

Contents

1	Introduction	1
1.1	Quantum Open Systems	1
1.2	Approaches to Continuous Measurements	2
1.3	Classical SDEs in Continuous Measurement Theory	3
1.4	The Plan of the Book	3
	References	4

Part I General Theory

2	The Stochastic Schrödinger Equation	11
2.1	Introduction	11
2.2	Linear Stochastic Differential Equations	12
2.2.1	An Homogeneous Linear SDE in Hilbert Space	13
2.2.2	The Stochastic Evolution Operator	14
2.2.3	The Square Norm of the Solution	17
2.3	The Linear Stochastic Schrödinger Equation	19
2.3.1	A Key Restriction	19
2.3.2	A Change of Probability	21
2.4	The Physical Interpretation	21
2.4.1	The POM of the Output and the Physical Probabilities	23
2.4.2	The A Posteriori States	25
2.4.3	Infinite Time Horizon	26
2.4.4	The Conservative Case	27
2.5	The Stochastic Schrödinger Equation	28
2.5.1	The Stochastic Differential of the A Posteriori State	28
2.5.2	Four Stochastic Schrödinger Equations	30
2.5.3	Existence and Uniqueness of the Solution	33
2.5.4	The Stochastic Schrödinger Equation as a Starting Point	38
2.6	The Linear Approach Versus the Nonlinear One	40
2.7	Tricks to Simplify the Equations	41
2.7.1	Time-Dependent Coefficients and Unitary Transformations	41

2.7.2	Complex Noise	42
2.8	Summary: The Stochastic Schrödinger Equation	43
2.8.1	The Linear Stochastic Schrödinger Equation	43
2.8.2	The Nonlinear Stochastic Schrödinger Equation	46
	References	47
3	The Stochastic Master Equation: Part I	51
3.1	From Hilbert Space to Statistical Formulation	51
3.1.1	A Mixed Initial State	51
3.1.2	The Linear Stochastic Master Equation – I	54
3.2	The Master Equation	54
3.2.1	The Mean Statistical Operator	54
3.2.2	The Mean Dynamics	55
3.2.3	Quantum Dynamical Semigroups	57
3.3	An Incomplete Observation	58
3.4	The Statistical Formulation	61
3.4.1	The Linear Stochastic Master Equation – II	61
3.4.2	The Physical Probabilities, the A Posteriori and the A Priori States	67
3.4.3	Back to the Hilbert Space Formulation	70
3.5	The Stochastic Master Equation	70
3.6	Summary of Linear Quantum Trajectories	71
3.6.1	The Master Equation	71
3.6.2	Reference Probability Space and Filtrations	72
3.6.3	The Linear Stochastic Master Equation for Non-normalised States	73
3.6.4	The Physical Probability and the POMs	74
3.6.5	The Nonlinear Stochastic Master Equation for Normalised States	74
	References	74
4	Continuous Measurements and Instruments	77
4.1	The Instruments	77
4.1.1	The Construction of the Instruments	77
4.1.2	Interpretation of the Instruments and the Output	79
4.2	Characteristic Functional and Characteristic Operator	83
4.2.1	Characteristic Function	83
4.2.2	Characteristic Functional and Finite Dimensional Laws	84
4.2.3	Characteristic Operator	86
4.2.4	One-to-One Correspondence Between Characteristic Operators and Instruments	88
4.3	Moments	90
4.3.1	The Mean	90
4.3.2	Higher Moments	91

4.4	Classical Post-measurement Processing	94
4.4.1	Time-Local Transformations	94
4.4.2	Response with Time Delay	99
4.5	Autocorrelation and Spectrum of the Output Process	100
4.5.1	The Spectrum of a Stationary Process	100
4.5.2	The Spectrum of an Asymptotically Stationary Process ..	102
4.5.3	The Spectrum of the Output of the Continuous Measurement	104
4.6	Summary	108
	References	109
5	The Stochastic Master Equation: Part II	111
5.1	Quantum Trajectories: The Nonlinear SDE Formulation	111
5.1.1	The Nonlinear SDE	112
5.1.2	Stochastic Master Equation, A Posteriori States and Output	115
5.1.3	The Linear SDE	115
5.1.4	Characteristic Functional and Instruments	116
5.2	Purification of the A Posteriori States	118
5.2.1	Preservation of Pure States	118
5.2.2	From Mixed to Pure States	119
	References	123
6	Mutual Entropies and Information Gain in Quantum Continuous Measurements	125
6.1	Introduction	125
6.2	States and Entropies in the Discrete Case	126
6.2.1	Algebras and States	126
6.2.2	Entropies and Relative Entropies	126
6.2.3	Mutual Entropy and χ -Quantities	127
6.3	The Continuous Measurement as a Channel	129
6.3.1	von Neumann Algebras and Normal States	129
6.3.2	Entropies	130
6.3.3	Channels	131
6.4	A Classical Continuous Information Gain	132
6.4.1	Product Densities	133
6.4.2	The Classical Information Gain	134
6.4.3	An Upper Bound on the Increments of the Classical Information gain	136
6.5	The Information Embedded in the A Posteriori States	137
6.6	Gain of Information on the Initial State: The Input/Output Classical Information	138
	References	142

Part II Physical Applications

7	Quantum Optical Systems	145
7.1	How to Construct Physical Models	145
7.1.1	From the Full Quantum Level to the Reduced Description by SDEs	145
7.1.2	Observed and Unobserved Channels — The Generators	146
7.2	Heterodyne and Homodyne Detection	148
7.2.1	The Measurement Scheme	148
7.2.2	Homodyning Versus Heterodyning	150
	References	150
8	A Two-Level Atom: General Setup	151
8.1	A Two-Level Atom Stimulated by a Laser	151
8.1.1	The Pauli Matrices	151
8.1.2	The Bloch Representation	152
8.1.3	The System Operators	153
8.2	The Reduced Dynamics of the Two-Level Atom	157
8.2.1	The Reduced Dynamics and the Rotating Frame	158
8.2.2	The Bloch Equations	159
8.2.3	The Evolution in the Generic Case	163
8.3	The Equilibrium State η_{eq}	165
8.3.1	Convergence to Equilibrium	165
8.3.2	The Explicit Expression of η_{eq}	167
8.3.3	Some Properties of η_{eq}	168
8.4	The SDEs for the Two-Level Atom	172
8.4.1	The SDEs in the Hilbert Space	172
8.4.2	The Stochastic Master Equation	175
8.4.3	Linear Entropy and Atomic Squeezing	177
8.4.4	The Instruments in the Rotating Frame	179
8.5	Summary	179
8.5.1	Bloch Representation of States and Terminology	179
8.5.2	The Model and the Parameters	180
	References	182
9	A Two-Level Atom: Heterodyne and Homodyne Spectra	183
9.1	Heterodyne Detection and Mollow Spectrum	183
9.1.1	The Output Current and the Electrical Power	183
9.1.2	The Fluorescence Spectrum	188
9.2	Homodyne Detection	199
9.2.1	The Spectrum of the Homodyne Current	199
9.2.2	Examples of Spectra and Squeezing	206
9.3	Summary	214
9.3.1	Atomic and Measurement Quantities	214

9.3.2	Heterodyne Spectral Density	214
9.3.3	Homodyne Spectral Density	216
	References	219
10	Feedback	221
10.1	Introduction	221
10.1.1	Feedback and Control of Quantum Systems	221
10.1.2	Feedback Control of a Quantum Optical System	222
10.2	The Feedback Scheme of Wiseman and Milburn	222
10.2.1	Observation Without Feedback	223
10.2.2	Introduction of the Feedback	224
10.3	The Two-Level Atom with Feedback	225
10.3.1	The Model	226
10.3.2	A Priori States	229
10.3.3	The Case of Many Stationary States	233
10.3.4	Equilibrium	234
10.3.5	The Nonlinear Stochastic Master Equation	234
10.4	Control of the Atomic State	235
10.5	Control of the Squeezing of Fluorescence Light	239
10.5.1	The Spectral Densities	240
10.5.2	Channel 1: The In-Loop Light	241
10.5.3	Channel 2	243
10.5.4	The Spectrum of the Cross-Correlations	245
10.5.5	Global Squeezing of Light and Atomic Squeezing	246
10.6	Control and Line Narrowing	248
10.6.1	A First Case of Line Narrowing	249
10.6.2	A Second Case of Line Narrowing	256
10.7	Summary	258
10.7.1	Atomic and Measurement Quantities	259
10.7.2	Homodyne Spectral Densities	259
10.7.3	Important Features of the Homodyne Spectral Densities	261
	References	261
A	Ordinary SDEs	263
A.1	Probability Spaces and Random Variables	263
A.1.1	Probability Spaces	263
A.1.2	Densities, Absolute Continuity, Conditional Expectations	267
A.2	Filtrations and Processes	269
A.2.1	Stochastic Processes	269
A.2.2	Filtrations	270
A.2.3	Adapted Processes	270
A.2.4	The Law of a Continuous Process	272
A.2.5	Martingales and Stopping Times	272

A.2.6	The Wiener Process	273
A.3	Stochastic Calculus	274
A.3.1	Classes of Integrands	275
A.3.2	Integrals on Time	275
A.3.3	Stochastic Integrals	276
A.3.4	Itô Formula	277
A.4	Ordinary Stochastic Differential Equations	279
A.4.1	The Main Class of SDEs	279
A.4.2	Types of Solutions	279
A.4.3	Sufficient Conditions for Existence and Uniqueness	280
A.4.4	L^p -Estimates on the Solution	281
A.5	Change of Measure and Girsanov Transformation	282
A.5.1	A Characterisation of the Wiener Process	282
A.5.2	Exponential of Itô Processes	283
A.5.3	Positive Martingales and Change of Measure	285
A.5.4	Girsanov Theorem	286
A.5.5	Extension of the Local Probability Measures	289
References	291
B	Some Notions of Quantum Mechanics	293
B.1	Notations	293
B.1.1	The Hilbert Space $\mathcal{H} = \mathbb{C}^n$	293
B.1.2	Operators	293
B.1.3	Dirac Notations	295
B.2	The Hilbert Space Formulation of Quantum Mechanics	296
B.2.1	Observables	296
B.2.2	The Abstract Schrödinger Equation	298
B.3	The Statistical Formulation of Quantum Mechanics	298
B.3.1	Statistical Operators	299
B.3.2	The von Neumann Equation	300
B.3.3	Master Equation and Quantum Dynamical Semigroups	301
B.4	Instruments	302
B.4.1	Operations and Events	302
B.4.2	Instruments and Observables	304
B.4.3	A Sequence of Measurements	309
References	311
References	313
Acronyms & Symbols	321
Index	323

Quantum Trajectories and Measurements in Continuous
Time

The Diffusive Case

Barchielli, A.; Gregoratti, M.

2009, XIV, 325 p. 30 illus., Hardcover

ISBN: 978-3-642-01297-6