Chapter 1
Introduction

The aim of this first chapter is to introduce basic notions of counterparty credit exposure, and to motivate with a few simple examples the problems and concepts we will be considering in more detail later in this book.

1.1 Basic Concepts

Consider two parties, A and B say, who enter into an OTC transactions portfolio.[1] This portfolio could consist of products ranging from interest-rate and cross-currency swaps in different currencies with various exotic features, to exotic options on equity, foreign exchange and commodity underlyings. It could also include various types of credit derivatives contracts, such as credit default swaps (CDS) on single names or collateral debt obligations (CDO) tranches in swap form on portfolios of reference entities, or credit indices.[2]

In general a given company, say a financial institution A, will have portfolios with many other counterparties, varying among sovereign entities, corporates, hedge funds, insurance companies (including for examples monolines[3]). It may also happen that the credit quality of the counterparty is not independent of the performance of the transaction entered into, such as what happens for example, when an electricity generating oil-powered plant bets on the price of oil.

Counterparty credit exposure is the amount a company, say A, could potentially lose in the event of one of its counterparties defaulting. It can be computed by simulating in different scenarios and at different times in the future, the price of the

[1] An OTC (Over The Counter) transaction is a transaction that is not traded through an exchange.

[2] A typical credit index is for example iTraxx; it is composed of the 125 most liquid CDS names referencing European investment grade credits.

[3] Monoline insurance are companies that guarantee to bond investors the payment of coupon and notional. They insure different type of securities, such as CDO, structured products and municipal bonds. Monolines have been affected in the recent credit crunch, raising counterparty risk issues.

G. Cesari et al., *Modelling, Pricing, and Hedging Counterparty Credit Exposure,*
Springer Finance, DOI 10.1007/978-3-642-04454-0_1,

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transactions with the given counterparty, and then by using some chosen statistic to characterise the price distributions that have been generated. Typical statistics used in the industry are (i) the mean, (ii) the 97.5% or 99% quantile, called *Potential Future Exposure* (PFE), and (iii) the mean of the positive part of the distribution, referred to as the *Expected Positive Exposure* (EPE). We will also have occasion to speak about less commonly used statistical measures that can be more appropriate for certain products.

As important as measuring counterparty exposure, via PFE or EPE, is the computation of the cost of hedging it, and the capability of having a dynamic hedging strategy, i.e. the computation of exposure sensitivities. In the financial industry the price of hedging is generally called *Credit Valuation Adjustment* (CVA). We will see that there are strong links between EPE and CVA computation.

1.2 Preliminary Examples

Some simple examples will help clarifying these points.

1.2.1 Vanilla Interest-Rate Swap

Consider counterparties A and B who enter into an interest-rate swap where A receives every six months the 6-month Libor rate on a notional of $100 million, while paying to B a fixed amount equal to the par 10-year swap rate on the same notional observed at inception.

This is a typical swap contract with value zero at inception. As time passes and market conditions change, the value of the swap changes accordingly. Thus, if the

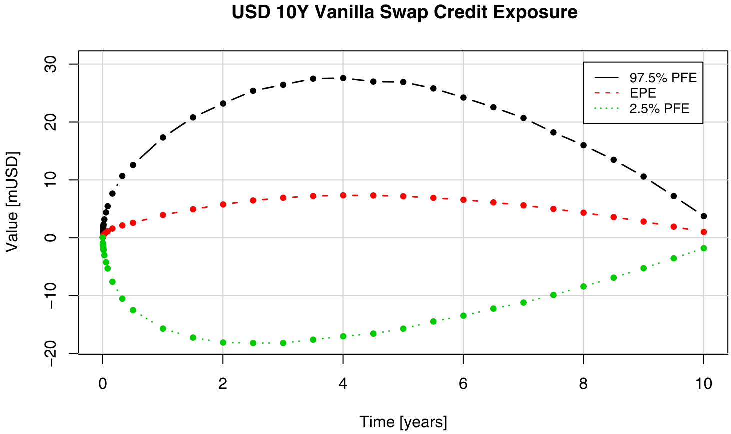

Fig. 1.1 Exposure profile for a typical USD 10-years swap contract, paying fix and receiving floating on a notional of 100 mUSD. The full distribution is shown in Fig. 1.2

swap rate decreases (resp. increases), the transaction will be out of the money (resp. in the money) as seen from A's point of view. Therefore, if B were to default at a point in the life of the trade when swap rates had increased, then A would need to replace in the market—*at higher cost than the fixed amount being paid to B*—the floating cashflows promised and not delivered by B.

To compute the credit exposure for the swap, we would need to estimate the values the swap could take in different market scenarios at points in the future. Figure 1.1 shows the 97.5%, the 2.5% quantiles and the EPE of the swap price distribution, over its entire life, as seen from party A's point of view. A plot like this is usually referred to as the exposure profile. Note that the 2.5% quantile seen from A's perspective, corresponds to the 97.5% quantile seen from B's perspective.

Figure 1.2 shows in the top panel the full price distribution over time. The bottom panel shows three slices of this distribution at three different points in time.

For this example, the 97.5% PFE quantile is a function that starts at zero, peaks at around the 4-year point and then decreases to zero. First, by definition, the fixed payment in the trade is the fair value for the swap, and this must therefore have value (and hence exposure level) identically equal to zero at inception. Similarly, towards the end of the transaction, when all payments but one due under the swap have been paid, the exposure remaining is that from only a single coupon exchange. This explains what happens at the right end of the profile. At intermediate times, the shape of the profile is the result of opposing effects. On the one hand, as the interest rates underlying the swap diffuse, there is more variability in the realised Libor rates, potentially leading to higher exposure. On the other hand, as time evolves there are fewer payments remaining under the swap, and this mitigates the effect of diffusing rates.

The profile therefore tells us that with 97.5% probability, the loss of A due to default of B will not exceed roughly \$28 million. Of course, this estimate is based on market information at inception of the swap, and would change if it were to be repeated at a different time.

1.2.2 Cancellable Swap

We can make our example slightly more interesting. It is common for swaps to trade with an additional callability feature, whereby one counterparty would have the option, at certain times in the life of the swap, to cancel ("call") the transaction for a fixed fee (which may be zero).[4]

Suppose that party A, from whose perspective we look at exposure, also holds the option to cancel the trade; one says that A is *long callability*. Assuming A behaves

[4]We define "cancellable swap" a swap which has an embedded option to terminate it at zero cost (or at a given predetermined fee). Sometimes these swaps are also called "callable". We use the term callable swap in a more generic way, considering the possibility that the swap is "called" into a new product. In this sense a cancellable swap is a simple example of a callable swap.

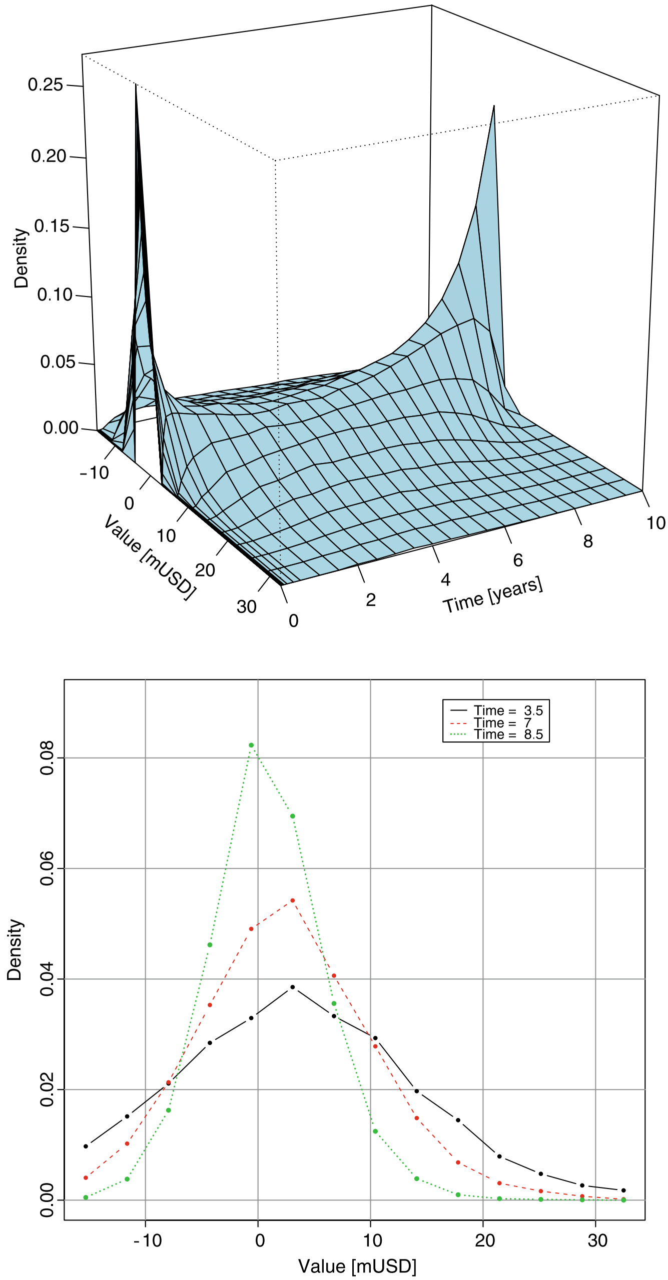

Fig. 1.2 Future value distribution for a typical USD 10-years swap contract, paying fix and receiving floating. The PFE and EPE are shown in Fig. 1.1

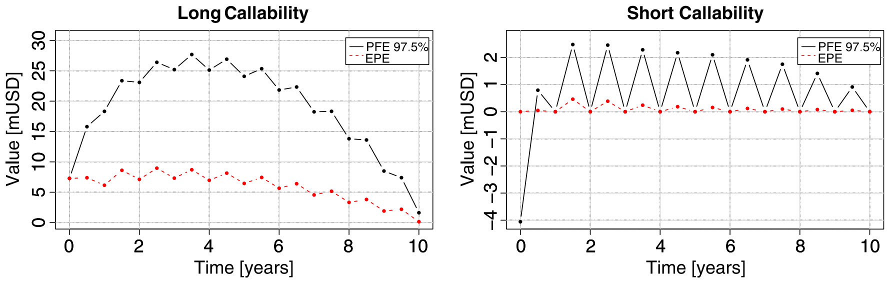

Fig. 1.3 Exposure of a typical cancellable 10-years swap, paying yearly the fair swap rate fixed at inception and receiving semi-annually the 6-month Libor rate on a notional of 100 mUSD. On the left (resp. right), the exposure represents long (resp. short) optionality to cancel the swap every year. The value of the swap at time-zero corresponds to the value of the option, which is assumed to be paid up-front by the counterparty

rationally, it would never decide to walk away from the swap in those scenarios where the swap has a high value (because the swap rate has increased and future receivables are worth more than at inception). This means that having the option to cancel, at zero cost, should not affect materially A's exposure. On the other hand, suppose A is *short callability*, meaning that it is B who has the option to walk away from the swap. Rational behaviour on B's part implies that B would cancel the swap when they are making a loss on the transaction, which is exactly when A would be in the money. Thus we would expect that with A being short callability, A's PFE (and EPE) to B is reduced to zero at each date where B has the option to cancel the transaction.

Figure 1.3 shows all this happening. In the left panel, we see that with A having the option to cancel the trade, the PFE profile is similar to that of a vanilla swap, with the exception of the time-zero level, which equates to the value of the cancellation option. On the right we see that A's exposure is reduced to zero at dates where B can cancel; on remaining dates, the PFE is reduced to that arising from coupons due until the next allowed cancellation date. Note that the time-zero point is not zero but negative from A's point of view, since it is B who holds the option in this case. Note that in practice the value of the option is often embedded in the fixed coupon of the swap, which has then value zero at inception.

From the computational point of view, there is a fundamental difference between the vanilla swap example of the previous section and the cancellable swap we have just described. A vanilla swap can in fact be priced analytically and in a model independent way, and therefore, as we will see, exposure could be computed in a classical Monte Carlo framework, where scenarios are generated and then products are priced at each scenario and each time step. On the other hand, a cancellable swap is *priced* using a lattice or Monte Carlo simulation, making therefore impractical the computation of credit exposure itself by Monte Carlo simulation.[5] This would entail in fact a Monte Carlo of Monte Carlo approach (with nested simulations), where one set of simulation is used for scenarios and one set, at each time step and scenario, for pricing. We will analyse this aspect in more details in the next chapters.

1.2.3 Managing Credit Risk—Collateral, Credit Default Swap

When structuring a new transaction (or portfolio of transactions), one of the criteria is the amount of acceptable credit exposure. This will depend on several factors including risk appetite and quality of the counterparty. The most common way to reduce counterparty exposure is to set up a collateral agreement, whereby the client is required to deposit collateral into a separate account at regular time intervals. A collateral agreement between counterparties can take one of several forms. For instance, it can be in the form of cash or securities, can be called daily or at other regular intervals, and there can be thresholds and minimum transfer amounts. In addition, since the point of holding collateral is to be able to liquidate it in case of the counterparty defaulting, market liquidity plays an important role in determining the amount of collateral needed. When collateral agreements are in place, therefore, credit exposure computation has to take into account features of that agreement together with the dynamics of the trade itself, in order to compute so-called *close-out risk*. Close-out risk measures the amount by which the value of a transaction could change during the period from when the counterparty is deemed to have defaulted, until the collateral has been liquidated and used to fund, at current market conditions, the replacement of the defaulted counterparty in the transaction. In general this computation should also include change in value of the collateral, possibly taking into account the correlation between collateral and transaction value.

A further possibility for A to manage counterparty credit exposure to B is to buy Credit Default Swap (CDS) protection on B from another counterparty C. The transaction between A and C is typically fully collateralised. This will transfer the risk of B to C. In case of default of B, the CDS would ensure that C will step in and make good any payments that were originally promised by B, or simply pay the value of the transaction. This should cover the value of the products (e.g. the interest-rate swap we described before) as calculated at the time of B's default. The value of the protection is called Credit Valuation Adjustment (CVA) and in principle should be charged to the client (in our case counterparty B) in order to reflect its credit risk.

For instance, suppose the credit spread of B is 100 bps,[6] the amount to protect \$100 million, the trade maturity 10 years. Under these market conditions the price

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[5]Note that a cancellable swap is the combination of a vanilla swap and a Bermudan option. If the option is European (i.e. the swap can be called only on one date), the cancellable swap can be priced in closed form.

[6]bps: basis points, a hundredth of a percent.

of buying protection on B will be in the order of $10 million.[7] Such protection is sufficient only at the time of calculation, and one would need to compute exposure sensitivities to the underlying factors in order to dynamically hedge the required amount of protection on B, as the future exposure to B will evolve with market conditions.

As we mentioned, the usage of CDS transfers the credit exposure from B to C. So, even if one assumes the exposure to B to be perfectly hedged via the CDS, there will be counterparty exposure to C, which offers protection.[8] Consider again the example above where A buys CDS protection on B on a notional amount of $100 million. Figure 1.4 shows a typical profile for such a transaction, assuming it is un-collateralised. For such a default protection product, the exposure one observes

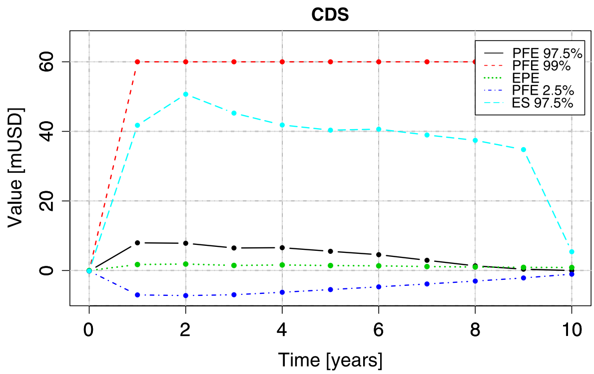

Fig. 1.4 Exposure of a typical credit default swap on a notional of 100 mUSD and spread about 100 bps. PFE (on different quantiles), EPE and ES are shown

results from the effects of (i) movements in the simulated credit spread of B and (ii) defaults. Clearly, the payment triggered by B's default, equal to about $1 - R = 60\%$ of notional, would imply that in a default scenario, A would have an exposure to C of $60 million. R is the recovery rate, i.e. the amount which can be recovered upon default of the counterparty. Now in Fig. 1.4, the PFE profile (which we recall is the 97.5% quantile of the distribution) does not show such high levels of exposure. This must mean that in the simulated scenarios, fewer than 2.5% of the scenarios involve B defaulting. Or in other words that the event of B defaulting is a rare event. To take into account this event one could display higher quantiles of the distribution, say the 99.9% quantile. Alternatively, one can calculate the *Expected Shortfall (ES)* of the distribution, which is simply the expected value of the tail of the distribution (see Chap. 12 for more details); this measure will uncover any large outliers in the distribution (such as the rare event of default of B, and hence large payment by C, in this case). Figure 1.4 displays this quantity, and clearly shows that defaults are indeed occurring even if they are not frequent enough to affect the 97.5% PFE.

[7]This is roughly equal to the 10-years duration multiplied by the 10-years spread.

[8]This is one of the reasons why, after the 2007–08 credit events, it is under discussion to use clearing houses when dealing with credit default swaps.

This example shows that with credit products, where events of *small* probability can lead to *large* payments, the PFE might not be the appropriate exposure measure to consider. We will have more to say on this in due course.

1.3 Why Compute Counterparty Credit Exposure?

Counterparty risk is at the root of traditional banking. Historically, the first form of financial instruments were bonds, and their value was mainly driven by the market's view of how creditworthy the issuers of these bonds were. However, today's financial world is much more complex, and the process of estimating counterparty risk much more challenging. While for loans and other traditional products the focus is mainly on estimating the capability of the borrower to repay its obligation, for derivative transactions, estimating accurately the future value of the transaction is as important and challenging as having a view on the ability of the counterparty to honour its obligations.

Accuracy is important because credit exposure models are used for several purposes in financial institutions, such as

(i) Setting limits on the amount of business allowed with a particular counterparty.
(ii) Dynamic hedging of counterparty risk, by buying credit protection on the counterparty. This in effect allows one to trade away counterparty credit risk.
(iii) Computation of risk weighted assets and capital requirements.
(iv) Obtaining insight about prices of complex transactions in potential future scenarios. For example, while counterparty risk is concerned with measuring how *high* the value of a transaction can go (and therefore how much a counterparty would owe), there are similarities between this and computing Value at Risk, or stress testing, where one would be interested in how much the value of a transaction could drop.

1.4 Modelling Counterparty Credit Exposure

In the previous sections we have introduced the concept of counterparty exposure and have provided some simple examples. We focus now on a more formal approach which will give a flavour of the mathematical tools we will need in the next chapters.

1.4.1 Definition

Given a portfolio of positions traded with a counterparty, the main quantity we need to model to compute the counterparty credit exposure at time t, is the distribution V_t of the portfolio prices, computed at time $t > 0$ and seen from today. We will see in the next chapter how V_t can be described in its full generality. For the moment we consider the case of products without callability features and where cashflow payments are performed at discrete time points (T_i), $i = 1, \dots, n$, with T_n being the maturity of the trade. Define X_t to be the (generally stochastic) payment made by the portfolio at any time t ($X_t = 0$ if t is not a member of (T_i), $i = 1, \dots, n$). Then at any time $t \geq 0$, V_t can be expressed as:

$$V_t = N_t \sum_{T_i > t}^{T_n} \mathbb{E}\left(\frac{X_{T_i}}{N_{T_i}} \,\middle|\, \mathscr{F}_t \right), \tag{1.1}$$

where N_t indicates the numeraire, $\mathbb{E}$ is the expectation in the numeraire measure and $\mathscr{F}_t$ the usual filtration. More details of the concept of numeraire, pricing measure, and filtration can be found in the literature (see for example Baxter & Rennie [10] for an intuitive description, or Rogers & Williams [93, 94] and Shreve [98] for a more formal approach) and will also be given later in this book (see Appendix B). For our purposes here it is enough to think of the numeraire as being the cash account, used to discount cashflows, and the filtration as the information available at time t.

At time $t = 0$ the distribution degenerates into the current price of the portfolio. We are interested in the distribution of V_t under either the real or the pricing (called also risk-neutral) measure. In general the price distribution V_t will change with time due to changes in market conditions, portfolio composition (for example due to payment of cashflows), and time value. If the portfolio is collateralised, it can be extended to take into account additional positions representing the collateral value. The computation of the price distribution V_t depends also on specific contractual features with the counterparty, e.g. netting agreements between short and long positions in the portfolio, or break clauses held by the counterparties.

The industry practice to compute exposure is to use a simple Monte Carlo framework implemented in three steps: (i) scenario generation, (ii) pricing, and (iii) aggregation.

The first step involves generating scenarios of the underlying risk factors at future points in time. Simple products can then be priced on each scenario and each time step, therefore generating empirical price distributions. From the price distribution at each time it is then possible to extract convenient statistical quantities. Exposure of portfolios can be computed by consistently pricing different products on the same underlying scenarios and aggregating the results taking into account possible netting and collateral agreement with the counterparty.

If taken literally, this approach works only for relatively simple products which can be priced analytically, or which can be approximated in analytical form, and which do not need complex calibrations depending on market scenarios. More exotic products requiring relatively complex pricing, cannot be treated in this way. As already mentioned, even a cancellable swap, which is a relatively simple product, cannot be computed easily in this framework.

In the next chapters we will show how (1.1) can be generalised and which algorithms can be implemented to compute exposure for more exotic products. We will also challenge the simple Monte Carlo approach we have just described, and see how more sophisticated modelling frameworks can provide answers to some of the common problems faced when building a counterparty exposure system.

1.4.2 Risk Measures

For practical reasons it can be useful to characterise the distribution V_t with some statistical quantities which can then be used for various risk controlling or risk management purposes. The Potential Future Exposure (PFE), computed at time t is defined as

$$\mathrm{PFE}_{\alpha,t} = q_{\alpha,t} = \inf\{x : \mathbb{P}(V_t \leq x) \geq \alpha\}, \tag{1.2}$$

where α is the given confidence level, and $\mathbb{P}$ indicates the probability distribution of V_t. Note that this is a function of time t and is the price of the obligation *in the future* given a set of scenarios. This pricing is called sometimes *Mark-to-Future*. The graph of $\mathrm{PFE}_{\alpha,t}$ as a function of t is known as the exposure profile of the trade.

Similarly the Expected Positive Exposure (EPE) will be computed as[9]

$$\mathrm{EPE}_t = \mathbb{E}\left[V_t^+\right], \tag{1.3}$$

where the expectation can be taken under the real or pricing measure depending on the usage of EPE.

An alternative measure to the quantile is the Expected Shortfall, called also Expected Tail Loss, defined as

$$\mathrm{ES}_{\alpha,t} = \mathbb{E}\left[V_t \mid V_t > \mathrm{PFE}_{\alpha,t}\right]. \tag{1.4}$$

Expected shortfall is used especially when it is convenient to have a measure which takes into account events of significant magnitude, which, however, can occur with only very small probability. As we have shown above, typical examples are credit derivatives, where the default of the reference entity protected by the derivative is a low probability event, which, however has significant impact.

1.4.3 Netting and Aggregation

In general, the credit exposure to a particular counterparty arises not from a single transaction but several ones. For any particular market scenario, some of these transactions will have positive, and others negative value. Consider, for example, a long and a short position on an option on highly correlated stocks, a portfolio of payers and receivers swaps[10] in different currencies, or, as a more sophisticated example,

[9] We will see later in Chap. 12 and Chap. 14 that other definitions of EPE are more appropriate to compute CVA.

[10] A payer (resp. receiver) swap, is a swap that pays (resp. receives) a fixed rate and receives (resp. pays) a floating Libor rate.

a long position on ABX[11] and a short position on a tranche of pool of MBS. One would expect that, at a given time as one position increases in value, the value of the other position decreases. Since both transactions are facing the same counterparty, it is natural to think about the possibility of *netting* these positive and negative values together, in order to reduce the overall exposure.

The possibility of treating risk in this way will depend on the legal agreement in place. Netting agreements can have different flavours. For example for a given counterparty it could be possible to net together interest-rate swaps, but not swaps with e.g. equity transactions.

From the quantitative and computational perspective netting and no-netting agreements will determine how aggregation is performed within a pool of transactions. The main challenge is the requirement of being scenario consistent across trades. This means that the price distributions of all transactions have to be computed together in order to choose the correct risk measure of the whole portfolio together with the correct netting agreements. This can pose significant constraints on the software architecture as well as on the computational capacity.

Once counterparty exposure is computed at portfolio level, one can be interested in assigning a portion of the exposure to each single transaction. It is interesting to note that this is not equivalent to computing exposure for each single transaction separately. This process of redistributing exposure is often called *exposure allocation* or *disaggregation* and can be performed in different ways leading to different results.

We will analyse quantitative aspects of both aggregation and allocation in Part IV where we discuss hedging and managing counterparty risk.

1.4.4 Close-Out Risk

Close-out risk refers to the possibility of loss during the time period between when a counterparty is deemed to be in default and when the transaction with that counterparty has been wound down or replaced in the market. The length of this time period, referred to as the *close-out period*, is typically assumed to be ten business days. In practice it may be shorter for liquid transactions or longer for specialised and bespoke transactions.

To mitigate close-out risk, a collateral agreement is often included in the transaction. Under such an agreement, the counterparty would have a commitment to post assets (be they in form of cash or other highly-rated assets) whenever the exposure from the transaction is observed to increase. There are several components that may be specified in a collateral agreement, such as (i) an initial upfront collateral amount called the *initial margin*, (ii) the threshold exposure above which extra collateral

[11]The ABX Index is a series of credit-default swaps based on 20 MBSs that relate to subprime mortgages.

would need to be posted, (iii) the minimum amount of collateral that may be posted on each collateral exchange date, and (iv) the frequency of the margin calls.

The collateral agreement is a legal agreement also referred to as the CSA (Credit Support Annex). Typically the terms of this agreement will depend on the jurisdiction where it applies. In Part IV we will analyse some quantitative and modelling aspects of close-out risk, without addressing all the intricacies of the legal aspects.

1.4.5 Right-Way/Wrong-Way Exposure

In all the examples we have analysed previously, we did not consider the quality of the counterparty, assuming in effect that counterparty exposure is equivalent to the future replacement value of the trade at time of counterparty default. In general, however, the level of exposure caused by the trade and the quality of the counterparty are not independent of each other. Information about one would force us to re-evaluate information we have about the other. We refer to such dependence as right-way or wrong-way exposure. The question is how to factor this effect into a credit quantification computation. Typical examples where such considerations are called for are when call or put options are written on the counterparty's own stock. These are limiting cases with practically no need of accurate modelling. The problem becomes more interesting when the product is complex and the correlation between counterparty quality and level of exposure cannot be clearly determined.

Consider for example an energy producer which swaps energy futures for a stream of coupons. In general, increases in energy prices could be beneficial to the company, therefore reducing its probability of default. One way of taking into account the Right Way risk is to measure correlation between energy prices and company credit spread.

Another interesting example[12] of wrong way risk are negative basis swaps performed with monoline insurance companies. Typically in this case the insurance company receives a premium and pays default protection on missing payments from a pool of mortgages. These swaps are called negative basis as, in normal market conditions, the price for the protection is lower than the value implied by the spread paid by the mortgages.

1.4.6 Credit Valuation Adjustment: CVA

Once counterparty exposure has been computed it is necessary to find ways of mitigating it. The simplest way is to compare the portfolio PFE with pre-defined limits and constrain the amount of transacted notional or, as we have seen previously, enter

[12]...especially in light of the 2007/2008 credit events....

into a collateral agreement. A possible alternative consists in buying credit protection on the counterparty. Its price corresponds to the value of the protection leg of a CDS that pays the exposure amount in case of default of the counterparty. This value is called in the industry credit valuation adjustment, CVA.

Intuitively we can see this as follows. Within a pricing framework the value of credit exposure can be seen as the expected value of the positive part of the price distribution weighted by the default probability. Assuming that prices are independent from defaults, we can separate expectations, obtaining that CVA is the value of a CDS with the notional being the EPE profile of the underlying transaction. Suppose for simplicity that the EPE profile is a piece-wise constant function over a time interval $(T_i - T_{i-1})$.

$$\mathrm{CVA} = \sum_i \mathrm{EPE}_i (T_i - T_{i-1}) D_{0,T_i} s_i, \tag{1.5}$$

where s_i is the spread corresponding to the time interval $T_i - T_{i-1}$ and D_{0,T_i} the discount bond maturing at time T_i. We can see that the CVA corresponds to a portfolio of forward starting CDSs (or equivalently long and short CDS positions) with piecewise constant notional. The availability of CDSs of different maturities will dictate how the EPE profile is discretized.

The CVA depends on the level of exposure as well on the credit spread of the counterparty. As counterparty exposure and spread change with time, the amount of credit protection needs to be adjusted accordingly. The process of balancing of exposure with CDSs and other instruments sensitive to market parameters corresponds to dynamically hedging counterparty credit exposure. More details on how to compute and hedge CVA are given in Chap. 14.

1.4.7 A Simple Credit Quantification Example

We will discuss in detail in Chap. 9 the computation of credit exposure for equity products. We consider here a very simple example where the form of the exposure profile and the maximum values of the PFE can already be deduced from an approximation.

Suppose company A has bought from counterparty B a call option of strike K on a stock S. Our goal is to compute the credit exposure and close-out risk company A is facing. As mentioned in the previous section we need to calculate the price distribution V_t. In the case of the call option, in a simplified context where rates are deterministic, (1.1) becomes

$$V_t = N_t \mathbb{E}\left[\frac{(S_T - K)^+}{N_T} \middle| \mathscr{F}_t \right] = e^{-r(T-t)} \mathbb{E}\left[(S_T - K)^+ \mid \mathscr{F}_t \right], \tag{1.6}$$

where S is the stock price, K the strike, and r the interest rate assumed to be constant. The notional (number of options) has been assumed equal to one. As mentioned previously, $\mathscr{F}_t$ is the usual filtration, N_t the numeraire, and the expectation is taken in the measure $\mathbb{N}$.

To solve this equation we need for the stock price S a model, with which simulate the stock value till maturity T. A simple model, which is often used in credit, is the geometric Brownian motion with constant volatility σ, interest rate r and dividend yield d.

$$\frac{dS}{S} = (r - d)dt + \sigma dW_t, \tag{1.7}$$

where W_t is a standard Brownian motion. As it is well known this stochastic differential equation can be solved analytically.

Thus, to compute exposure we need to simulate the stock with (1.7), and then, using the Black and Scholes formula [15] we can price the option at each time step and in each scenario.

As the exposure of an equity option is generally monotonic in the underlying and is growing with time, and a vanilla stock option depends only on the current stock value (the product is not path dependent), the max PFE will be in general at maturity T.[13] We can compute it at let's say 97.5% confidence level as (see also Appendix A)

$$\mathrm{PFE}_T = S_0\left(e^{(r-d-\frac{\sigma^2}{2})T+1.965\sigma\sqrt{T}}\right) - K, \tag{1.8}$$

assuming it is a positive quantity. The expected exposure can be computed in this simple case as

$$\mathrm{EPE}_t = V_0 e^{rt}, \tag{1.9}$$

where V_0 is the option premium. We can see this as following. Given that the value of a call option is always positive, we can write (in our simplified set-up),

$$\mathrm{EPE}_t = \mathbb{E}[V_t^+] = \mathbb{E}[V_t] = \mathbb{E}[\mathbb{E}[e^{-r(T-t)}(S_T - K)^+|\mathscr{F}_t]] = V_0 e^{rt}. \tag{1.10}$$

As for approximating the close-out exposure for a short close-out period, one can use a first-order Taylor approximation.

$$\mathrm{CloseOut}_t \approx V_0 + \varDelta(S_t - S_0), \tag{1.11}$$

where $\varDelta$ is the first order derivative of the call option price with respect to the stock and S_t is the value of the stock during the close-out period. This is the close-out risk for the initial period, i.e. for the time between time-zero and t. We will see later in this book that the computation of close-out risk presents subtleties which go far beyond this simple computation.

[13]The exact shape of the PFE curve will depend on the interest rate, dividend curve, and option characteristics.

1.4.8 Computing Credit Exposure by Simulation

Within a Monte Carlo framework, to compute exposure we could simulate the stock price from today to maturity using (1.7), and then price the option on each path and each time step using Black and Scholes, again with constant rate, dividend, and volatility. As we will see in the next chapters, it is more convenient to simulate martingale processes, for which only the volatility structure is relevant, while the drift (and thus the dividends) does not need to be specified (for a definition of martingale see Appendix B and for more details see for example the books by Baxter & Rennie [10], Rogers & Williams [93, 94] and Shreve [98]). In practice a convenient quantity we can simulate are forward prices. By considering our example in these terms, we can write (1.6) as

$$V_t = N_t \mathbb{E}\left[\frac{(F_{T,T} - K)^+}{N_T} \middle| \mathscr{F}_t \right], \tag{1.12}$$

where $F_{t,T}$ is the t-value of the T-forward. The link with the notation in the previous section is,

$$F_{t,T} = S_t e^{(r-d)(T-t)}. \tag{1.13}$$

As before, assume for simplicity that the numeraire N is independent from the stock price, and impose also the simple specification

$$dF_{t,T} = F_{0,T} \sigma dW_t, \tag{1.14}$$

with the volatility being a constant $\sigma > 0$ and with W being a standard $\mathbb{N}$-Brownian Motion. This is the quantity we simulate in t. The paths generated by integrating this SDE are our scenarios which we show in Fig. 1.5 in a stylised representation.

Note that (1.14) can be integrated analytically at each time step, thus avoiding discretisation errors,

$$F_{t,T} = F_{0,T} \exp\left(-\frac{\sigma^2}{2} t + \sigma W_t \right). \tag{1.15}$$

We can then price at each scenario and each time step the stock option using again Black and Scholes, expressed in terms of the forward price $F_{t,T}$.

PFE can be computed analytically at maturity, where the option price is given by the stock price minus the strike.

$$\mathrm{PFE}_T = \left[F_{0,T} \exp\left\{ \sigma \sqrt{T} \tilde{q}_{\alpha,T} - \frac{1}{2}\sigma^2 T \right\} - K \right]^+, \tag{1.16}$$

where $\tilde{q}_\alpha = \Phi^{-1}(\alpha)$ is the α-quantile of the standard normal distribution. This is the equivalent of (1.8) generalised for any quantile. We have also floored at zero the exposure, as in some cases one is interested only at the amount the counterparty should pay. Negative exposure represents the amount we owe to the counterparty.

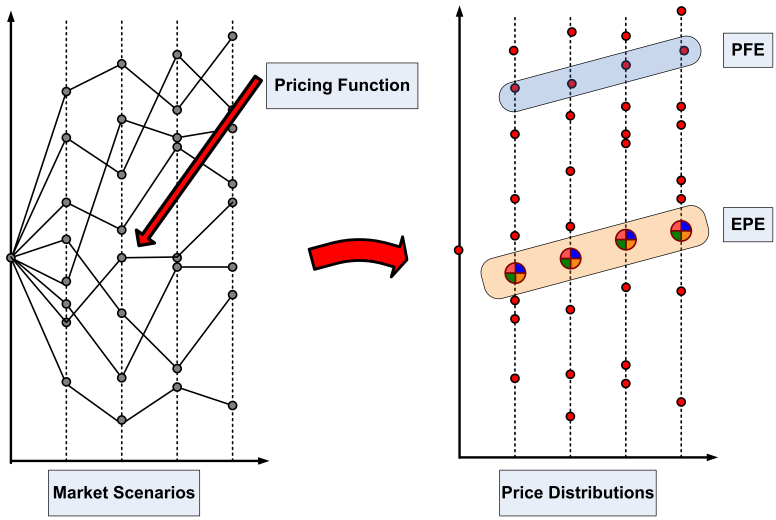

Fig. 1.5 Computing exposure by Monte Carlo simulation. The paths on the left panel represent stock prices. At each scenario and each time step, the price of the option is computed using the analytical Black and Scholes formula. Resulting prices are represented on the right panel. From the price distributions generated in this way at each time step, various statistical quantities (e.g. PFE and EPE) can be extracted. The bigger circles indicate a mean

1.4.9 Implementation Challenges

The Monte Carlo framework we have shown in the previous section, seems to give a good implementation recipe. For a given portfolio of transactions we could (i) identify the underlying risk factors and simulate forward (or spot) prices, taking into account correlations if required, (ii) use functions already implemented to price each product, and then (iii) derive statistical quantities. As we have mentioned already, this could be the approach followed by a financial institution to assess the counterparty credit risk of its OTC derivatives portfolios.

In the implementation phase, however, there can be issues which need to be addressed.

(i) The generation of correlated scenarios is not trivial, as there can be thousands of different risk factors driving the dynamics of products in the portfolio. Consider for example an equity portfolio, where each underlying stock needs, at least in principle, a specific simulation.

(ii) The scenarios have to be consistent across systems to build a counterparty view. This is a requirement which is much more stringent than what is generally specified in the design of a Front Office system used for pricing or a Risk system used to monitor the Profit and Loss (P&L) of a bank. Basically what we need here is the same underlying models, or the same family of models, for all types of products. In fact, even if the correlation between asset classes can be in some cases ignored (e.g. equity could be considered not correlated with interest rate), still all these models need to be expressed using the same numeraire (the discount factors in equity have to be consistent with the discount factors used in FX or rates). This consistency can be difficult to achieve, as often large financial companies have different systems to book and value, for example, interest-rate, equity, or FX products.

(iii) Pricing functions developed in various libraries are not necessarily designed to be integrated in a counterparty exposure framework. This has implications from both a software and architecture, as well as from a methodological point of view. Consider for example path dependent products. Counterparty exposure depends on the whole scenario history, which could be in different formats across different pricing systems.

(iv) Not all products can be computed in analytical form. Most exotics are priced on grids using PDEs or using Monte Carlo approaches. In these cases the exposure computation would require a Monte Carlo simulation for scenarios and a Monte Carlo simulation, or a PDE computation, for each scenario and time step to price the instrument. This becomes quickly unfeasible from a computational point of view. In addition, depending on the model used for pricing, calibration could also become problematic, as it has to be performed at each scenario.

In practice credit systems based on the classical Monte Carlo scheme approximate products using a simplified representation. While these approximations could have their justification in a risk environment, they are difficult to use when counterparty risk has to be priced and hedged.

1.4.10 An Alternative Approach: The AMC Algorithm

The points highlighted in the previous section clearly show that the classical Monte Carlo scheme has intrinsic limitations and that we need an alternative approach. As we will see at length in the rest of the book, there are possibilities to circumvent in a systematic way some of the problems related to valuation and architecture.

The basic idea is to approach the counterparty exposure problem as a pricing problem, and thus to use *pricing* algorithms, which generate not just the value of a trade at inception, but rather a price distribution at predetermined time steps. One possibility is to use the so called American Monte Carlo algorithm, which we will refer to as, simply, the AMC algorithm. The main feature of this algorithm is that, instead of building a price moving forward in time, it starts from maturity, where the value of the transaction is known, and goes backwards, till the inception.

In general AMC is used for pricing products with callability, i.e. products whose values depend on a strategy which can be determined by only knowing future states of the world. From a counterparty exposure perspective, the benefit of this approach is that, not only a price at time-zero is provided, but also the price distribution at each time step. In addition, the algorithm is generic, in the sense that using simply a payoff description, we can obtain the information needed to compute counterparty

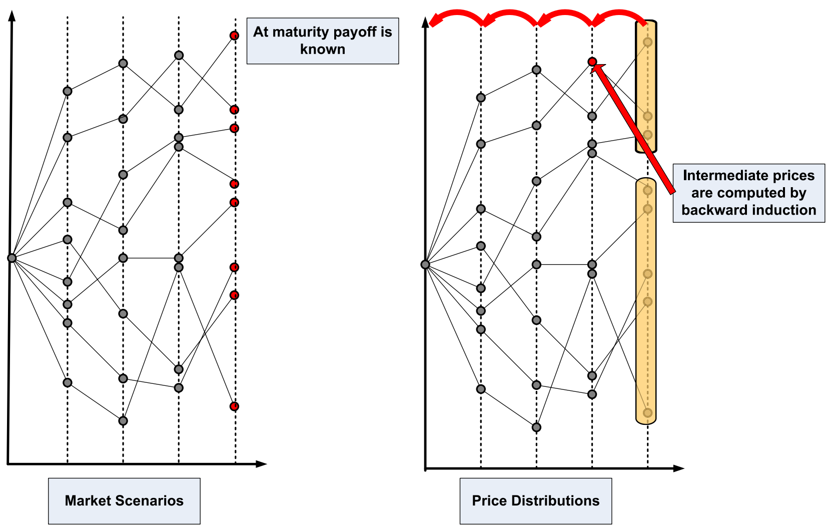

Fig. 1.6 A simplified graphical representation of the AMC algorithm. In the left hand panel we show the scenarios generated according to some underlying model. At maturity the payoff of the trade is known. To estimate the value at intermediate scenarios we need to proceed with a backward induction step

exposure. This suggests the possibility of having a generic trade representation and thus the possibility of having a modular software architecture that incorporates trade descriptions without explicit knowledge of each type of product. The challenge is that we need to develop an underlying model capable of pricing a *hybrid product*, consisting potentially of a large portfolio of transactions. This hybrid model will need to take into account all stochastic drivers of the portfolio in a consistent, arbitrage free way.

It is natural to ask what is the performance of the AMC algorithm for vanilla products. We will see that by a careful implementation the prices computed via AMC are very close to those computed using for example closed form formulae.

1.5 Which Architecture?

Building a system that computes credit valuation adjustments and counterparty exposure for the book of a large financial firm is a very challenging task, not only from the modelling and algorithmic perspective, but also from the technical and IT point of view. One of the problems is that often in large institutions such as Investment Banks, products are not booked on one system. They are in general recorded on several systems, which do not necessarily communicate between each other. To overcome this situation we suggest developing not only a common modelling platform, but also a programming language, which allows the representation of different types of products. As we will see later in this book, we have called our language

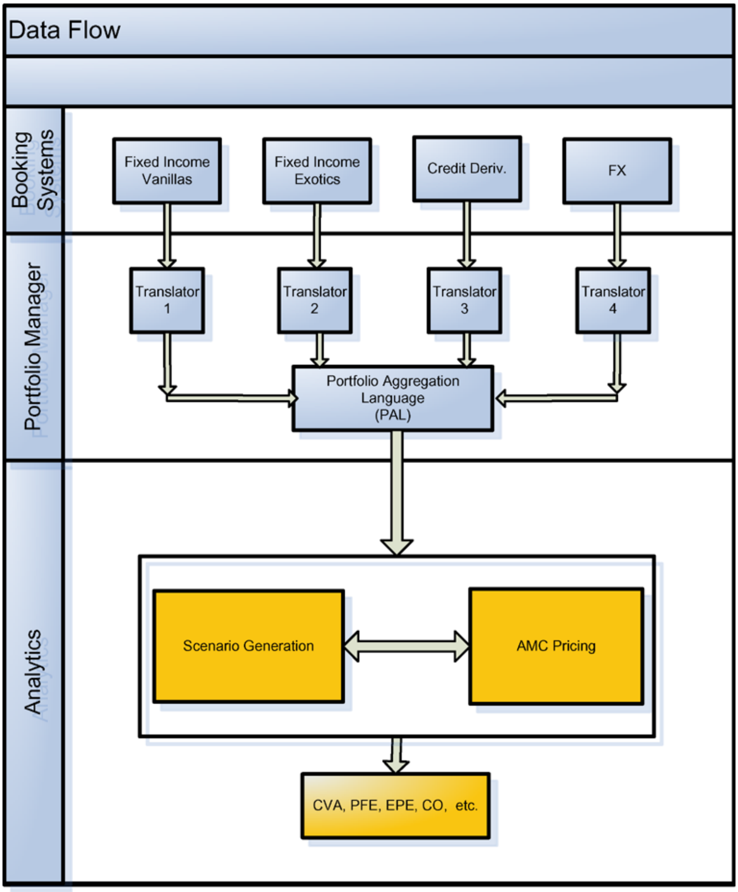

Fig. 1.7 High level architecture description

PAL, Portfolio Aggregation Language, to highlight the fact that we need to aggregate trades at counterparty (or at netting pool) level.

Once we have this common booking language, we can translate bookings made in other systems into PAL, bridging the difference between these systems. In the figure below we show how the system architecture could be implemented.

1.6 What Next?

We have introduced all basic concepts needed to understand counterparty credit exposure. We have now to analyse in detail the steps necessary to build a system designed to compute and hedge counterparty risk for large portfolios of exotic transactions. This is what we do in the rest of this book. We start from a generic modelling and simulation framework based on American Monte Carlo techniques, and then we present a software architecture, which, with its modular design, allows the computation of credit exposure in a portfolio-aggregated and scenario-consistent way.

http://www.springer.com/978-3-642-04453-3

Modelling, Pricing, and Hedging Counterparty Credit Exposure
A Technical Guide
Cesari, G.; Aquilina, J.; Charpillon, N.; Filipovic, Z.; Lee, G.; Manda, I.
2009, XX, 254 p. 70 illus., Hardcover
ISBN: 978-3-642-04453-3