

Contents

| | |
|--|-----|
| Introduction | 1 |
| 1 The Discrete Time Case | 7 |
| 1.1 Normal Martingales | 7 |
| 1.2 Stochastic Integrals | 8 |
| 1.3 Multiple Stochastic Integrals | 11 |
| 1.4 Structure Equations | 13 |
| 1.5 Chaos Representation | 15 |
| 1.6 Gradient Operator | 18 |
| 1.7 Clark Formula and Predictable Representation | 22 |
| 1.8 Divergence Operator | 24 |
| 1.9 Ornstein-Uhlenbeck Semi-Group and Process | 28 |
| 1.10 Covariance Identities | 32 |
| 1.11 Deviation Inequalities | 36 |
| 1.12 Logarithmic Sobolev Inequalities | 42 |
| 1.13 Change of Variable Formula | 49 |
| 1.14 Option Hedging | 53 |
| 1.15 Notes and References | 58 |
| 2 Continuous Time Normal Martingales | 59 |
| 2.1 Normal Martingales | 59 |
| 2.2 Brownian Motion | 60 |
| 2.3 Compensated Poisson Martingale | 63 |
| 2.4 Compound Poisson Martingale | 71 |
| 2.5 Stochastic Integrals | 74 |
| 2.6 Predictable Representation Property | 84 |
| 2.7 Multiple Stochastic Integrals | 86 |
| 2.8 Chaos Representation Property | 89 |
| 2.9 Quadratic Variation | 90 |
| 2.10 Structure Equations | 93 |
| 2.11 Product Formula for Stochastic Integrals | 96 |
| 2.12 Itô Formula | 102 |
| 2.13 Exponential Vectors | 107 |

| | | |
|----------|--|------------|
| 2.14 | Vector-Valued Case | 109 |
| 2.15 | Notes and References | 111 |
| 3 | Gradient and Divergence Operators | 113 |
| 3.1 | Definition and Closability | 113 |
| 3.2 | Clark Formula and Predictable Representation | 114 |
| 3.3 | Divergence and Stochastic Integrals | 119 |
| 3.4 | Covariance Identities | 121 |
| 3.5 | Logarithmic Sobolev Inequalities | 123 |
| 3.6 | Deviation Inequalities | 125 |
| 3.7 | Markovian Representation | 127 |
| 3.8 | Notes and References | 130 |
| 4 | Annihilation and Creation Operators | 131 |
| 4.1 | Duality Relation | 131 |
| 4.2 | Annihilation Operator | 134 |
| 4.3 | Creation Operator | 138 |
| 4.4 | Ornstein-Uhlenbeck Semi-Group | 144 |
| 4.5 | Deterministic Structure Equations | 146 |
| 4.6 | Exponential Vectors | 151 |
| 4.7 | Deviation Inequalities | 154 |
| 4.8 | Derivation of Fock Kernels | 158 |
| 4.9 | Notes and References | 160 |
| 5 | Analysis on the Wiener Space | 161 |
| 5.1 | Multiple Wiener Integrals | 161 |
| 5.2 | Gradient and Divergence Operators | 166 |
| 5.3 | Ornstein-Uhlenbeck Semi-Group | 171 |
| 5.4 | Covariance Identities and Inequalities | 173 |
| 5.5 | Moment Identities for Skorohod Integrals | 177 |
| 5.6 | Differential Calculus on Random Morphisms | 180 |
| 5.7 | Riemannian Brownian Motion | 186 |
| 5.8 | Time Changes on Brownian Motion | 192 |
| 5.9 | Notes and References | 194 |
| 6 | Analysis on the Poisson Space | 195 |
| 6.1 | Poisson Random Measures | 195 |
| 6.2 | Multiple Poisson Stochastic Integrals | 203 |
| 6.3 | Chaos Representation Property | 212 |
| 6.4 | Finite Difference Gradient | 218 |
| 6.5 | Divergence Operator | 226 |
| 6.6 | Characterization of Poisson Measures | 231 |
| 6.7 | Clark Formula and Lévy Processes | 234 |
| 6.8 | Covariance Identities | 237 |

| | | |
|----------|--|------------|
| 6.9 | Deviation Inequalities | 241 |
| 6.10 | Notes and References | 245 |
| 7 | Local Gradients on the Poisson Space | 247 |
| 7.1 | Intrinsic Gradient on Configuration Spaces | 247 |
| 7.2 | Damped Gradient on the Half Line | 255 |
| 7.3 | Damped Gradient on a Compact Interval | 263 |
| 7.4 | Chaos Expansions | 267 |
| 7.5 | Covariance Identities and Deviation Inequalities | 270 |
| 7.6 | Some Geometric Aspects of Poisson Analysis | 272 |
| 7.7 | Chaos Interpretation of Time Changes | 277 |
| 7.8 | Notes and References | 280 |
| 8 | Option Hedging in Continuous Time | 281 |
| 8.1 | Market Model..... | 281 |
| 8.2 | Hedging by the Clark Formula | 284 |
| 8.3 | Black-Scholes PDE | 288 |
| 8.4 | Asian Options and Deterministic Structure | 290 |
| 8.5 | Notes and References | 293 |
| 9 | Appendix..... | 295 |
| 9.1 | Measurability | 295 |
| 9.2 | Gaussian Random Variables..... | 295 |
| 9.3 | Conditional Expectation | 296 |
| 9.4 | Martingales in Discrete Time..... | 296 |
| 9.5 | Martingales in Continuous Time | 297 |
| 9.6 | Markov Processes..... | 298 |
| 9.7 | Tensor Products of L^2 Spaces | 299 |
| 9.8 | Closability of Linear Operators | 300 |
| | References..... | 301 |
| | Index | 309 |

Stochastic Analysis in Discrete and Continuous Settings
With Normal Martingales

Privault, N.

2009, XVI, 282 p., Softcover

ISBN: 978-3-642-02379-8