Chapter 2
The Pure Theory of Spatial Markets

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2.1 Introduction

Spatial phenomena which had been topics in theoretical Geography and Location Economics were seen as common objects in the new Regional Science of the 1950s.

A point of crystallization was the notion of spatial markets going back to Wilhelm Launhardt (1886), who perceived them, as market (and supply) areas. Market areas are territories in which a given firm is the nearest and ceteris paribus the cheapest and hence exclusive supplier.

This essay develops a systematic theory of spatial markets. It is thus an expository, drawing on previous work of T.C. Koopmans, Martin Beckmann and others of the "efficient allocation" school.

The underlying theme is the familiar one of showing how prices (indexed by location) obtained in competitive markets can guide allocation of resources in space, and the deviations from this optimum that occur under various types of institutions.

While thus backward looking, spatial markets can still provide an organizing framework to our contemporary interest in innovation.

When buyer and seller are not in the same place, distance intervenes and transaction costs for transportation and/or communication arise, ordinary market theory no longer applies, e.g., "law of the single price" is violated. We are then, faced with spatial markets. This is actually among the oldest subjects treated in location theory.

In Von Thünen's "Isolated State" land use, production and sales to a metropolitan market are investigated as functions of distance (Von Thünen 1826). Wilhelm Launhardt's location theory is grounded on market areas as sets of exclusive sales in a territory surrounding a firm or localized industry (Launhardt 1886).

In this paper we offer a modern perspective of the theory of spatial markets in perfect competition. We exclude spatial price policies under monopoly, which have a rich literature of their own (Greenhut and Ohta 1975; Beckmann 1976). Market

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strategies of oligopolists aimed at defence or penetration of market areas, in which pricing is confounded with locational choice are also beyond the scope of this paper. The pure theory of spatial markets considers only those questions that exist when locations are given. In essence, it is location theory without locational choice.

2.2 The Transportation Problem of Linear Programming

Traditionally, spatial markets have been classified as either market or supply areas centered on an exporting or importing location. But these do not exhaust all conceivable market configurations. At any rate spatial structures like these should be discovered, not assumed, by spatial economic theory.

The modern approach to spatial markets takes off from the Transportation Problem of Linear Programming (Koopmans 1949). We reformulate it in terms of given constant excess supplies q. In a set of locations $i, j, k = 1, \ldots, n$, allowing transhipments in the presence of necessary restrictions of flows to a given transportation network N, so that in all summations it is understood that indices run only over locations in the network.

Our interest is in flows x_{ij} of a commodity from point location i to point location j at unit transportation costs r_{ij} that will satisfy a commodity balance condition to meet excess demand q_i

$$\sum_j x_{ji} - x_{ij} = q. \tag{2.1}$$

Competitive market equilibrium, achieving a Pareto optimum, must then minimize total transportation costs

$$\min_{x_{ij} \geq 0} \sum_{i,j} r_{ij} x_{ij}. \tag{2.2}$$

This linear program (2.1) and (2.2) is feasible provided

$$\sum_i q_i = 0, \tag{2.3}$$

i.e., aggregate excess demand must be zero.

An optimal solution, i.e., a market equilibrium is characterized by necessary and sufficient "efficiency conditions" (Koopmans 1949). The equilibrium involves "dual variables" p_i that can be interpreted as competitive market prices. The efficiency conditions are

$$\hat{x}_{ij} \left\{ \begin{matrix} = \\ \geq \end{matrix} \right\} 0 \Leftrightarrow p_j - p_i \left\{ \begin{matrix} < \\ = \end{matrix} \right\} r_{ij}. \tag{2.4}$$

In equilibrium the transactions $\hat{x}_{ij}$ are "efficient" and thus recover their transportation cost r_{ij}, while any of the excluded inefficient transactions do not. The remarkable thing is not that this should be necessary, but that this is sufficient and together with constraints (2.1) will determine all valid equilibria.

The efficiency conditions (2.4) now imply

$$p_i = \max_j p_j - r_{ij}, \tag{2.4a}$$

$$p_j = \min_i p_i + r_{ij}. \tag{2.4b}$$

These equations state that sellers in i export to buyers j in order to maximize prices p_i received in i. Buyers in j look for the cheapest sources i after transportation costs r_{ij}.

A trading post j may both buy from cheapest sources i and sell to best buyers k

$$\min_i p_i + r_{ij} = p_j = \max_k p_k - r_{jk}. \tag{2.4c}$$

Geographically the areas containing sellers (sources) or buyers (sinks) may, but need not, overlap. In the fur trade the sources were Indians in the wilderness and the buyers, residents of Europe.

Observe, that since only price differences occur in (2.4) the price level is indeterminate, a consequence of the feasibility equation (2.3). The "primal variables" $\hat{x}_{ij}$ satisfying (2.1)–(2.4) need not be unique. A theorem assures the existence of an optimal trade pattern with at most $n-1$ flows, where n is the number of locations. This means that the flow system can form a tree or a set of trees, each being an independent market by itself.

A set of demand locations j receiving from a single supplier i is *i's* market area and *a* set of supply locations i for a single demand location j is *j's* supply area.

In addition there may be locations that will both import and export (forward). These may be called transit points or trading posts: they are common on given transportation networks.

An interesting identity relates the minimal transportation cost T to the price system

$$T = \sum_{i,j} r_{ij}\hat{x}_{ij} = \sum_{i,j} \left(p_j - p_i\right)\hat{x}_{ij} = \sum_i p_i \sum_j \left(\hat{x}_{ji} - \hat{x}_{ij}\right) = \sum_i p_i q_i \tag{2.5}$$

using (2.1) and (2.4).

This identity is a part of the Duality Principle of LP (Dantzig 1959)

$$T = \min_{X_{ij}} \sum r_{ij}x_{ij} = \max_{p_j} \sum_j p_j q_j$$

$$\text{s.t.} \sum_j x_{ji} - x_{ij} = q_i$$

$$\text{s.t.}\, p_j - p_i \leq r_{ij}. \tag{2.5a}$$

2.3 Braess's Paradox

Expanding a program by simultaneously adding an amount a to supply at some location i and to demand at some location j can reduce total transportation cost T.

Proof: Change q_i at a supply location $q_i < 0$ with $p_i > p_j$ to $q_i - a$ and at a demand location $q_j > 0$ to $q_j + a$ to obtain $T = -ap_i + ap_i = a\ (p_j - p_i) < 0$.

Figure 2.1 shows an example. Increasing supply at location d by one unit and demand in a by one unit increases "cheap" shipments from d to c and reduces (expensive) shipments from b to c by one unit while raising "cheap" shipments from b to a for a total saving of 2.

Conversely decreasing supply and demand simultaneously, even to the point of eliminating locations can increase system transportation cost.

2.4 Relaxation

The rigid relationship between excess demand and net imports may be relaxed so that demand requirements are always covered and supply availabilities are never exceeded by setting

$$\sum_j x_{ji} - y_{ij} \geq q_i. \tag{2.1a}$$

The feasibility condition (2.3) is now relaxed to

$$\sum_i q_i \leq 0. \tag{2.3a}$$

Aggregate excess demand must be non-positive.

This causes (efficiency) prices to be non-negative, and zero in the case of supply under-utilization through an additional efficiency condition

$$p_i \begin{Bmatrix} = \\ \geq \end{Bmatrix} 0 \Leftrightarrow \sum_j x_{ji} - x_{ij} \begin{Bmatrix} > \\ = \end{Bmatrix} q_i. \tag{2.6}$$

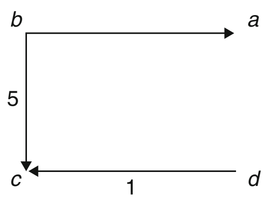

Fig. 2.1 Point $a=1$, point $b=0$, point $c=5$, point $d=4$

If "$<$" in (2.3a), this means that at some location $\hat{\imath}$the following holds

$$\sum_j x_{j\hat{\imath}} - x_{\hat{\imath}j} > q_{\hat{\imath}}. \tag{2.6a}$$

In view of (2.4b) and $r_{ij}>0$ this cannot happen at locations of positive (excess) demand; it must occur in a location of excess supply. The event (2.6a) will fix the price level $p_{\hat{\imath}} = 0$ and make prices uniquely determined.

Because of the "complementary slackness" in conditions (2.4) and (2.6) the identity (2.5) of (minimal) transportation cost and priced excess demand remains valid.

The inclusion of production cost

$$\min_{\substack{x_{ij}\geq 0 \\ z_i \geq 0}} \sum_{i,j} r_{ij}x_{ij} + \sum_i h_i(z_i)z_i, \tag{2.7}$$

$$\sum_j x_{ji} - x_{ij} \geq q_i - z_i, \tag{2.7a}$$

where z_i is the production in location and $h_i(z_i)$ is the cost function, or more simply, with constant unit cost h_i in the cost minimization description of spatial market equilibrium (2.2) is straightforward. The results are included in the next section.

2.4.1 Flexible Demand

As long as demand is given, independent of price, competitive market equilibrium can be described as achieved by aggregate cost minimization. When demand is flexible, viz. price dependent, we must resort to welfare maximization.

There are two ways of constructing an appropriate welfare function: either using the inverse of a price dependent excess demand function, or more directly as total utility, in money terms, minus aggregate costs.

That utility may be expressed in money units and hence made interpersonally comparable while perhaps objectionable to economic purists, is accepted practice in applied micro-economics. We sketch the first and elaborate the second approach. Let

$$q_i(p) \tag{2.8}$$

be the excess demand function for location i, assumed to be strictly decreasing with price p. Its inverse exists, say,

$$p_i(q) \tag{2.8a}$$

and is also strictly decreasing. The integral

$$v_i(q_i) = \int_0^{q_i} p_i(q)\mathrm{d}q \tag{2.9}$$

can then be shown to represent the sum of consumers' and producers' surplus v_i when excess demand equals q_i. Purists' objections to the consumers' surplus notwithstanding, it is this welfare measure

$$\max_{x_{ij}\geq 0} \sum_i v_i\left(\sum_j x_{ji} - x_{ij}\right) - \sum r_{ij}x_{ij} \tag{2.10}$$

which is maximized with respect to the flows x_{ij} in spatial market equilibrium (Samuelson 1952). In fact (2.10) yields

$$\hat{x}_{ij}\begin{Bmatrix} = \\ < \end{Bmatrix}0 \Leftrightarrow p_j - p_i\begin{Bmatrix} < \\ = \end{Bmatrix}r_{ij}, \tag{2.4}$$

$$\sum_j \hat{x}_{ji} - \hat{x}_{ij} = q_i(p_i). \tag{2.11}$$

In the second approach utility $u_i(q)$ of consuming a quantity q in location i of the good under consideration is considered, with the usual assumptions

$$u_i' > 0, \qquad u_i'' \leq 0. \tag{2.12}$$

Together with convex costs (say) we consider the welfare function

$$\sum_j u_j(q_j) - \sum_i h_i(z_i) - \sum_{i,j} r_{ij}x_{ij}. \tag{2.13}$$

Maximizing welfare function with the linear constraints

$$q_i = z_i + \sum_j x_{ji} - x_{ij} \tag{2.1b}$$

is a well-behaved concave nonlinear program (NLP) yielding the Kuhn–Tucker conditions in terms of dual variables or prices p_i

$$q_i\begin{Bmatrix} = \\ \geq \end{Bmatrix}0 \Leftrightarrow u_i'\begin{Bmatrix} < \\ = \end{Bmatrix}p_i, \tag{2.14}$$

$$z_i\begin{Bmatrix} = \\ \geq \end{Bmatrix}0 \Leftrightarrow h_i'\begin{Bmatrix} < \\ = \end{Bmatrix}p_i, \tag{2.15}$$

$$\hat{x}_{ij}\begin{Bmatrix} = \\ \geq \end{Bmatrix}0 \Leftrightarrow p_j - p_i\begin{Bmatrix} < \\ = \end{Bmatrix}0. \tag{2.4}$$

For simplicity and in the tradition of location theory let utility be a square of consumption q and thus demand a linear function, $q_i = a_i - p_i$, and costs be linear $h(z_i) = b_i + h_i z_i$

$$z_i\begin{Bmatrix} = \\ \geq \end{Bmatrix}0 \Leftrightarrow h_i\begin{Bmatrix} > \\ = \end{Bmatrix}p_i \tag{2.16}$$

(for standardized quantity and price units). Then (2.14) becomes

$$q_i\begin{Bmatrix} = \\ \geq \end{Bmatrix}0 \Leftrightarrow a_i - q_i\begin{Bmatrix} < \\ = \end{Bmatrix}p_i \tag{2.14a}$$

or simply

$$q_i = \max(0, a_i - p_i). \tag{2.14b}$$

Demand is a (piece-wise) linear function of price.

Depending on climatic or other regional conditions, or by longstanding local custom, low values a_i of utility may prevail which exclude local demand for a good even when its price is low. By contrast in places of poor accessibility even desirable goods of high (local) utility a_i may not be consumed due to high prices p_i.

When production is subject to capacity limits $z_i \leq c_i$ say (2.7a), then (2.15) is modified to

$$\begin{Bmatrix} z_i = 0 \\ 0 \leq z_i \leq c_i \\ z_i = c_i \end{Bmatrix} \Leftrightarrow h_i\begin{Bmatrix} > \\ = \\ \geq \end{Bmatrix}p_i \tag{2.16a}$$

production is a step function of price, and $(p - h)$ is a capacity rent.

The study of spatial markets in discrete locations for supply and demand has also been studied for supply and demand in continuously extended market areas, characterized by isotims (lines of equal price) and fields of flow (Beckmann 1952; Puu 1977; Beckmann and Puu 1985).

The flow variable x_{ij} becomes the vector v, the commodity balance equation (2.1) the divergence equation

$$\text{div } v(x) + q(x) = 0 \tag{2.1c}$$

the efficiency condition (2.4) became the gradient equation

$$k\frac{v}{|v|} = \text{grad } p. \tag{2.4c}$$

But this will not be pursued here.

2.5 Uniform Pricing

Monopolistic price strategies include, besides mill pricing (or f.o.b.) uniform delivered pricing and discriminatory pricing such as zonal tariffs or basing point (Pittsburgh plus) pricing. Under uniform pricing, buyers pay the same price inclusive of transportation costs, if they live within a specified area.

Under competitive conditions, which we consider here, only mill or uniform pricing are viable. Uniform pricing is viable only if within the specified area (or radius) all customers must be served within the specified area (or radius). Otherwise price cutting will cause the areas of free delivery to shrink, ultimately to a point, which is the suppliers' (common) location.

The special character of uniform pricing is apparent also from the fact that market equilibrium can no longer be derived from maximization of welfare (or minimization of cost).

Uniform pricing, although a form of price discrimination, has the advantage of simplicity and attractiveness and thus is the most common type in consumer product markets (Greenhut and Ohta 1975).

Under mill pricing

$$p_j = h_i + r_{ij} \tag{2.4b}$$

the market radius R is limited by the consumers' willingness to pay. Assume the same demand function f in all locations. Then at the market limit R

$$0 = a_j - p_j = a - h - R_M. \tag{2.17}$$

Under uniform pricing $\bar{p}$ the market radius R is determined by the suppliers' ability to cover their production and transportation costs

$$\bar{p} \geq h + r \tag{2.18}$$

and perfect competition will cause the "=" sign to hold

$$\bar{p} \geq h + R_u. \tag{2.18a}$$

Under mill pricing, competition drives the mill price to marginal (or constant) production costs, say at $i=0$

$$p_o = h. \tag{2.19}$$

From (2.4b), (2.17), (2.18a), (2.19)

$$a - h - R_M = 0 < a - \bar{p} = a - h - R_u \tag{2.20}$$

it follows that

$$R_u < R_M \tag{2.20a}$$

since f is decreasing.

The market radius under uniform pricing cannot be larger than under mill pricing.

When marginal production cost = average production cost = h = constant, mill pricing in equilibrium does not allow firms to recover any fixed cost F, but uniform pricing under competitive pressure does. For profits in a market of radius R from prices $\bar{p}$ set to cover marginal production and transportation costs (2.18a) are

$$\int_0^R (R - r)m(r)\mathrm{d}r = \int_0^R M(r)\mathrm{d}r > 0, \quad \text{where} \quad M(r) = \int_0^r m(r)\mathrm{d}r, \tag{2.21}$$

where $m(r)$ is the density of demand in a ring of width dr at distance r. Thus for uniform population density m

$$m(r) = 2\pi mr \tag{2.22}$$

one has

$$M(r) = \pi mr^2,$$

$$\pi mR^2 = F, \tag{2.23}$$

If the area which suppliers must serve can be expanded beyond the minimal radius R needed to recover fixed cost,

$$R = \sqrt{\frac{F}{\pi m}} \tag{2.23a}$$

both $\bar{p}$ and each firm's profits are raised until free entry reduces the latter by lowering the representative firm's demand density m so that (2.23a) holds once more.

2.6 Heterogeneous Products

Let the varieties of a good be identified with the locations of their production i. The set of product varieties is then a subset of all locations i. We assume an additive logarithmic utility function

$$u_j = \sum_i a_{ij} \log q_{ij}. \tag{2.24}$$

We denote consumption of i in location j by q_{ij} and standardize

$$\sum_i a_{ij} = 1. \tag{2.24a}$$

Also, assume a budget constraint

$$\sum_i p_{ij} q_{ij} = y_j, \tag{2.25}$$

where y_j is the budget allocated to the class of goods i in location j, where p_{ij} are prices.

A straightforward calculation for this well-known type of utility function yields the nice demand functions

$$q_{ij} = a_{ij} \frac{y_j}{p_{ij}}. \tag{2.26}$$

As before, assume mill pricing so that

$$p_{ij} = p_i + r_{ij}. \tag{2.27}$$

Demand functions of the type

$$q_{ij} = \frac{a_{ij} y_j}{p_i + r_{ij}} \tag{2.26a}$$

are close to a gravity model of spatial interaction, except for the addition of mill prices p_i to the distant terms r_{ij}. Equation (2.26a) shows that sales are approximately proportional to inverse distance.

The ratio of sales of two rival product i, k in location j is approximately

$$\frac{q_{ij}}{q_{kj}} \cong \frac{a_i}{a_k} \times \frac{r_{kj}}{r_{ij}}, \tag{2.28}$$

where "attractions" a_{ij} and a_{kj} are independent of buyer locations j and prices are small compared to distances (transportation costs).

Attractions are all the same $a_{ij} \equiv a_{kj}$, this implies that the closest seller has the dominant market share. If we redefine conventional market areas – the set of points closest to a given supplier – as areas of dominant market share, they will once more cover the entire region as mutually exclusive and exhaustive market areas. When attractions differ, these market areas are still exhaustive and exclusive, but distorted from the case where distance alone matters.

The logarithmic utility function leads to a particularly nice and simple resolution of the budget constraint. When the heterogeneous good is inexpensive enough to leave out any budgetary restrictions, additive power functions as utility functions will also generate a gravity distance effect. Only utilities of the entropy type will generate a (negative) exponential distance effect in spatial markets.

2.7 Innovation

Spatial markets are a convenient vehicle to study the regional impact of innovations. In particular they offer a natural classification of the spatially relevant types of innovation.

2.7.1 Demand

An innovation in demand can take the following forms: A known product is introduced in new locations or market areas. Inventive sales managers in search of new outlets will eventually sell refrigerators even to the Eskimos. Secondly, new uses may be found for a product in some locations (perhaps in conjunction with price reductions). Most importantly, new products may be introduced, which of course will have to be advertised. It is a well established practice to test the acceptance of a new product in some test location(s) that are considered to be normal enough to serve as good market predictors.

2.7.2 Supply

New (and better) locations – with better access to labor or other inputs, better climate, etc. – may be discovered for the production of a known good, or better methods may be found in the existing locations. The locations of a firm may also be restructured to generate new varieties of a product. The invention of a new product will always require locational choice. New products with an initially small national demand are best started from the metropolis.

An innovation improving supply over extended areas was the so-called green revolution, the introduction of higher yielding and better resistant crops.

Another important innovation is the discovery of new resource deposits (oil, coal, ores) or sources of labor supply in overlooked settlements.

2.7.3 Distribution

While geographical distances remain unchanged, their impact through transportation and communication cost has been strongly reduced through innovations.

Transportation costs have shown a secular trend to fall. This has been due to both technological innovations and to changes in organization (regulation). In our times the greatest changes have occurred in communication, particularly through the internet. As a result market areas no longer need to be contiguous or close to the point of supply. The limitation on demand that is imposed by information deficits about a product has been lifted and this has vastly expanded the potential markets of many products.

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