

Contents

1	The Case of Manifolds	1
1.1	Poincaré–Hopf Index Theorem	1
1.1.1	Poincaré–Hopf Index at Isolated Points	1
1.1.2	Poincaré–Hopf Index at Nonisolated Points	3
1.2	Poincaré and Alexander Dualities	5
1.3	Chern Classes via Obstruction Theory	6
1.3.1	Chern Classes of Almost Complex Manifolds	6
1.3.2	Relative Chern Classes	8
1.4	Chern–Weil Theory of Characteristic Classes	11
1.5	Čech-de Rham Cohomology	15
1.5.1	Integration on the Čech-de Rham Cohomology	16
1.5.2	Relative Čech-de Rham Cohomology – Alexander Duality	17
1.6	Localization of Chern Classes	19
1.6.1	Characteristic Classes in the Čech-de Rham Cohomology	19
1.6.2	Localization of Characteristic Classes of Complex Vector Bundles	20
1.6.3	Localization of the Top Chern Class	22
1.6.4	Hyperplane Bundle	23
1.6.5	Grothendieck Residues	25
1.6.6	Residues at an Isolated Zero	26
1.6.7	Examples	28
2	The Schwartz Index	31
2.1	Isolated Singularity Case	31
2.2	Whitney Stratifications	33
2.3	Radial Extension of Vector Fields	34
2.4	The Schwartz Index on a Stratified Variety	37
2.4.1	Case of Vector Fields with an Isolated Singularity...	37
2.4.2	Case of Vector Fields with Nonisolated Singularity..	38

3	The GSV Index	43
3.1	Vector Fields Tangent to a Hypersurface	43
3.2	The Index for Vector Fields on ICIS	46
3.3	Some Applications and Examples	49
3.4	The Case of Isolated Smoothable Singularities	52
3.5	Nonisolated Singularities	53
3.5.1	The Strict Thom Condition for Complex Analytic Maps	54
3.5.2	The Hypersurface Case	56
3.5.3	The Complete Intersection Case.....	57
3.6	The Proportionality Theorem	58
3.7	Geometric Applications	61
3.7.1	Topological Invariance of the Milnor Number	61
3.7.2	The Canonical Contact Structure on the Link	62
3.7.3	On the Normal Bundle of Holomorphic Singular Foliations.....	68
4	Indices of Vector Fields on Real Analytic Varieties	71
4.1	The Schwartz Index on Real Analytic Varieties	71
4.2	The GSV Index on Real Analytic Varieties.....	73
4.3	A Geometric Interpretation of the GSV Index	77
4.4	Topological Invariants and Curvatura Integra.....	78
4.5	Relation with the Milnor Number for Real Singularities	81
5	The Virtual Index	85
5.1	The Virtual Tangent Bundle of a Local Complete Intersection	85
5.2	Chern–Weil Theory for Virtual Bundles	86
5.3	Characteristic Numbers on Singular Varieties	88
5.4	The Virtual Index	91
5.5	Identification with GSV Index When Singularities are Isolated.....	92
5.6	A Generalization of the Adjunction Formula	93
5.7	An Integral Formula for the Virtual Index	95
6	The Case of Holomorphic Vector Fields	97
6.1	Baum–Bott Residues of Holomorphic Vector Fields	98
6.2	One-Dimensional Singular Foliations.....	101
6.3	Residues of Holomorphic Vector Fields on Singular Varieties	104
6.3.1	Grothendieck Residues Relative to a Subvariety	104
6.3.2	Residues for the Ambient Tangent Bundle (Generalized Variation)	105
6.3.3	Residues for the Normal Bundle (Residues of Type Camacho–Sad)	107
6.3.4	Residues for the Virtual Tangent Bundle (Singular Baum–Bott)	110

7	The Homological Index and Algebraic Formulas	115
7.1	The Homological Index.....	116
7.2	The Hypersurface Case.....	119
7.3	The Index of Real Analytic Vector Fields	123
7.3.1	The Signature Formula of Eisenbud– Levine–Khimshiashvili	124
7.3.2	The Index on Real Hypersurface Singularities	126
8	The Local Euler Obstruction	129
8.1	Definition of the Euler Obstruction. The Nash Blow Up	129
8.1.1	Proportionality Theorem for Vector Fields	131
8.2	Euler Obstruction and Hyperplane Sections.....	133
8.3	The Local Euler Obstruction of a Function	136
8.4	The Euler Obstruction and the Euler Defect	137
8.5	The Euler Defect at General Points.....	139
8.6	The Euler Obstruction via Morse Theory	140
9	Indices for 1-Forms	143
9.1	Some Basic Facts About 1-Forms	143
9.2	Radial Extension and the Schwartz Index	147
9.3	Local Euler Obstruction of a 1-Form and the Proportionality Theorem	149
9.4	The Radial Index	151
9.5	The GSV Index	153
9.5.1	Isolated Singularity Case.....	154
9.5.2	Nonisolated Singularity Case	155
9.6	The Homological Index.....	158
9.7	On the Milnor Number of an Isolated Singularity	159
9.8	Indices for Collections of 1-Forms	161
9.8.1	The GSV Index for Collections of 1-Forms	165
9.8.2	Local Chern Obstructions.....	165
10	The Schwartz Classes	167
10.1	The Local Schwartz Index of a Frame	167
10.2	Proportionality Theorem.....	171
10.3	The Schwartz Classes.....	175
10.4	Alexander and Other Homomorphisms	176
10.5	Localization of the Schwartz Classes	178
10.5.1	The Topological Viewpoint	179
10.5.2	The Differential Geometric Viewpoint	180
10.6	MacPherson and Mather Classes	182

11	The Virtual Classes	185
11.1	Virtual Classes	185
11.2	Lifting a Frame to the Milnor Fiber	187
11.3	The Fulton–Johnson Classes	189
11.4	Localization of the Virtual Classes	191
12	Milnor Number and Milnor Classes	193
12.1	Milnor Classes	194
12.2	Localization of Milnor Classes	195
12.3	Differential Geometric Point of View	196
12.4	Generalized Milnor Number	200
13	Characteristic Classes of Coherent Sheaves on Singular Varieties	201
13.1	Local Chern Classes and Characters in the Čech-de Rham Cohomology	201
13.2	Thom Class	206
13.3	Riemann-Roch Theorem for Embeddings	207
13.4	Homology Chern Characters and Classes	210
13.5	Characteristic Classes of the Tangent Sheaf	212
	References	215
	Index	223

Vector fields on Singular Varieties

Brasselet, J.-P.; Seade, J.; Suwa, T.

2009, XX, 232 p., Softcover

ISBN: 978-3-642-05204-0