

Chapter 1

Preface

This book aims to convert the noble art of constructing an entire function with prescribed asymptotic behavior to a handicraft.

For this you should just consider the limit set that describes the asymptotic behavior of the entire function, i.e., you should consider the set $U[\rho, \sigma]$ of subharmonic functions (that is, $\{v \text{ is subharmonic} : v(re^{i\phi}) \leq \sigma r^\rho\}$) and pick out the subset U which characterizes its asymptotic properties.

How to do it? The properties of limit sets are listed in Section 3. All the standard growth characteristics are expressed in terms of limit sets in Sections 3.2, 3.3, 5.7. Examples of construction are to be found in Sections 5.4–6.3. So you can use this book as a reference book for construction of entire functions.

Of course, you need some terms. All the terms that we use in this book are listed on pages 249–253.

If you want to study the theory, I recommend that you solve the exercises that are in the text. Most of them are trivial. However, I recommend that you do all of them by the moment that they appear trivial to you.

A few words about the history of this book. It arose from a course of lectures that I gave at Kharkov University in 1977. After some time, under pressure and with active help of Prof. I.V. Ostrovskii, a rotaprint edition (Edition of KhGU) of this course appeared: the first part in 1978, the second one in 1982. Mathematical Reviews did not notice this fact.

Since that time lots of new and important results have been obtained. Some of them were presented in Chapter 3 of the review [GLO].

In 1994, when I started to work in the Bar-Ilan University and obtained a personal computer, my first wish was to study typing on it in English. This was the first impulse for translating this course into English (there are no more than five copies of this book in the world, I believe, one of them being mine). I continued this project while working in Bar-Ilan (1994–2006) but there was not much time for this. And now I have finished.

Acknowledgements

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I am indebted to Prof's L. Hörmander and R. Sigurdsson who have sent me the preprints of their papers that were not yet published. I am indebted to Prof. I.F. Krasichkov-Ternovskii, who explained to me many years ago the connection between the multiplier problem and completeness of the exponent system in a convex domain.

I am indebted to Prof's. M.I. Kadec and V.P. Fonf for proving Theorem 4.1.5.2, which is rather far off my speciality.

I am indebted to my coauthors Prof's D. Drasin and P. Poggi-Corradini; I have exploited the results of our joint paper in Section 6.2.

Of course, I am indebted to my late teacher Prof. B.Ya. Levin, who taught me entire and subharmonic functions and gave me the first problems in this area. Actually, the theory of limit sets is a generalization of the theory of functions of completely regular growth.

I am also indebted very much to my grandson Sasha Sodin, who transformed "my English" into English.

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