

Preface

Magnetohydrodynamics, or MHD, is a theoretical way of describing the statics and dynamics of electrically conducting fluids. The most important of these fluids occurring in both nature and the laboratory are ionized gases, called *plasmas*. These have the simultaneous properties of conducting electricity and being electrically charge neutral on almost all length scales. The study of these gases is called *plasma physics*.

MHD is the poor cousin of plasma physics. It is the simplest theory of plasma dynamics. In most introductory courses, it is usually afforded a short chapter or lecture at most: Alfvén waves, the kink mode, and that is it. (Now, on to Landau damping!) In advanced plasma courses, such as those dealing with waves or kinetic theory, it is given an even more cursory treatment, a brief mention on the way to things more profound and interesting. (It is *just* MHD! Besides, *real* plasma physicists do kinetic theory!)

Nonetheless, MHD is an indispensable tool in all applications of plasma physics. Even the simplest experiment will not be built unless it has first passed muster with MHD. The reason is that MHD deals with fundamental force balance and deviation from it, concepts that are surprisingly subtle and complex. MHD also provides the machinery for understanding the basic properties of global structure of magnetized plasmas, how they can sustain themselves, and why they share a small number of global properties. In this course we will look at many of these important issues in what I hope is sufficient detail as to reveal some of the elegance that underlies this immensely useful theory, and gain some appreciation for the skill of the pioneers of the field.

While MHD is the simplest mathematical model of a plasma, it is difficult to justify as a valid description of any interesting plasma. Surely plasmas know that they are made of individual ions and electrons (or at a minimum of separate ion and electron fluids) and that they are so hot that collisions between particles are relatively rare events! MHD completely ignores both of these issues. Nonetheless, it is a fact that MHD provides a remarkably accurate description of the low-frequency, long-wavelength dynamics of real plasmas. MHD seems to *work* (even when it shouldn't)!

The validity of MHD as a mathematical model for magnetized plasmas has been discussed in great detail elsewhere, particularly in the book by Freidberg.² It cannot be said better, so it will not be attempted here. Instead, from the beginning we adopt the point of view that MHD describes the dynamics of a continuum fluid that is capable of conducting an electric current, that this fluid can be characterized by a few parameters such as mass density, velocity, and pressure, and that the material properties of this fluid are independent of the physical size of the sample. That is, *the material looks exactly the same no matter how finely it is subdivided*, and behavior arising from the atomic structure of matter is not considered. This is precisely the approach taken with hydrodynamics, which is one of the most complex and difficult topics in classical physics. We will find that MHD is more complex, and even more difficult, primarily as a result of spatial anisotropy introduced by the magnetic field.

In this course, MHD will be treated as a topic in theoretical physics. There will be little attempt at experimental justification or motivation; these will be taken as self-evident. Mathematics is the language of theoretical physics, and a rudimentary knowledge of what is normally called “methods of theoretical physics” (e.g., linear algebra, ordinary differential equations, Fourier analysis, and some special functions) is assumed. However, the subject requires some additional emphasis on vectors, tensors and dyads, the calculus of variations, and what is called “matched asymptotic expansions,” and these will be reviewed briefly as necessary. Of course, and as always, a certain amount of algebraic fortitude is useful.

The broad areas chosen for presentation are the derivation and properties of the fundamental equations, equilibrium, waves and instabilities, turbulence, self-organization, and dynamos. The latter topics require the inclusion of the effects of electrical resistivity and nonlinearity. Together, these span the range of MHD issues that have proven to be important for understanding magnetically confined plasmas, as they all describe either force balance, or some small deviation from it. Most have also been important in some space and astrophysical applications. It will take some effort to do them justice in the course of a single semester. Unfortunately, issues related to large deviations from force balance, such as strong flows and shocks, will not be covered, perhaps for no better reason than you cannot present what you do not know!

One of the areas where MHD has been useful is in the design and analysis of toroidally confined plasmas, in particular the tokamak configuration. The theory of these toroidal plasmas is extremely well developed and mathematically nuanced. It is unfortunate that this elegant theory has led to a jargon, a patois, that is difficult to penetrate while providing few additional physical insights. (Further, these theories of tokamak plasmas are applicable only in a vanishingly small volume of the universe, namely, the total volume of all the tokamaks!) Most of the important concepts can be illustrated in other, simpler geometries, and that is the plan here. (See also the last comment in the previous paragraph.) One exception is the equilibrium of an

² Jeffrey P. Freidberg, *Ideal Magnetohydrodynamics*, Plenum Press, New York (1987).

axially symmetric toroidal plasma, which provides an introduction to the necessary concepts and terminology. The plan is to provide sufficient background to enable further individual study.

Nonetheless, each of the topics to be presented in the course can be (and has been) pursued in significantly more (often mind-numbing) detail than can be presented here. These topics are somewhat analogous to a constellation of black holes (or perhaps rabbit holes); it is possible to disappear into any one of them for the rest of your life, never to be seen again. The best I can do is act as a guide around this universe and to try to provide you with a “star chart” so that you can avoid the most obvious traps and pitfalls in your future careers. Another goal is to provide sufficient background in MHD so as to make the published literature in these areas at least initially accessible.

In preparing this course, I have assumed that all students are at the level of advanced graduate education and that this may be one of their last formal courses before embarking on full-time research. These students have already proven that they are excellent at taking written exams. It is also conceivable, perhaps probable, that for each topic there will be some students in the class with more knowledge than the instructor. Therefore, it is useless to try to devise tricky exams or to try to be too didactic about any particular topic. The fundamental working assumption is that all students are sufficiently mature and motivated to accept and appreciate *guidance* rather than instruction and examination, and that is how I shall proceed.

Lectures in Magnetohydrodynamics

With an Appendix on Extended MHD

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