Foreword

In Wilhelm Traugott Krug's *General Handbook of the Philosophical Sciences* of 1827 ("Allgemeines Handwörterbuch der philosophischen Wissenschaften"), we find the following entry for the terms "mathematics" and "mathematical":

"Mathematics ... only [deals with] magnitudes which appear in time and space and which therefore can be represented, counted and measured as numbers or figures... A philosopher should familiarize himself with mathematics and a mathematician with philosophy, as far as their talent, interests, time and surroundings will permit. But one should not confuse and throw into one pot what the progress of scientific knowledge has separated, and rightly so. ...mathematical philosophy and philosophical mathematics – in the commonly accepted sense of the terms, namely as a mixture of both – are scientific or, rather, unscientific monsters. They no more satisfy and please the educated mind than could a human body consisting of a mixture of man and woman."[1]

But this view did not prevent Hermann Graßmann[2], a 35 year-old secondary school teacher from the Prussian town of Stettin, from publishing a work of mathematics which, as he later remarked, "is certain to be more pleasing to more philosophically inclined readers".[3] Graßmann's book also claimed to have founded a new branch of science "which extends and intellectualizes the sensual intuitions of geometry into general, logical concepts, and, with regard to abstract generality, is not simply one among the other branches of mathematics, such as algebra, combination theory, and function theory, but rather far surpasses them, in that all fundamental elements are unified under this branch, which thus as it were forms the keystone of the entire structure of mathematics."[4]

Hermann Graßmann, a novice in mathematics whose name was completely unknown in the mathematical community of his day, did not hesitate to send his book – *Linear Extension Theory, A New Branch of Mathematics* (1844) – to the most famous math-

ematicians of his time. But their assessment of his work remained completely within the framework of the Kantian view of mathematics, which we find in the quotation above. This was a disaster for Graßmann. Among German mathematicians, August Ferdinand Möbius was closest to Graßmann's scientific perspective. Möbius told Apelt in a letter that he had repeatedly attempted to understand Graßmann's book, "...but I never got beyond the first pages ... since [the book] ... lacks all intuitive clarity, which is the essential characteristic of mathematical insight."[5] In a letter to Gauß, Möbius wrote that Graßmann had "strayed from the firm foundations of mathematics"[6]. Johann August Grunert wrote Graßmann: "I also would have hoped that you would have refrained from getting so involved in philosophical reflections."[7] Ernst Friedrich Apelt, a friend of Möbius, remarked that "Graßmann's peculiar *Extension Theory* ... seems to be built on a wrong understanding of the philosophy of mathematics. ...The *abstract* extension theory he is looking for could only be developed from concepts. Concepts are not the source of mathematical knowledge, but intuition."[8] Finally, Richard Baltzer came to the following conclusion: "...I begin to feel dizzy in the head and disoriented when I read it."[9] Moritz Cantor summed up the fate of *Extension Theory* in one simple sentence: "The book was published in 1844 by O. Wigand in Leipzig, nobody reviewed it, nobody bought it, and therefore the publisher destroyed the entire first edition!"[10] Half a century later, nobody doubted the importance of Graßmann's mathematical work. On Felix Klein's initiative, a six-volume collection of Graßmann's writings in mathematics and physics was published between 1894 and 1911.[11] Thanks to mathematicians such as Hermann Hankel, Alfred Clebsch, Felix Klein and Friedrich Engel, Graßmann's achievements concerning the foundations of vector and tensor calculus, the development of n-dimensional affine and projective geometry and his fundamental work in algebra and in other areas were recognized in retrospect. Today, Graßmann has become a familiar name in mathematics. Nevertheless, many mathematicians are quite unfamiliar with his *magnum opus* in mathematics. Even though "a general feeling of respect for this mathematician from Stettin has spread in the scientific community", as F. Engel remarked in 1911, "this feeling of respect usually does not arise from knowledge of *Graßmann's* writings but, rather, is based on hearsay."[12] Graßmann's *Extension Theory* of 1844 was ignored for over a quarter of a century. Among other reasons, the general rejection of its philosophical approach and its philosophical mode of presentation led to this lack of recognition. Unhappily, this anti-philosophical attitude blinded mathematicians to the true value of *Extension Theory*. A closer analysis of the book will show that Graßmann's philosophical and, to put it more precisely, *dialectical* approach to mathematical problems is exactly what gave him the inspiration he needed to create and elaborate a new mathematical discipline, namely vector and tensor calculus. What is more: Graßmann was capable of building an unheard-of vector-algebraic theory of n dimensions because he was familiar with the philosophical thinking of his time and because he consciously used dialectics,

the philosophy of the increasingly dominant German bourgeoisie, as a method for establishing and presenting new insights.

The present book aims to critically appreciate and explain the life and work of Hermann Graßmann (1809 – 1877).

Notwithstanding the fact that Graßmann has entered into the history of mathematics as the founder of vector algebra, he still remains a relatively unknown figure. The hundredth anniversary of his death in 1977 passed almost completely unnoticed. A conference held in Germany on the occasion of the 150th anniversary of the publication of *Linear Extension Theory* in May 1994 was one of the last major attempts to save his name from oblivion. In September 2009 the Graßmann Bicentennial Conference in Potsdam will commemorate the 200th anniversary of Graßmann's birth and attempt to contextualize his work from a present-day perspective.

The 19th century, in which Graßmann's scientific creativity blossomed, is still a highly promising area for future research. Few scholars have attempted to analyze the historical interactions between philosophy and mathematics. Very much remains to be done.[13]

These are plenty of reasons to have another look at Graßmann.

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