

Contents

Preface	vii
Motivation	vii
Overview	viii
Acknowledgements	xi
1 Borel Sets, Measures and All That	1
1.1 Introduction	1
1.2 Categories and Functors	2
1.2.1 Bisimilarity and Behavioral Equivalence	3
1.2.2 Yoneda’s Lemma	4
1.3 σ -Algebras and Their Generators	4
1.4 Polish and Analytic Spaces	9
1.4.1 Metric Spaces	10
1.4.2 Polish Spaces: Elementary Properties	13
1.4.3 Manipulating Polish Topologies	15
1.4.4 Analytic Spaces	17
1.5 Measurable Selectors	23
1.6 Probability Measures	25
1.6.1 Weak Topology	28
1.6.2 Metrizability of the Weak Topology	30
1.6.3 The Weak*- σ -Algebra	32
1.6.4 Applications of the π - λ -Theorem	39
1.7 Smooth Equivalence Relations	43
1.7.1 Invariant Borel Sets	47
1.7.2 Operations on Smooth Relations	49
1.7.3 Congruences for Stochastic Relations	53
1.8 Grounded Relations on Subprobabilities	55
1.8.1 A Borel Isomorphism	59
1.8.2 Characterizing Groundedness	61
1.9 Bibliographic Notes	67

2	Modal Logics	69
2.1	Introduction	69
2.2	Bisimulations	70
2.3	Modal Logics: Syntax and Semantics	76
2.3.1	Examples	79
2.3.2	Bisimulations for Kripke Models	82
2.4	Temporal Logics: $\mu\mathbf{CSL}$	84
2.4.1	The Logic $\mu\mathbf{CSL}$	84
2.4.2	Defining Models and Their Morphisms	85
2.4.3	Interpreting $\mu\mathbf{CSL}$	88
2.4.4	Congruences	91
2.4.5	Logical Equivalence and Bisimilarity	93
2.5	Bibliographic Notes	99
2.6	Appendix: Behavioral and Logical Equivalence Reconsidered	100
2.6.1	Discussing the Strategy	101
2.6.2	The Equivalence Relation Induced by the Logic \mathfrak{L}	102
2.6.3	Logical Equivalence	106
2.6.4	Logical vs. Behavioral Equivalence	107
3	The Giry Monad: Randomized Morphisms	113
3.1	Introduction	113
3.2	The Giry Monad	115
3.3	Randomized Congruences and Morphisms	117
3.3.1	Randomizing Morphisms	120
3.3.2	Relating Randomized Morphisms and Congruences	123
3.3.3	Factoring Through a Randomized Congruence	128
3.4	Randomized Morphisms for Modal Logic	131
3.4.1	A Simple Hennessy-Milner Logic, Again	133
3.4.2	Borel Isomorphisms	138
3.4.3	Distributional Equivalence	141
3.4.4	Bisimulations	148
3.5	Bibliographic Notes	154
4	Coalgebraic Logic	157
4.1	Introduction	157
4.2	Predicate Liftings	159
4.3	Left Coalgebras	164
4.3.1	Stochastic Left Coalgebras	165
4.3.2	The Logic	167
4.3.3	Congruences	171
4.3.4	Application to Modal Logic	179
4.3.5	A Remark on Extending the Logic	185
4.4	Right Coalgebras	186
4.4.1	Stochastic Right Coalgebras	187
4.4.2	Models, Lifting Valuations	190

4.4.3	Logical vs. Behavioral Equivalence	196
4.4.4	Behavioral Equivalence vs. Bisimilarity	199
4.4.5	A Local Construction	208
4.5	Bibliographic Notes	213
Logics: The Grammars	215
	Basic Modal Language: $\mathfrak{M}_b(\tau, P)$	215
	The Extended Modal Language: $\mathfrak{M}_s(\tau, P)$	215
	Hennessy-Milner Logic: $\mathfrak{L}(\text{Act}, \mathbb{Q} \cap [0, 1])$	216
	Continuous Time Stochastic Logic With Fixed-Point Operators: $\mu\text{CSL}(\text{AP}, \text{SV}, \text{PV})$	216
	Coalgebraic Logic (Left Case): $\mathfrak{L}^b(\Lambda, \Theta, \Gamma)$	216
	Coalgebraic Logic (Right Case): $\mathfrak{L}^\sharp(\Lambda, \text{Br}, \text{Inf}, \mathbb{V})$	217
References	219
List of Symbols	225
Index	227



<http://www.springer.com/978-3-642-02994-3>

Stochastic Coalgebraic Logic

Doberkat, E.-E.

2009, XV, 231 p. 81 illus., Hardcover

ISBN: 978-3-642-02994-3