Invited Paper

Coherence Holography: A Thought on Synthesis and Analysis of Optical Coherence Fields

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1 Introduction

Coherence plays fundamental role in holography and interferometry, and control of coherence is essential in an illumination system for microscopy and lithography. This talk is intended to give a tutorial introduction to the concept, the principle, and the applications of a new technique of unconventional holography, called *coherence holography*, which was developed recently for synthesis and analysis of a spatial coherence function [1]. The basic principle of the new technique was inspired by the fact the coherence function $\Gamma(\boldsymbol{r}_1, t_1; \boldsymbol{r}_2, t_2)$ and the optical field $u(\boldsymbol{r}, t)$ bear a formal analogy in the sense that both obey the same wave equation:

$$\begin{pmatrix}\text{Coherence}\\ \text{Function}\end{pmatrix} \qquad \nabla^2_{i=1,2}\Gamma(\boldsymbol{r}_1, t_1; \boldsymbol{r}_2, t_2) = \frac{1}{c^2}\frac{\partial^2 \Gamma(\boldsymbol{r}_1, t_1; \boldsymbol{r}_2, t_2)}{\partial t^2_{i=1,2}}; \tag{1}$$

$$\begin{pmatrix}\text{Optical}\\ \text{Field}\end{pmatrix} \qquad \nabla^2 u(\boldsymbol{r}, t) = \frac{1}{c^2}\frac{\partial^2 u(\boldsymbol{r}, t)}{\partial t^2}.$$

This formal mathematical analogy suggests that the basic principle of holography already established for the wave of optical field can also be applied for the wave of coherence function. Just as a conventional computer-generated hologram can create an arbitrary 3-D optical field, a computer-generated coherence hologram (CGCH) can synthesize an optical field with a desired 3-D coherence function. Furthermore, the time-space symmetry in the wave equation indicates that the role of a temporal coherence function can be replaced by a spatial coherence function. This leads to a new concept of dispersion-free optical coherence tomography

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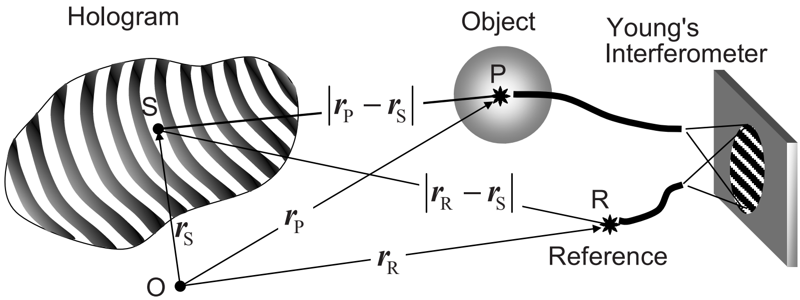

Fig. 1. Schematic geometry for holographic recording and reconstruction of a coherence hologram

and profilometry [2], which makes use of a spatial (rather than temporal) coherence function of quasi-monochromatic light synthesized by CGCH.

2 Principle of coherence holography

2.1 Reciprocity in spatial coherence and hologram recording

The principle of coherence holography can be explained on the basis of reciprocity in spatial coherence and holographic recording. Referring to Fig.1, let us first consider recording of a conventional hologram. We note an arbitrary point P at $\boldsymbol{r}_P$ on a 3-D object recorded with a reference beam from a point source R at $\boldsymbol{r}_R$. Suppose a point S at $\boldsymbol{r}_S$ on the hologram is located on one of the bright fringes resulting from constructive interference between the object and reference beams from points P and R. Then the optical path difference (OPD) between the two beams becomes integer multiples of the wavelength λ:

$$OPD(\boldsymbol{r}_P, \boldsymbol{r}_R; \boldsymbol{r}_S) = |\boldsymbol{r}_P - \boldsymbol{r}_S| - |\boldsymbol{r}_R - \boldsymbol{r}_S| = n\lambda \tag{1}$$

Next we illuminate the hologram with spatially incoherent quasi-monochromatic light. This generates an extended incoherent source with its irradiance distribution proportional to the intensity distribution of the hologram $I(\boldsymbol{r}_S)$. Let us now consider a reciprocal process in which light emitted from point S on the source reaches points P and R to create fields $u(\boldsymbol{r}_P)$ and $u(\boldsymbol{r}_R)$, respectively. From reciprocity of wave propagation and because of Eq. (1), $u(\boldsymbol{r}_P)$ and $u(\boldsymbol{r}_R)$ are always in phase with the phase difference $2n\pi$, irrespective of the initial phase and the position of the point source S, as far as S is located on one of the bright fringes; this condition is automatically satisfied for all points on the hologram because no lights are emitted from the location of the dark fringes. Thus the incoherently illuminated hologram creates an optical field which is in phase and has high coherence between this particular pair of points at P and R. If we fix one point as a reference at R and move the other point P over the 3-D space with $\boldsymbol{r}_P$ as a variable, we can reconstruct the object as a 3-D distribution of the mutual intensity $J(\boldsymbol{r}_P,\boldsymbol{r}_R)=\langle u(\boldsymbol{r}_P)u^*(\boldsymbol{r}_R)\rangle$, which can be detected with a suitable interferometer. In Fig. 1, a Young's interferometer using an optical fibre is shown just for a conceptually simplest example.

2.2 Formal analogy between the diffraction integral and van Cittert-Zernike theorem

Another explanation of the principle can be made on the basis of formal analogy between the diffraction integral and van Cittert-Zernike theorem. Let us consider a conventional reconstruction process in which the hologram is illuminated from backward with a coherent spherical wave $\exp(-ik|\boldsymbol{r}_S-\boldsymbol{r}_R|)/|\boldsymbol{r}_S-\boldsymbol{r}_R|$ converging into the reference source point R. The reconstructed optical field, which forms a real image of the 3-D object at point P, is given by the diffraction integral for the Fresnel-Huygens secondary waves

$$u(\boldsymbol{r}_P,\boldsymbol{r}_R)=\iint I(\boldsymbol{r}_S)\frac{\exp[ik(|\boldsymbol{r}_P-\boldsymbol{r}_S|-|\boldsymbol{r}_R-\boldsymbol{r}_S|)]}{|\boldsymbol{r}_P-\boldsymbol{r}_S|\cdot|\boldsymbol{r}_R-\boldsymbol{r}_S|}d\boldsymbol{r}_S \tag{2}$$

It should be noted that Eq. (2) has exactly the same form as the formula for mutual intensity given by van Cittert-Zernike theorem

$$J(\boldsymbol{r}_P,\boldsymbol{r}_R)=\iint I(\boldsymbol{r}_S)\frac{\exp[ik(|\boldsymbol{r}_P-\boldsymbol{r}_S|-|\boldsymbol{r}_R-\boldsymbol{r}_S|)]}{|\boldsymbol{r}_P-\boldsymbol{r}_S|\cdot|\boldsymbol{r}_R-\boldsymbol{r}_S|}d\boldsymbol{r}_S \tag{3}$$

except the difference that $I(\boldsymbol{r}_S)$ in Eq. (2) is the amplitude transmittance of the hologram whereas $I(\boldsymbol{r}_S)$ in Eq. (3) is the intensity distribution of the spatially incoherent source. The implication of this formal analogy is that if a hologram, having intensity transmittance $I(\boldsymbol{r}_S)$ proportional to the recorded intensity, is illuminated with spatially incoherent light, an optical field will be generated, for which the mutual intensity between observation

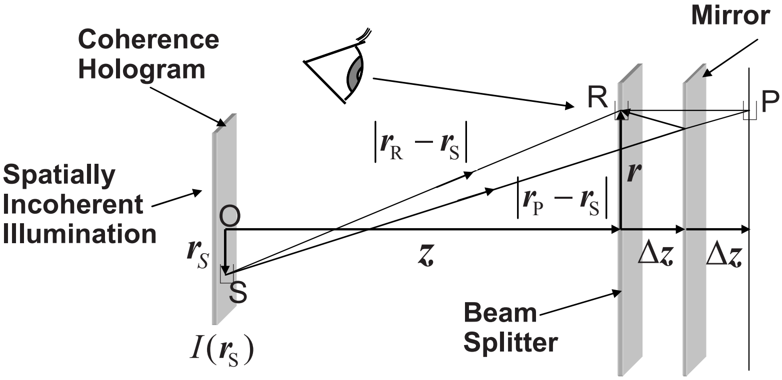

Fig. 2. Schematic geometry for reconstruction of a coherence hologram

point P and reference point R is equal to the optical field that would be reconstructed if the hologram with the same amplitude transmittance were illuminated with a phase-conjugated version of the reference beam. Unlike conventional holography, the reconstructed coherence image is not directly observable. It can be visualized only as the contrast and the phase of an interference fringe pattern by using an appropriate interferometer.

3 Experiment

Detection of coherence image by scanning the probe point of Young's interferometer, as shown in Fig.1, is impractical. We used a simple optical geometry shown in Fig.2, which is in effect a Fizeau interferometer but we realized it with a Michelson interferometer in our experiment. Light from an incoherently illuminated coherence hologram is guided into the interferometer to form interference fringes on the plane of the beam splitter as the result of interference between the fields at point R and point P.

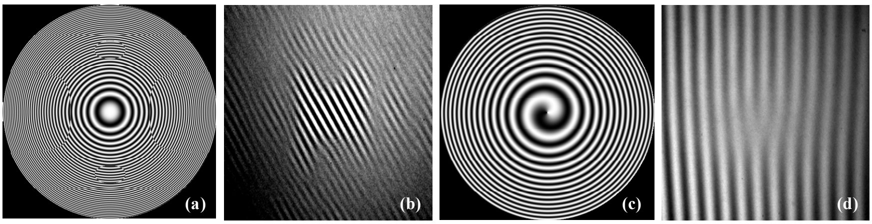

Fig. 3. (a) Computer-generated coherence hologram for a letter H. (b) Coherence image with the high coherence region representing letter H. (c) Computer-generated coherence hologram for generating a coherence vortex. (d) Interferogram of the coherence vortex with phase singularity in the coherence function.

The fringe contrast reflects the mutual intensity between the two points given by van Cittert-Zernike theorem in Eq. (3). Using the position vectors defined in Fig. 2, the mutual intensity is now approximated by

$$\begin{aligned} J(\boldsymbol{r}_P,\boldsymbol{r}_R) &\propto \iint I(\boldsymbol{r}_S)\exp[ik(|\boldsymbol{r}_P-\boldsymbol{r}_S|-|\boldsymbol{r}_R-\boldsymbol{r}_S|)]d\boldsymbol{r}_S \\ &= J(\boldsymbol{r},\Delta z)=\iint I(\boldsymbol{r}_S)\exp\left[ik(|\boldsymbol{z}+2\Delta \boldsymbol{z}+\boldsymbol{r}-\boldsymbol{r}_S|-|\boldsymbol{z}+\boldsymbol{r}-\boldsymbol{r}_S|)\right]d\boldsymbol{r}_S \\ &\approx \exp\left[i\alpha(\Delta z)\right]\iint I(\boldsymbol{r}_S)\exp\left[-ik\left(\frac{\Delta z}{z}\right)\times\frac{|\boldsymbol{r}-\boldsymbol{r}_S|^2}{z}\right]d\boldsymbol{r}_S \end{aligned} \tag{4}$$

where $\alpha(\Delta z)=k(2\Delta z)\left[1-2(\Delta z/z)^2\right]$ and the distance z between the hologram and the observation plane is assumed to be much larger than other parameters. It should be noted that the mutual intensity is given by the Fresnel transform of the incoherently illuminated hologram. If we record a Fresnel hologram with coherent light for an object at distance $\bar{z}=z^2/2\Delta z$ from the hologram, and illuminate the hologram with spatially incoherent light from behind, we will observe on the beam splitter a set of interference fringe patterns whose fringe contrast and phase represent, respectively, the field amplitude and the phase of the original object recorded with coherent light. Just as a computer-generated hologram (CGH) can create a three-dimensional image of a non-existing object, a computer-generated coherence hologram (CGCH) can create an optical field with a desired three-dimensional distribution of spatial coherence function. This CGCH gives a new possibility of optical tomography and profilometry [2] based on a synthesized spatial coherence function, and serves as a generator of coherence vortices [3]. An example of an on-axis (Gabor-type) CGCH for an object of a letter H is shown in Fig.3 (a). Figure 3 (b) shows the reconstructed coherence image, in which the letter H is displayed by the region of high contrast fringes representing the designed high coherence area [1]. Figures 3 (c) and 3(d) show, respectively, a coherence hologram for generating a coherence vortex and an interferogram of the coherence vortex [3].

4 McCutchen theorem and dispersion-free depth sensing with a spatial frequency comb generated by coherence holography

When the incoherent source (representing the coherence hologram in Fig.1) is at a distance very large compared to that between the two observation points P and R, the mutual intensity $J(\boldsymbol{r}_P, \boldsymbol{r}_R)$ in Eq. (3) can be approximated by a formula in the form of the Debye integral, which we call McCutchen theorem [4]:

$$J(\boldsymbol{r}_P - \boldsymbol{r}_R) \approx \iint I(\boldsymbol{r}_S)\exp\left[-ik(\frac{\boldsymbol{r}_S - \boldsymbol{r}_R}{|\boldsymbol{r}_S - \boldsymbol{r}_R|})\cdot(\boldsymbol{r}_P - \boldsymbol{r}_R)\right]\frac{d\boldsymbol{r}_S}{|\boldsymbol{r}_S - \boldsymbol{r}_R|^2} \tag{5}$$
$$= \iint I(\boldsymbol{c}_S)\exp\left[-ik\boldsymbol{c}_S\cdot(\boldsymbol{r}_P - \boldsymbol{r}_R)\right]d\Omega_S$$

where $\boldsymbol{c}_S = (\boldsymbol{r}_S - \boldsymbol{r}_R)/|\boldsymbol{r}_S - \boldsymbol{r}_R|$ is a unit vector pointing from point R toward the source element $d\boldsymbol{r}_S$, $d\Omega_S = d\boldsymbol{r}_S/|\boldsymbol{r}_S - \boldsymbol{r}_R|^2$ is an increment of solid angle subtending the source element $d\boldsymbol{r}_S$ (see, Fig.4). The source distribution projected onto the unit sphere (*c* sphere) is called a generalized source. MacCutchen theorem can have a more simplified form of 1-D Fourier transform:

$$J(L) = \int_{-\infty}^{\infty} I(l)\exp(-ikLl)dl \tag{6}$$

where $L\boldsymbol{d} = \boldsymbol{r}_P - \boldsymbol{r}_R$ and $l = \boldsymbol{c}_S\cdot\boldsymbol{d}$, with $\boldsymbol{d}$ being a unit vector in the direction of $\boldsymbol{r}_P - \boldsymbol{r}_R$, and $I(l)$ is the projection of the generalized source upon a line oriented in the same $\boldsymbol{d}$ direction in the **c** sphere; $I(l)$ is called the source distribution. McCutchen theorem gives an insight into the new concept of a spatial frequency comb and a spatial coherence comb which can be applied for dispersion-free depth sensing [5]. Suppose a coherence hologram with a Fresnel-zone-plate-like incoherent ring source structure as shown in Fig.5. The thin ring sources are first projected onto the McCutchen *c* sphere and then onto a line connecting a reference point R and an observation point R, to form a periodic impulse source distribution with period Δl, expressed by a comb function $I(l) = \text{comb}(l/\Delta l)$. This source distribution is called a spatial frequency comb because the

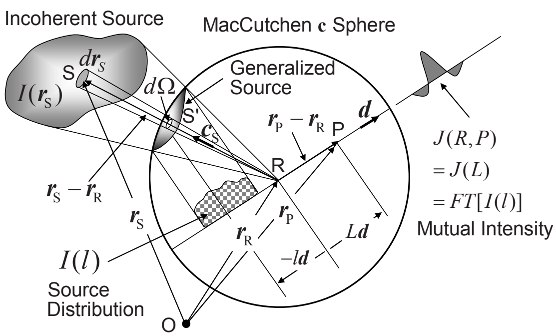

Fig.4. McCutchen theorem for generalized source and mutual intensity

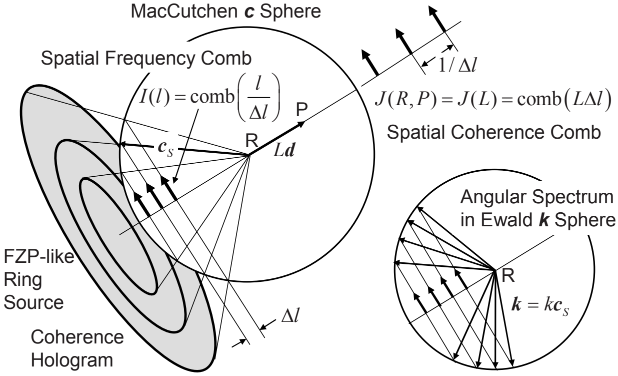

Fig.5. Spatial frequency comb generated by a coherence holography

directional unit vector $\boldsymbol{c}_s$ pointing the ring source can be related to a ***k***-vector by $\boldsymbol{k} = k\boldsymbol{c}_s$ which represents an angular spectrum in an Ewald sphere as shown in Fig.5. From the Fourier relation in Eq. (6), the mutual intensity of the optical field between points R and P (separated by L on the line parallel to ***d***) becomes a comb function $J(L) = \mathrm{comb}(L\Delta l)$, called a spatial coherence comb, and exhibits a selective high spatial coherence in the direction of ***d*** with period inversely proportional to Δl . By controlling the FZP-like ring source in the coherence hologram with a spatial light modulator (SLM), we can scan the coherence comb function in the depth direction ***d*** to obtain a 3-D tomography image. Unlike a conventional optical frequency comb (which is composed of equally spaced multiple polychromatic line spectra), the spatial frequency comb is generated by monochromatic light and opens up a new possibility of spatial coherence tomography completely free from dispersion problem [5].

An example of depth sensing with a variable longitudinal spatial coherence comb function created by an SLM-generated tunable spatial frequency comb is shown in Fig.6 [5]. Shown in Fig.6 (a) is a coherence hologram with a FZP-like source distribution, in which each ring source is composed of a same number of point sources to equalize the comb heights. The interval of the spatial coherence comb is varied by SLM, and fringe contrast on the surface of two gauge blocks with a 400 μm height gap was observed with a Michelson interferometer. In Fig. (b), fringe contrast is low because the coherence comb does not match the surfaces of the gauge blocks, whereas high-contrast fringes are observed in Fig. (c) and Fig. (d) because the coherence comb matches the surfaces I and II, respectively.

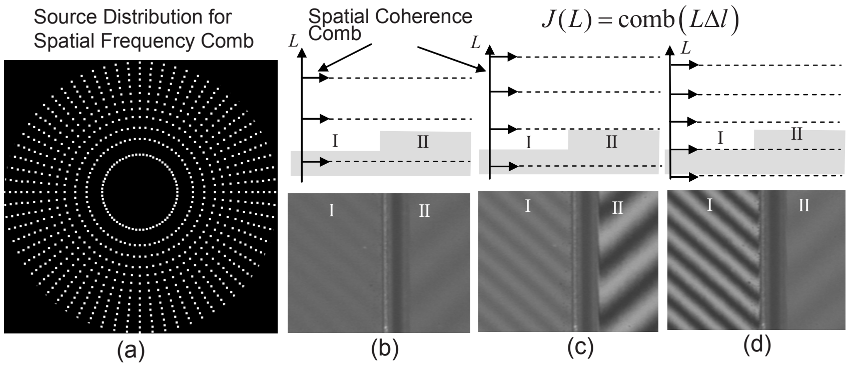

Fig.6. Sensing of the depth of block gauge surfaces by gating with a spatial coherence comb. (a) Source distribution for spatial frequency comb; (b)-(d) spatial coherence gating function is scanned by changing the interval of the spatial coherence comb.

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