Preface

The purpose of this Handbook is to highlight both theory and applications of weighted automata.

Weighted finite automata are classical nondeterministic finite automata in which the transitions carry weights. These weights may model, e.g., the cost involved when executing a transition, the amount of resources or time needed for this, or the probability or reliability of its successful execution. The behavior of weighted finite automata can then be considered as the function (suitably defined) associating with each word the weight of its execution. Clearly, weights can also be added to classical automata with infinite state sets like pushdown automata; this extension constitutes the general concept of weighted automata.

To illustrate the diversity of weighted automata, let us consider the following scenarios. Assume that a quantitative system is modeled by a classical automaton in which the transitions carry as weights the amount of resources needed for their execution. Then the amount of resources needed for a path in this weighted automaton is obtained simply as the sum of the weights of its transitions. Given a word, we might be interested in the minimal amount of resources needed for its execution, i.e., for the successful paths realizing the given word. In this example, we could also replace the "resources" by "profit" and then be interested in the maximal profit realized, correspondingly, by a given word. Furthermore, if the transitions carry probabilities as weights, the reliability of a path can be formalized as the product of the probabilities of its transitions, and the reliability of a word could be defined again as the maximum of the reliabilities of its successful paths. As another example, we may obtain the multiplicity of a word, defined as the number of paths realizing it, as follows: let each transition have weight 1; for paths take again the product of the weights of its transitions (which equals 1); then the multiplicity of a word equals the sum of the weights of its successful paths. Finally, if in the latter example we replace sum by "maximum," weight 1 is associated to a word if and only if it is accepted by the given classical automaton.

In all of these examples, the algebraic structure underlying the computations with the weights is that of a semiring. Therefore, we obtain a uniform and powerful automaton model if the weights are taken from an abstract semiring. Here the multiplication of the semiring is used for determining the weight of a path, and the weight of a word is then obtained by the sum of the weights of its successful paths. In particular, classical automata are obtained as weighted automata over the Boolean semiring. Many constructions and algorithms known from classical automata theory can be performed very generally for such weighted automata over large classes of semirings. For particular properties, sometimes additional assumptions on the underlying semiring are needed.

Another dimension of diversity evolves by considering weighted automata over discrete structures other than finite words, e.g., infinite words, trees, traces, series-parallel posets, or pictures. Alternatively, in a weighted automaton, the state set needs not to be finite, so we can consider, e.g., weighted pushdown automata with states being pairs of states (in the usual meaning) and the contents of the pushdown tape. Moreover, weighted context-free grammars and algebraic systems arise from weighted automata over trees by using the well-known equivalence between frontier sets of recognizable tree languages and context-free languages.

For the definition of weighted automata and their behaviors, matrices and formal power series are used. This makes it possible to use methods of linear algebra over semirings for more succinct, elegant, and convincing proofs.

Weighted finite automata and weighted context-free grammars were first introduced in the seminal papers of Marcel-Paul Schützenberger (1961) and Noam Chomsky and Marcel-Paul Schützenberger (1963), respectively. These general models have found much interest in Computer Science due to their importance both in theory as well as in current practical applications. For instance, the theory of weighted finite automata and weighted context-free grammars was essential for the solution of classical automata theoretic problems like the decidability of the equivalence: of unambiguous context-free languages and regular languages; of deterministic finite multitape automata; and of deterministic pushdown automata. For the variety of theoretical results discovered, we refer the reader to the indispensable monographs by Samuel Eilenberg (1974), Arto Salomaa and Matti Soittola (1978), Wolfgang Wechler (1978), Jean Berstel and Christophe Reutenauer (1984), Werner Kuich and Arto Salomaa (1986), and Jacques Sakarovitch (2003). (See Chap. 1 for precise references.) On the other hand, weighted automata and weighted context-free grammars have been used as basic concepts in natural language processing and speech recognition, and recently, weighted automata have been used in algorithms for digital image compression.

Since the publication of the mentioned monographs, the field of weighted automata has further developed both in depth and breadth.[1] The editors of this Handbook are very happy that international experts of the different areas agreed to write survey articles on the present shape of their respective field. The chapters of this Handbook were written such that a basic knowledge of automata and formal language theory suffices for their understanding.

Next, we give a short overview of the contents of this Handbook. Part I provides foundations. More specifically, in Chap. 1, Manfred Droste and Werner Kuich present basic foundations for the theory of weighted automata, in particular, semirings, formal power series, and matrices. As is well known, regular and context-free languages can be obtained as least solutions of suitable fixed point equations. In Chap. 2, Zoltán Ésik provides an introduction to that part of the theory of fixed points that has applications to weighted automata and their behaviors, and to weighted context-free grammars in the shape of algebraic systems.

Part II of this Handbook investigates different concepts of weighted recognizability. In Chap. 3, Zoltán Ésik and Werner Kuich develop the theory of finite automata starting from ideas based on linear algebra over semirings. In particular, they derive the fundamental Kleene–Schützenberger characterization of the behaviors of weighted automata over Conway semirings. In Chap. 4, Jacques Sakarovitch presents the theory of rational and recognizable formal power series over arbitrary semirings and graded monoids. As a consequence, he derives that the equivalence of deterministic multitape transducers is decidable. A seminal theorem of J. Richard Büchi (1960) and Calvin C. Elgot (1961) shows the equivalence in expressive power between classical finite automata (over finite and infinite words) and monadic second-order logic. In Chap. 5, Manfred Droste and Paul Gastin present a weighted version of monadic second-order logic and derive corresponding equivalence results for weighted automata. In Chap. 6, Mehryar Mohri presents several fundamental algorithms for weighted graphs, weighted automata, and regulated transducers as, e.g., algorithms for shortest-distance computation, ε-removal, determinization, minimization, and composition.

In Part III of this Handbook, alternative types of weighted automata and various discrete structures other than words are considered. In Chap. 7, Ion Petre and Arto Salomaa present the core aspects of the theory of algebraic power series in noncommuting variables, weighted pushdown automata, and their relationship to formal languages. In Chap. 8, Juha Honkala extends the theory of algebraic power series by considering Lindenmayerian algebraic systems and several restricted such systems. The following two chapters consider weighted automata acting on extensions of finite words. In Chap. 9, Zoltán Fülöp and Heiko Vogler survey the theory of weighted tree automata and weighted tree transducers. This combines classical results of weighted au-

[1] For instance, see the biennial workshops on "Weighted Automata: Theory and Applications" (WATA) since 2002.

tomata and transducers on words and of unweighted tree automata and tree transducers. In Chap. 10, Ina Fichtner, Dietrich Kuske, and Ingmar Meinecke present different weighted automata models for concurrent processes, formalized by traces and series-parallel posets, and analyze their relationships. They also consider two-dimensional extensions of words, namely pictures.

Part IV deals with applications of weighted automata. In Chap. 11, Jürgen Albert and Jarkko Kari present the use of weighted automata and transducers for digital image compression and give comparisons with the image compression standard JPEG. In Chap. 12, George Rahonis describes the theory of fuzzy recognizable languages. This theory arises by considering weighted automata over particular semirings, namely bounded distributive lattices. In Chap. 13, Christel Baier, Marcus Größer, and Frank Ciesinski present the main concepts of Markov decision processes as an operational model for probabilistic systems, and basic steps for the (qualitative and quantitative) analysis against linear-time properties. In Chap. 14, Kevin Knight and Jonathan May address the reawakened interest in string and tree automata among computational linguists. The chapter surveys tasks occurring in natural language processing and shows their solutions by using weighted automata.

Some of the chapters contain open problems. We hope that this will stimulate further research.

Finally, we would like to express our thanks to all authors of this Handbook and to the referees for their careful work. Moreover, warm thanks go to Carmen Heger for her support in the technical compilation of the chapters.

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