

Preface

This Edited Volume is based on the workshop on “Recent Developments in Quantum Field Theory” held at the *Max Planck Institute for Mathematics in the Sciences* in Leipzig (Germany) from July 20th to 22nd, 2007. This workshop was the successor of two similar workshops held at the Heinrich-Fabri-Institute in Blaubeuren in 2003 on “Mathematical and Physical Aspects of Quantum Field Theories” and 2005 on “Mathematical and Physical Aspects of Quantum Gravity¹”.

The series of these workshops was intended to bring together mathematicians and physicists to discuss basic questions within the non-empty intersection of mathematics and physics. The general idea of this series of workshops is to cover a broad range of different approaches (both mathematical and physical) to specific subjects in mathematical physics. In particular, the series of workshops is intended to also discuss the conceptual ideas on which the different approaches of the considered issues are based.

The workshop this volume is based on was devoted to *competitive methods* in quantum field theory. Recent years have seen a certain crisis in theoretical particle physics. On the one hand there is this phenomenologically overwhelmingly successful Standard Model which is in excellent agreement with almost all of the experimental data known to date. On the other hand this model also suffers from conceptual weakness and mathematical rigorousness. In fact, almost all the experimentally confirmed statements derived from the Standard Model are based on perturbation theory. The latter, however, uses renormalization theory which actually still is not a mathematically rigorous theory. Despite recent progress, it is clear that a deeper understanding of this issue has to be achieved in order to gain a more profound understanding in elementary particle dynamics. Moreover, it seems almost embarrassing that we have no idea what more than 90% of the energy in the universe may look like. Even more demanding are the conceptual differences between the basic ideas of a given quantum field theory and general relativity. There is not yet a theory available which allows to combine the basic principles of these two cornerstones of theoretical physics and which also reproduces (at least) some of the experimentally verified predictions made by the Standard Model. A quantum theory of gravity should cover or guide an extension or re-modelling of the particle physics side. These problems are among the driving forces in recent developments in quantum field theory.

¹See the volume “Quantum Gravity – Mathematical Models and Experimental Bounds”, B. Fauser, J. Tolksdorf, and E. Zeidler Eds., Birkhäuser Basel, 2007.

One competitive candidate for a unifying theory of quantum fields and gravity is string theory. The present volume features a particular scope on these activities. It turned out that the flow of communication between string theory and other approaches is not entirely free, despite of a great effort of the organizers to allow this to happen. The workshop showed, in very lively discussions, that there is a need for exchanging ideas and for clarifying concepts between other approaches and string theory. This might be a well suited topic for a following workshop. The first chapter, by Bert Schroer, dwells partly on some of the difficulties to achieve a better understanding between the ideas of string theory and algebraic quantum field theory. In addition to Bert Schroer's view, the editors are glad to point out, that in several chapters of this book, and especially in the long last chapter, string motivated ideas do entangle and interact with quantum field theory and provide thereby competitive approaches.

The present volume covers several approaches to generalizations of quantum field theories. A common theme of quite a number of them is the belief that the structure of space-time will change at very small distances. The basic idea is that probing space-time with quantum objects will yield a fuzzy structure of space-time and the concept of a point in a smooth manifold is difficult to maintain. Whatever the fuzzy structure of space-time may look like on a (very) small scale, any such description of fuzzy space-time would have to yield a smooth structure on a sufficiently large scale. How to resolve the discrepancy? One may start from the outset with a discrete set and expect space-time and the causal structure to be emergent phenomena. One might use q -deformation to introduce a, however rigid, discrete structure, or one might study locally deformed space-times using deformation quantization, a presently very much pursued approach. A very radical approach is proposed by the topos approach to physical theories. It allows to reestablish a (neo-)realist interpretation of quantum theories and hence goes conceptually far beyond the usual generalizations of quantum (field) theories.

Other activities in quantum field theory are tied to issues that are more mathematical in nature. While path integrals are suitable tools in particle, solid state, and statistical physics, they are notoriously ill defined. This volume contains a thorough mathematical discussion on path integrals. This discussion demonstrates under what circumstances these highly oscillatory integrals can yield rigorous results. These methods are also used in the AdS/CFT infrared problem and thus have implications for quantum holography, a major topic for discussions during the workshop.

A number of contributions to this volume discuss different aspects of perturbative quantum field theory. Approaches include causal perturbation theory, allowing to formulate quantum field theories more rigorously on curved space-time backgrounds and Hopf algebraic methods, which help to clarify the complicated process of renormalization.

The last and by far most extensive contribution to this volume presents a detailed mathematical discussion of several of the above topics. This article is motivated by string theory covering categorical issues.

The idea of the third workshop was to provide a forum to discuss different approaches to quantum field theories. The present volume provides a good cross-section of the discussions. The refereed articles are written with the intention to bring together experts working in different fields in mathematics and physics who are interested in the subject of quantum field theory. The volume provides the reader with an overview about a variety of recent approaches to quantum field theory. The articles are purposely written in a less technical style than usual to encourage an open discussion across the different approaches to the subject of the workshop.

Since this volume covers rather different perspectives, the editors thought it might be helpful to start the volume by providing a brief summary of each of the various articles. Such a summary will necessarily reflect the editors' understanding of the subject matter.

Holography, especially in the form of AdS/CFT correspondence, plays a vital role in recent developments in quantum field theory. The connection between a bulk and a boundary quantum field theory has fascinating consequences and may provide us with a pathway to a realistic interacting quantum field theory. A further important point is that it can be used to derive *area laws* much alike Bekenstein's area law for black holes.

In his discussion of holography **Bert Schroer** also highlights several critical aspects of quantum field theory. Furthermore, his contribution to this volume provides quite a bit of historical details and insights into the original motivation of the introduction of such concepts as light-cone quantization, the ancestor of holography.

Schroer's reflections on some socially driven mechanisms in the development of physics are surely subjective and controversial. His pointed contributions during the workshop made it, however, clear that his criticism should not be misunderstood as a no-go paradigm against other approaches, as also the variety of chapters in this book suggest.

A very radical way to avoid concepts like 'space-time points', which is used in general relativity but is in conflict with the uncertainty principle, is give up the assumption of a continuum. Also Bernhard Riemann, when he introduced his differential geometric concepts, was careful enough to note that the assumption of a continuum at very small scales is an untested idealization. Topos theory allows the usage of 'generalized points' in algebraic geometry. Lawvere studied elementary topoi to show that the foundation of mathematics is not necessarily tied to set theory. Moreover, Lawvere showed that the logic attached to topoi is strong enough to provide a foundation of the whole building of mathematics. The corresponding chapter by **Andreas Döring** summarizes and explains very clearly how topos theory might be useful to describe physical theories. He shows that topos theory produces an internal logic and is capable to assign to all propositions of the theory truth values. In that sense the topos approach overcomes foundational

problems of quantum theory, sub-summarized by the Kochen-Specker theorem. Eventually, topos theory may also open a doorway to unify classical and quantum physics. This in turn may yield deeper insights into a quantization of gravity.

Feynman path integrals are a widely used method in quantum mechanics and quantum field theory. These integrals over a path space are relatives of Wiener integrals and provide a stochastic interpretation as also the “sum over histories” interpretation. However, path integrals in quantum field theory are known to be notoriously mathematically ill defined. In their contribution, **Sergio Albeverio** and **Sonia Mazzucchi** present an introduction to a mathematical discussion of Feynman path integrals as oscillatory integrals. Due to the oscillating integrand these integrals may converge even for functions which are not Lebesgue integrable.

Using a stochastic interpretation, constructive quantum field theory deals with path integrals of non-Gaussian, type. **Hanno Gottschalk** and **Horst Thaler** apply this stochastic interpretation of path integrals to investigate the AdS/CFT correspondence that is motivated by string theory. Especially the infra-red problem and the triviality results of ϕ^4 theory are discussed in their contribution. A comprehensive discussion of the encountered problems is presented and four possible ways to escape triviality are discussed in the conclusions of their chapter.

Originally, mirror symmetry emerged from string theory as a duality of certain 2-dimensional field theories. Mirror symmetry has very remarkable mathematical properties. In his contribution, **Karl-Georg Schlesinger** very clearly explains how mirror symmetry can be extended to the noncommutative torus. Such a generalization of mirror symmetry to a noncommutative setting is motivated, for example, by string theoretical considerations. The present work leads to decisive statements and a conjecture about the algebraic structure of cohomological field theories and deformations of the Fukaya category attached to commutative elliptic functions. Higher n -categories and fc-multi-categories appear naturally in such a development.

Using quantum field theoretical methods, Edward Witten made a number of remarkable mathematical statements. Among them he presented an expression for the volume of the moduli space of flat $SU(2)$ bundles on a compact Riemann surface of general genus. From this result follow the cohomology pairings of intersections. Many heuristically obtained results, that is using formal path-integral methods, were rigorously proved later on by mathematicians. In his contribution to the volume, **Partha Guha** presents a route to obtain similar results for flat $SU(3)$ bundles using the Verlinde formula. The results employ Euler-Zagier sums and multiple zeta values in an intriguing and surprising way.

θ -deformed space-times are another approach to quantize gravity. Such a description of space-time, however, suffers from several shortcomings. For example, it is not Lorentz invariant. Moreover, such a deformation produces ‘quantum effects’ on any scale, invalidating the theory on the classical level.

An interesting description of θ -deformed space-times is provided by deformation quantization. Such a description allows to introduce locally noncommutative spaces which may fit more with the physical intuition. **Stefan Waldmann** expertly reviews in his contribution the deformation quantization description of θ -deformed space-times. He also presents some motivation for the concepts used in this approach and discusses the range of validity of these concepts.

Renormalization is known to be the salt which makes quantum field theory digestible, i.e. to produce finite results. The scheme of renormalization was established by physicists in the years 1950-80. The basic ideas of renormalization have been made more mathematically concise by the work of Kreimer, Connes-Kreimer and others using Hopf algebras. However, this approach was established only on toy model QFTs. **Walter D. van Suijlekom** pushes the Hopf algebraic method into the realm of physically interesting models, like non-Abelian gauge theories. The corresponding chapter of this volume contains a clear and relatively nontechnical description how the Hopf algebra method can be applied to Ward identities and Slavnov-Taylor-identities.

Perturbative quantum field theory is well-known to be quite successful when applied to the Standard Model. However, there is some belief that perturbation theory is not fundamental. Recent developments exhibited a Hopf algebraic structure which may help to understand renormalization of Abelian and non-Abelian Yang-Mills quantum field theories. Gravity has a rather different gauge theoretical structure and is not amenable to the usual techniques used in Yang-Mills gauge theories. **Dirk Kreimer** explains similarities between perturbatively treated quantum Yang-Mills theories and Einstein's theory of gravity. These similarities might eventually allow to quantize gravity using standard perturbative methods.

Quantum field theory, celebrated presently as the fundamental approach to formulate and describe quantum systems, has weak points when applied to systems having a degenerate lowest energy sector. Such systems *do occur* in solid-state physics, for examples when studying the colours of gemstones, and cannot be treated by the usually applied standard methods of quantum field theory. **Christian Brouder** develops a method to deal with such degenerated quantum field theories. Firstly the degenerated state is described via its cumulants, then these cumulant correlations are turned into interaction terms. This extends to the edge the combinatorial complexity but reestablishes valuable tools from standard nondegenerate quantum field theory, such as the Gell-Mann Low formula. The soundness of the method exhibits itself in a short and clear proof of Hall's generalized Dyson equation.

The article by **Ferdinand Brennecke** and **Michael Dütsch** presents a summary of the present state of the art of renormalization techniques in causal perturbation theory. The main tools are the master action Ward identity and the quantum action principle, which finally allow to use local interactions in the renormalization process. A nice recipe style guide to the method is given in the conclusions.

In string theory certain dualities are known to play a crucial role in connecting strongly coupled theories with weakly coupled theories. While the strong coupled case is difficult to treat, the weakly coupled dual theory may admit a perturbative regime. One such setting is found in matrix string theory in a non-Abelian Yang-Mills setting. In his contribution to the volume, **Matthias Blau** sets up a quantum mechanical toy model to discuss the geometry behind such dualities. He shows that plane wave metrics play a certain role in the solution of the time dependent harmonic oscillator. His discussion may serve as a blue print for the much more complex noncommutative non-Abelian Yang-Mills case.

Loop quantum gravity is one of the approaches to gain insight into a theory of quantum gravity. Technical problems like the resolution of the Hamiltonian constraint make it difficult to evaluate loop quantum gravity in realistic situations. **Martin Bojowald** reviews in his article an approach that uses effective actions in canonical gravity to study quantum cosmology. He explains how solutions can be obtained for an anharmonic oscillator model using integrability. The analogous treatment of canonical quantum gravity yields a bouncing cosmological solution which allows to avoid the big bang singularity.

Many proposals have been made to establish a mathematical modelling of non-smooth structures on the Planck scale. One such model is developed by **Felix Finster** starting from a discrete set of points. All additional structures like causality, Lorentz symmetry and smoothness at large scales have to be established as emergent phenomena. Finster explains in his article how such structures may occur (in principle) in a continuum limit of a set described by a specific discrete variational principle. Several small systems of this type are analyzed and the structure of the emergent phenomena is discussed.

A recurrent theme in physics is the question: “How can space-time be mathematically modelled at very short distances?” Lattices emerging from a “ q -deformation” might provide one such candidate. **Hartmut Wachter** shows in his contribution how a q -calculus approach to a non-relativistic particle can be worked out.

The method of q -deformation was originally motivated as a regularization scheme. Similarly, the idea of supersymmetry originated from the hope that super-symmetric theories may have a better ultraviolet behaviour. In his contribution, **Alexander Schmidt** presents a discussion on how q -deformation can be extended to a super-symmetric setting.

The book closes with a rather long chapter by **Hisham Sati**, **Urs Schreiber** and **Jim Stasheff**. String theory replaces point-like particles by extended objects, strings and in general branes. Such objects can still be described via differential geometry on a background manifold, however, higher degree fields, like 3-form fields, emerge naturally. Higher categorical tools prove to be advantageous to investigate these higher differential geometric structures. Major ingredients are

parallel n -transport, higher curvature forms etc. and therefore the algebra of invariant polynomials, which embeds into the Weil algebra, which in turn projects to the Chevalley-Eilenberg algebra. These algebras are best studied as differentially graded commutative algebras (DGCAs). On the Lie algebra level this structure is accompanied by L_∞ -algebras which carry for example a (higher) Cartan-Ehresmann connection. Natural questions from differential geometry, such as classifying spaces and obstructions to lifts etc. can now be addressed. The higher category point of view generalizes, unifies and thereby explains many of the standard constructions.

The chapter is largely self contained and readable for non-experts despite being densely written. It develops the relevant structures, gives explicit proofs, and closes with an outlook how to apply these intriguing ideas to physics.

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Bertfried Fauser, Jürgen Tolksdorf and Eberhard Zeidler
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