifically, have grown tremendously. Starting from the speculations on Davydov's soliton in biophysics and nonlinear optical couplers and proposals for waveguide arrays in the 1970s and 1980s, the studies of the DNLS-type systems passed to a different realm in the 1990s through the experimental realization of optical waveguide arrays and the observation of key theoretical predictions including discrete solitons, diffraction, Peierls barriers, multipulse features, and diffraction management. They reemerged in yet another entirely different physical incarnation in Bose–Einstein condensates (BECs) in optical lattices in the 2000s, representing one of the most exciting aspects of nonlinear phenomena in this novel state of matter. This was even more remarkable in view of the wide range of attention that BECs garnered due to their realization being awarded the Nobel prize in Physics in 2001 and their being intimately connected to superfluidity and the Nobel Prize in Physics in 2003. In the meantime, additional related aspects arose in some of the earlier settings, including but not limited to, for instance, the realization of periodic media in photorefractive crystals.

I first came across DNLS-type equations during my time as a Ph.D. student, and started working on them during my summer visits at the Center for Nonlinear Studies, at LANL, which has always been a guiding center for work around this theme (even since its original inception!). After the end of my dissertation work, the DNLS became an even stronger focal point, among other reasons due to the increasing visibility of the physical realizations of this deceptively simple-looking mathematical model. Over the years, I have had the privilege to work with a wonderfully diverse, and physically, as well as mathematically gifted set of collaborators from whom I learnt a great deal about this topic; however, I have always been surprised by the fact that despite the level of maturity that this field has arrived at, there has not been a more comprehensive publication (i.e., a book) that focuses on this main theme at the interface of nonlinear science, lattice systems, and wave phenomena. It is in that spirit that I decided to dedicate my first sabbatical from UMass in an effort to sum up some of the main axes of the phenomenology of this equation, at least as I view it and as it has been distilled through my own research efforts on the subject over the years; undoubtedly, this brings a considerable personal flavor. However, in an

effort to broaden the scope of the work, as well as to offer some of its most recent developments, I decided to partition the book into two segments. The first part, comprising the first seven chapters, consists of some of the principal and general features of the DNLS system. The second part, consisting of chapters 8–22, is a set of minireviews on more specialized, as well as often more recent topics, written by a number of friends and collaborators, who kindly offered their expert help.

At this point, I would like to thank all the contributors to this volume for their excellent coordination, and their important and informative contributions. I should also express my gratitude to all my collaborators over the years on this theme of work, for all of what they have taught me. Among them, I should especially mention Ricardo Carretero-González for his invaluable editorial and consulting assistance throughout this project. I should also thank the National Science Foundation for its support through the CAREER program and the Alexander von Humboldt Foundation for offering me the opportunity to work, as undistracted as possible, toward the completion of this book, at the University of Heidelberg, through one of its Research Fellowships. Lastly, but most importantly, I am grateful to Maria, and our two daughters, Despina and Athena, for helping me, in more ways than I can enumerate, to complete this project.

I hope that the result, despite its personal flavor and its strong parallels with my own journey through this intriguing dynamical system, will offer the reader, be they a novice, or a seasoned researcher in this field, a useful reference point where many of the fundamental ideas are explored and references are given to more specialized publications. Furthermore, I hope it will provide not only a perspective of the mathematical tools and techniques, but also a view toward the numerical computations/methods and importantly key connections to the physical realizations of this class of models. As the cycle of this work is closing, and new ones are opening up, perhaps it is relevant that I conclude this journey with one of my favorite verses of Tennyson's *Ulysses*:

“Tho’ much is taken, much abides; and though we are not now that strength which in old days moved earth and heaven; that which we are, we are; one equal temper of heroic hearts, made weak by time and fate, but strong in will to strive, to seek, to find, and not to yield.”

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*Panayotis Kevrekidis*

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