

Preface

In this monograph we apply scattering theory methods to calculations in quantum field theory, with a particular focus on properties of the quantum vacuum. These methods will provide efficient and reliable solutions to a variety of problems in quantum field theory. Our approach will also elucidate in a concrete context many of the subtleties of quantum field theory, such as divergences, regularization, and renormalization, by connecting them to more familiar results in quantum mechanics.

We will use tools of scattering theory to characterize the spectrum of energy eigenstates in a potential background, hence the term *spectral methods*. This mode spectrum comprises both discrete bound states and a continuum of scattering states. We develop a powerful formalism that parameterizes the effects of the continuum by the density of states, which we compute from scattering data. Summing the zero-point energies of these modes gives the energy of the quantum vacuum, which is one of the central quantities we study. Although the most commonly studied background potentials arise from static soliton solutions to the classical equations of motion, these methods are not limited to such cases.

A novel and central feature of this approach is its ability to make direct contact between perturbative and non-perturbative treatments of quantum field theory. Although we will study field configurations corresponding to strong potentials that cannot be treated perturbatively, we will nonetheless be able to implement standard perturbative renormalization conditions specified in terms of experimental inputs. Using these conventional renormalization conditions allows us to make direct comparisons to the perturbative sector of the quantum field theory and thus compute quantum effects for non-perturbative field configurations in an accurate and unambiguous way.

The spectral method is exact to one-loop order because it sums all one-loop Feynman diagrams with any number of insertions of the background field. This property allows us to proceed where perturbation theory or the derivative expansion would not be valid. For example, in a model with no classical solitons we can demonstrate the existence of a non-topological soliton stabilized at one-loop order by quantum fluctuations.

Our methods are also efficient for practical computations: The quantities entering a numerical calculation are cutoff independent and do not involve

differences of large numbers, so the algorithms are highly convergent. In general the quantum fluctuations are described by coupled partial differential equations. For technical reasons we are restricted to systems with sufficient symmetry so that these equations of motion allow a partial wave decomposition. Fortunately, for physically interesting systems this is often the case.

The outline of the monograph is as follows: In the first chapter, we explain the basic ideas intuitively, postponing the precise and rigorous derivations to the following two chapters. In Chap. 2 we review the technical results from scattering theory that we will need. In Chap. 3 we establish the central techniques of the spectral method. On a first reading, one may want to skip the technical details in Chap. 2 and 3 and proceed to the discussion of applications in the subsequent chapters.

In Chap. 4 we employ the spectral method to compute energies of the quantum vacuum for configurations in one spatial dimension. We consider both bosonic and fermionic quantum fields and combine these results in a simple supersymmetric model. For a system with fermionic fluctuations, we show that quantum contributions to the energy can stabilize classically unstable field configurations. In Chap. 5 we show how the spectral method is able to count states as the vacuum is distorted by the background field. In particular, it allows us to compute integer and fractional charges that are induced by bosonic background fields coupled to charged fermions. Hedgehog configurations play a major role in particle physics and the spectral method is particularly well suited for their investigation. In Chap. 6 we discuss them for two renormalizable models.

The spectral method provides an elegant means to disentangle divergences that arise in quantum field theory calculations. This property is particularly valuable for the famous Casimir effect because its study requires proper identification not only of ultraviolet divergences of quantum field theory but also of divergences that originate from singular boundary conditions. We will explore these issues in Chap. 7 in our calculation of Casimir forces and stresses. In Chap. 8 we return to particle physics models similar to those already discussed in Chap. 6. However, this time we will explore string type configurations that are translationally invariant in one spatial dimension, using the formalism developed in Chap. 3 for this purpose. Finally, in Chap. 9 we consider Q -balls as an example of quantum corrections for background fields with simple time dependence.

The spectral method is an advanced technique in the theory of quantum fields. We have tried to keep the exposition as simple and self-contained as possible, given the restricted space of a lecture note. In particular, we have added a non-technical introduction to the main aspects of the method and devoted separate chapters to the more sophisticated formalism from scattering and quantum field theory. Overall, we anticipate that the volume can be mastered with a profound knowledge of quantum mechanics alone, although familiarity with the concepts of quantum field theory will be very useful. We

expect that this monograph can be used profitably as a companion to the more standard presentation of quantum field theory, since it provides explicit examples of concepts such as dimensional regularization and gauge theory anomalies in the context of ordinary quantum mechanics. It can thus serve both as a textbook for graduate students with some background in quantum field theory and as a summary report for researchers in the field. In particular, we hope that this monograph spreads the appreciation of the many advantages that spectral methods provide for the study of quantum field theories and stimulates further developments of the subject.

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